Conversion in matter may account for missing solar neutrinos

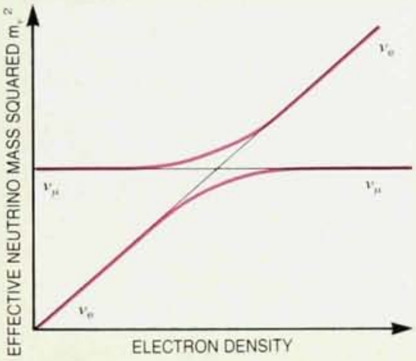
In 1938 Hans Bethe propounded the nuclear-fusion sequences we confidently assume to describe the energy sources of the Sun. Almost a half-century later, a 24 March *Physical Review Letter*[1] by this same Hans Bethe takes a swipe at the one conspicuous fly remaining in the ointment of the "standard solar model" that has grown up around his seminal explanation of what makes the stars shine.

The fly in the ointment is of course the mystery of the missing solar neutrinos (PHYSICS TODAY, December 1978, page 19). For almost two decades Raymond Davis (Brookhaven) and his colleagues have been monitoring with exquisite care the flux of solar neutrinos reaching their detector a mile underground in a South Dakota gold mine. The standard solar model confidently predicts that Davis's 615 tons of dry-cleaning fluid (C_2Cl_4) should detect about one solar neutrino a day. The experimenters, however, see only about one-third the neutrino flux the theorists tell them they ought to see.

Given the confidence people have in Davis's data, this state of affairs might well shake the confidence reposed in the standard solar model. Short of invoking some rather exotic and speculative astrophysics, it is very difficult to tinker with the standard model enough to get anything like a threefold suppression of the neutrino flux. It's disturbing when a clear prediction of the accepted astrophysical theory turns out to be seriously wrong for the one star we can examine close-up.

Bethe's new contribution is to an alternative approach to the solar-neutrino dilemma, set in motion a year ago[2] by S. P. Mikheyev and A. Yu. Smirnov at the Institute for Nuclear Research in Moscow. Theorists loath to concede ignorance of the physics of ordinary stars are quite willing to admit considerable ignorance of the physics of neutrinos. The hope is to explain the neutrino anomaly in terms of exotic elementary-particle phenomena occurring somewhere between the source and the observer, leaving the solar model essentially intact.

Last summer Mikheyev and Smirnov pointed out a previously unsuspected resonant phenomenon implicit in the theory of neutrino oscillations in matter. Just as one can regenerate short-lived neutral kaons by passing a beam of long-lived kaons through matter, one could in principle do the same with different "flavors" of neutrinos if the different neutrinos—electron, muon and tau—have different masses, and their vacuum eigenstates involve some mixing of these leptonic flavors. But before Mikheyev and Smirnov uncovered this resonant amplification effect, the general opinion of the experts had been that neutrino oscillations—either inside the Sun or in vacuum, on the way here—constituted much too weak an effect to account for the missing solar neutrinos.



Resonant neutrino conversion in the Sun is treated by Hans Bethe like an adiabatic level-crossing transition. In the absence of flavor mixing, the effective m^2 eigenvalues for ν_e and ν_μ as functions of electron density in matter would be intersecting straight (black) lines, the former increasing with density because of charged-current interaction. When a bit of mixing couples the flavors, the eigenvalue curves are given by the curved red lines, no longer intersecting. If the density change is adiabatically slow, a ν_e produced in the solar core stays on the upper curve, emerging as a ν_μ. The near-crossing point is the resonant density of S. P. Mikheyev and A. Yu. Smirnov.

The neutrinos produced in the fusion and decay processes in the Sun are electron neutrinos, and all solar-neutrino detectors depend on nuclear transmutations that are initiated only by electron neutrinos. If solar neutrinos are missing, an obvious suspicion is that they may have been lost by neutrino oscillation—transmutation into muon neutrinos, or perhaps even tau neutrinos. There is, however, a strong prejudice among particle physicists that the mixing of different neutrino flavors is small. When the "mixing angles" characterizing such flavor mixing are small, one can readily dismiss oscillation in vacuum as an insufficient effect. Oscillation between neutrino flavors while traversing matter is a more complex affair, but again it was thought that a small mixing angle would render oscillation in matter also uninteresting for the solar-neutrino problem. In his 1978 paper developing the formalism for neutrino oscillations in matter, Lincoln Wolfenstein (Carnegie–Mellon University) had written, ". . . if one is considering a discussion of solar neutrinos [one should] ignore the passage through matter. . . . The effective distance over which neutrino oscillations take place is from the solar surface to the Earth's surface." But Wolfenstein and everyone else had failed to notice the resonant effect hidden in his equations. Now, as a result of the Russian paper, all bets were off.

