# Group theory: Who needs it?

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In the late 1930s, when I was a graduate student, the application of group theory to quantum mechanics was not commonly a part of the curriculum except for obvious examples of spatial symmetry in solid-state and molecular physics. We prided ourselves that we could obtain the results for atomic spectra without using the formal apparatus of group theory. Edward Condon and George Shortley, in the introduction to their treatise *The Theory of Atomic Spectra*, explicitly reveal this attitude. They write: "We wish finally to make a few remarks concerning the place of the theory of groups in the study of the quantum mechanics of atomic spectra. The reader will have heard that this discipline is of great importance for the subject. We manage to get along without it."

The situation is strikingly different today. Group theory is such a pervasive component of research and study that it is not unlikely that many are more conversant with group theory than with the Schrödinger equation, the reverse of the situation that prevailed 50 years ago.

What makes group theory such a powerful tool? The traditional answer—that group theory permits the formulation as well as the exploitation of the symmetries of the system under study—begs the question. Surely what one would like to know is what features of the observed phenomena reveal the presence of a symmetry and an associated group structure. In fact, one can point to close, direct connections between experiment and group structure. It is from these that group theory ultimately derives its effectiveness.

Where does one search for symmetry? Generally among the low-lying states, where the levels are few and well separated and their properties measurable. One can expect that their wavefunctions will have a high degree of symmetry, reflecting the symmetry of the system and thereby achieving a greater binding. Moreover, as a consequence, these wavefunctions may be simply related to each other.

The search for symmetry is a search for simply related sets of states. A first indication of such states lies in the energy spectrum. A multiplet structure suggests an underlying degeneracy, one hallmark of symmetry. A familiar example is the near equality of the neutron and proton masses, which suggested the internal symmetry of isospin. This multiplet has been enlarged to the baryon octet with the addition of the particles $\Lambda$, $\Sigma^{+,-,0}$ and $\Xi^{0,-}$. However, their mass differences are of the order of hundreds of MeV, indicating the need for additional evidence before one can identify the underlying SU(3) symmetry.

Another type of regularity indicating group structure is provided by the deformed nuclei, which exhibit rotational spectra. The group is the rotation group. When other degrees of freedom such as shape and volume are included, the group is enlarged to U(6).

A simple relation connecting the energies of the low-lying levels is not sufficient to establish a group structure. There must be relations as well among the associated wavefunctions. For each group, these are provided by operators that relate the basic states for a group representation. The raising and lowering operators for the harmonic-oscillator wavefunctions are examples.

Observationally, these relations manifest themselves through transitions, induced by a probe such as the photon. A signal of the existence of a symmetry, which will be most visible for a special probe or probes, is the enhanced transition rates for some sets of transitions and their reduced values for other sets. These results suggest groupings of related states, while the operators associated with the transitions are closely related to the group operators I just mentioned. Once these levels and groupings are known, one searches for the underlying group by comparing the predicted level scheme, branching ratios and selection rules with experimental data. These predictions follow from the properties of the group and do not depend upon dynamical calculations. The confrontation of theory and experiment is immediate. In summary, once we know the probe that induces the transitions, which reveal the symmetry as I have described it, we have the key to unlock the nature of the group operators and the group.

An example is furnished by the enhancement of quadrupole transitions between the levels of a rotational band of a deformed nucleus and their inhibition for interband transitions. The quadrupole operator inducing the transitions is the operator connecting the wavefunction of the band and is thus the representation of an element of the underlying group.

We rarely find the ideal situation, in which group properties and experiment are in exact accord. Violations occur and the symmetry is said to be broken. The unequal masses of the members of the baryon octet are an example of symmetry breaking. Nevertheless one can still identify the underlying group in this case because the component subgroups of SU(3) maintain their identities. Symmetry breaking is accomplished by assigning different masses to each subgroup. This assumes that the symmetry breaking does not lead to significant matrix elements among different representations of the group. If they are large, symmetry is badly broken, and the advantages of the group description lessen. In the case of the isobar analog states of nuclear physics, the energies of states of differing isospin are shifted by the isospin-symmetry-breaking Coulomb interaction, but little mixing of different isospins occurs. Isospin remains a good quantum number.

I have focused on internal symmetries principally because of the strikingly close relation of group structure to experiment. That comparatively elementary relationship provides the ultimate basis for more complex uses of group theory. I will discuss these uses in a subsequent column. □

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