

letters

Multifractals and fractals

Bravo for your April Search and Discovery story (page 17). In your February Reference Frame column (page 6), Leo Kadanoff asked, "Why all the fuss about fractals?" Now we have that part of the answer which involves his own work and that of his close associates concerning "multifractals." This answer exhibits two of the main attractions of fractal geometry. First, the physical problems it tackles are very important, difficult, diverse and apparently unrelated (as exemplified by the new problem of diffusion-limited aggregation and the old problems of the onset of turbulence and of mature turbulence). Second, the set of tools of fractal geometry is powerful and unified, and we see that it is developing fast.

Kadanoff's query could receive many other answers, all equally impressive but centered on the work of other scholars. I trust you will come to them.

Now that the process of recording the history of fractals in physics has begun, allow me to add to the record. The first mention of multifractals was in 1968, in my work on mature turbulence,¹ and I published²⁻⁶ extensively on them in the 1970s. There was enough material for surveys in a special chapter of my book *Les Objets Fractals*,⁴ and in Section 6 of my Haifa *StatPhys* paper.⁶ My contribution to *Dimensions and Entropies in Chaotic Systems*⁷ is a handy summation of this early work. In my contribution to *Turbulence and Navier Stokes Equations* I acknowledged⁵ that "penetrating comments by Uriel Frisch... motivated me to substantial further development." Frisch and Giorgio Parisi (reference 3 of the April news story) coined the term "multifractals" and went on to their reformulation of this topic. Similarly, R. Benzi, G. Paladin, Parisi and V. Vulpiani describe their work (reference 4 of the April news story) as "in the spirit of Mandelbrot's weight(ed) curdling," and their first results were precisely as my work and that of J. Peyriere⁸ had predicted. Also, the papers by H. G. E. Hentschel and I. Procaccia⁹ and by P. Grassberger¹⁰ generated much later work, but started from things already known. In reference 9, note 17 states

that inequalities among a multifractal's infinity of exponents were published³ by me in the *Journal of Fluid Mechanics*.

Thus the link between my old work and your news item was documented repeatedly, yet these discreet footnotes deserve an airing. Anyhow, the latest about (multi)fractals from Chicago fills me with the pride of a father—soon to be a grandfather?

While my papers from the 1970s are no easier to read than they were to write, they include items that have not been superseded or rediscovered. In particular, my contribution to *Statistical Models and Turbulence*² includes useful illustrations of multifractal measures: the densities of the overprinted dots on page 17 of your April issue and the closely related plates 198 and 199 of my book *The Fractal Geometry of Nature*.¹¹

More of history. The physicists' present wide involvement with fractals owes a great deal to specific events at Harvard in 1980 that led to papers I wrote¹² with Y. Gefen, A. Aharony and assorted accomplices.

Kadanoff asked in February, "Where is the physics of fractals?" It is true that diffusion-limited aggregation and its innumerable variants have "only" been discovered and described, and they have not yet been fully accounted for. Description coming before theory is the usual pattern of science. But look at all the hard physics that has become thoroughly understood in these scant six years. Look at the explosion of knowledge about percolating networks, and the wonderful and diverse ways in which the fractal shapes influence and mold physics!

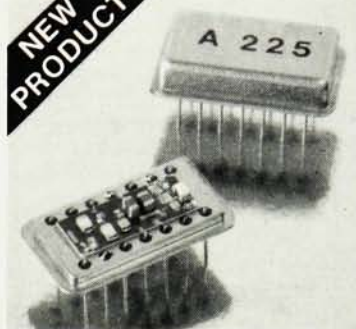
We all want more, and expect the future of fractals in physics—just as in engineering—to dwarf its past, but the breadth and health of the present do deserve mention.

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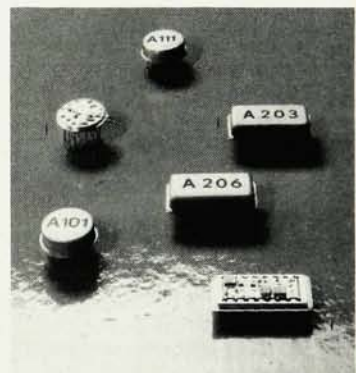
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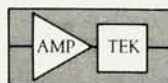
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Apartheid

I have often found it bitterly amusing to receive an appreciative nod from acquaintances who discover that I have lived for over a year in the Soviet Union, followed by a murderous frown when they learn that I have spent an equal amount of time in South Africa. It is thus distressing to read the letter of A. Bhattacharya, Murray Goldman and T. Schaffer, and that of Charles Hyde-Wright, in the October PHYSICS TODAY (page 148).

Bhattacharya and his colleagues are probably correct when they say that "the violations of fundamental rights in South Africa are not on a par with those of other African states or the Soviet Union." If one were perverse enough to construct a scale of human-rights violations, at least one could say that the South African government oppresses only three-quarters of the population, whereas the same sort of laws in the USSR oppress virtually everybody. (Is this acceptable because it is no longer discriminatory?) And if you were one of the 100 000-300 000 victims of the recent Ugandan genocide, you might be hard pressed to rank South African atrocities above all others.

Bhattacharya, Goldman and Schaffer write that "it is totally irrelevant whether these universities claim to be racially neutral. They are subject to the same laws of apartheid and must comply. It is hard for us to imagine that a freedom-loving American physicist would accept a position anywhere in South Africa. . . . If this hurts South African physicists maybe they should think of emigration." I am skeptical that even they believe these statements. First, they are condemning certain South Africans because they were born there. Emigration is not easy, emotionally or legally. (Are Bhattacharya and his colleagues offering jobs?) Indeed, some love this troubled land and want to stay in order to help as they are able. Second, I doubt that Bhattacharya and his colleagues would object to an American who came to teach at the University of the Western Cape, a nonwhite and somewhat radical institution in Cape Town, which has recently been the scene of many protests against government actions. Neither have I heard complaints that Gandhi spent 20 years in South Africa. Third, while discriminatory policies do prevail at the Afrikaner universities (Stellenbosch, Orange Free State, Pretoria, Potchefstroom), other South African universities (Witswatersrand, Rhodes, Cape Town) are surprisingly liberal.

The University of Cape Town, of which I have firsthand experience, has a nonracial admissions policy, an affirmative-action program and a 15% nonwhite student population. This figure shows that Cape Town is more integrated than, say, the University of Texas, even if one adjusts for the percentage of nonwhite population in the country or state. Cape Town's liberal traditions appear to me genuine. For instance, the vice-chancellor, Stuart Saunders, has recently been elected president of the South African Institute of Race Relations. Several members of my department also belong to this organization, and one, cosmologist George Ellis, is chairman of its Cape Town division. Protests against repressive government actions are frequent, are supported by the faculty and are held in dangerous circumstances. Students and staff have been teargassed, beaten by the police and arrested. Clearly, the racial policies of an individual institution are reflected in its response to the apartheid laws.

In regard to Hyde-Wright's letter, I should mention that the University of South Africa at Pretoria, as distinguished from the University of Pretoria, is primarily a correspondence school and has a very large black enrollment. To the best of my knowledge it accepts students from any country in the world, regardless of race.

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