The resonant effect in matter had in fact been hinted at in numerical calculations carried out in 1980 by Vernon Barger (University of Wisconsin) and colleagues. "But the brilliance of Mikheyev and Smirnov is that they applied this effect to the solar-neutrino puzzle," Bethe told us. Their paper has as yet appeared in English only in preprint form. Bethe's *Physical Review Letter* rederives the resonant effect from a somewhat different physical perspective and presents us with a transparently straightforward calculation of the ν_e–ν_μ mass splitting from the application of the theory to the discrepancy between the Davis data and the predictions of the standard solar model.

This "masterpiece of back-of-the-envelope calculation," as Peter Rosen (Los Alamos) describes it, yields a ν_μ mass of about 0.01 electron volts, a thousand times lighter than the mass quoted for the (presumably lighter) ν_e in 1980 by the group that measured the end point of the tritium beta-decay spectrum at the Institute for Theoretical and Experimental Physics in Moscow. That determination of a ν_e mass on the order

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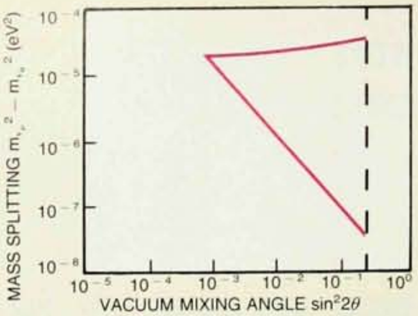
of 30 eV is no longer taken very seriously, Bethe told us. Starting with the 0.01-eV mu-neutrino mass he got from the resonance theory and the Davis data, Bethe takes a stab at the electron-neutrino mass by invoking the highly speculative, "but very interesting 'seesaw model'" proposed in 1979 by Murray Gell-Mann, Richard Slansky, Pierre Ramond and T. Yanagida. This model then yields a ν_e mass of 2×10^{-7} eV, hopelessly beyond the resolution of the new tritium-spectrum end-point experiments currently under way at the University of Zurich, Los Alamos, IBM, Livermore and several other labs. Although these new experiments avoid the thick, solid target that complicates the Soviet result, they will clearly not be sensitive to a ν_e mass much smaller than an electron volt.

Bethe concludes that the resonant conversion of electron neutrinos to muon neutrinos in the Sun could explain the Davis data even if the mixing angle θ were as small as 0.4°. Prior to the work of Mikheyev and Smirnov it had been assumed that significant oscillation between the two neutrino flavors would not occur unless the mixing angle were nearly maximal (45°), a supposition that would certainly not appeal to the inclinations of the elementary-particle theorists.

The Homestake Gold Mine detector, built by Davis and his Brookhaven colleagues, observes a neutrino by its capture in a Cl^{37} nucleus, which it converts to Ar^{37}. The argon thus produced is periodically removed from the detector liquid by chemical means, and the quantity of this unstable nuclear species is measured by its decay radioactivity. The threshold neutrino energy for the production of Ar^{37} in chlorine is 0.814 MeV.

John Bahcall (Institute for Advanced Study) and his collaborators have for many years been carrying out[3] prodigious calculations of what solar-neutrino detectors ought to see, based on the standard model of the Sun and the energy dependence of neutrino capture cross sections in detector materials. For the Brookhaven chlorine detector, the only one thus far in the business, they calculate that one should see a flux of about 6 "solar-neutrino units," where one SNU equals 10^{-36} captures per target atom per second.

The bulk of these captures—about 4 SNU—are expected to come from the decay of B^8, a relatively rare species in the center of the Sun, because the more abundant solar neutrino sources (proton–proton fusion, N^{13} decay and so forth) produce mostly lower-energy neutrinos. The p–p reaction, the principal source of solar neutrinos, falls entirely below the chlorine detector's threshold, and only B^8 neutrinos are energetic enough to reach the excited



Neutrino mass splitting and mixing angle, as calculated by Mikheyev and Smirnov, are limited to two solutions (red curves) by the observation that two-thirds of the expected solar neutrinos are missing. Bethe's solution approximates the upper curve. The interior of the triangle, a region of very efficient neutrino conversion, is excluded by the fact that we see any solar neutrinos at all. The right border (dashed) is simply a mixing-angle limit beyond which vacuum oscillation becomes significant.

states of Ar^{37} that make the chlorine detector really efficient.

To explain the fact that the Davis experiment sees[4] only 2.1 ± 0.3 SNU, one seeks a mechanism by which about twice that number of neutrinos would have been lost along the way. Neutrino interactions being as weak as they are, the electron-neutrino flux is clearly not significantly attenuated by collisions in the Sun or Earth. (Steven Weinberg points out that one can frighten small children by telling them how many light years of lead a β-decay neutrino can traverse without collision.) But the unscattered passage of a neutrino through matter is coherently related to its forward scattering amplitude. Because electron neutrinos can interact with electrons by the exchange of the $W^{\pm}$, the charged intermediate vector boson, and muon neutrinos cannot, these two kinds of neutrinos have significantly different forward scattering amplitudes in matter.

If oscillations between the two kinds of neutrinos occur in vacuum as a result of flavor mixing and a nonvanishing mass difference, the vacuum oscillations will be modified in matter because of this difference in the forward scattering amplitudes. The usual scenario for vacuum oscillation supposes that ν_e and ν_μ, the states of well-defined charged-current ($W^{\pm}$ exchange) weak interaction, are not precisely the vacuum eigenstates of definite mass. In terms of these definite mass states, the states defined by the charged-current interaction would be linear superpositions

$$|\nu_e\rangle = |\nu_1\rangle\cos\theta + |\nu_2\rangle\sin\theta$$
$$|\nu_\mu\rangle = -|\nu_1\rangle\sin\theta + |\nu_2\rangle\cos\theta$$

of the two vacuum eigenstates with different masses m_1 and m_2. Assuming that the mixing angle θ is small, one can speak sloppily of m_1 as "the mass of the electron neutrino."

In vacuum, if one starts with a pure electron neutrino of energy E, say from a beta decay, the probability that this neutrino would behave like a muon neutrino in a collision after traveling a distance x is given by

$$P_{e\to\mu} = \tfrac{1}{2}\sin^2 2\theta(1 - \cos 2\pi x/l)$$

where l, the characteristic "vacuum oscillation length," is given by $4\pi E/(m_2^2 - m_1^2)$, in units that set $\hbar = c = 1$. In this, the most commonly supposed version of neutrino oscillations, one gets an oscillating probability of metamorphosis to a neutrino of another flavor unless the mass difference or the mixing angle vanishes.

In passage through matter, however, all this is modified by the fact that a ν_μ can scatter off electrons only via neutral-current interaction (the exchange of a Z^0). This difference in forward scattering amplitudes between the two neutrino flavors introduces a second characteristic length, $l_0 = \sqrt{2}\pi/GN_e$, where GN_e, the product of the Fermi coupling constant and the density of electrons in the material, is effectively an extra potential seen only by electron neutrinos. One must then replace l in the formalism for neutrino oscillation by

$$l_m = l[1 + (l/l_0)^2 - 2(l/l_0)\cos 2\theta]^{-1/2}$$

and $\sin^2 2\theta$ by

$$\sin^2 2\theta_m = (l_m/l)^2 \sin^2 2\theta$$

What Mikheyev and Smirnov noticed was the resonant character of the expression for l_m, the oscillation length in matter. As one increases l/l_0, say by increasing the density of the material through which the neutrino is passing, l_m goes through a sharp, resonancelike maximum as l/l_0 goes through $\cos 2\theta$. So does the neutrino-oscillation amplitude. And, more importantly, θ_m, the effective mixing angle in matter, passes through 90°, much like the phase shift of a resonant scattering amplitude.

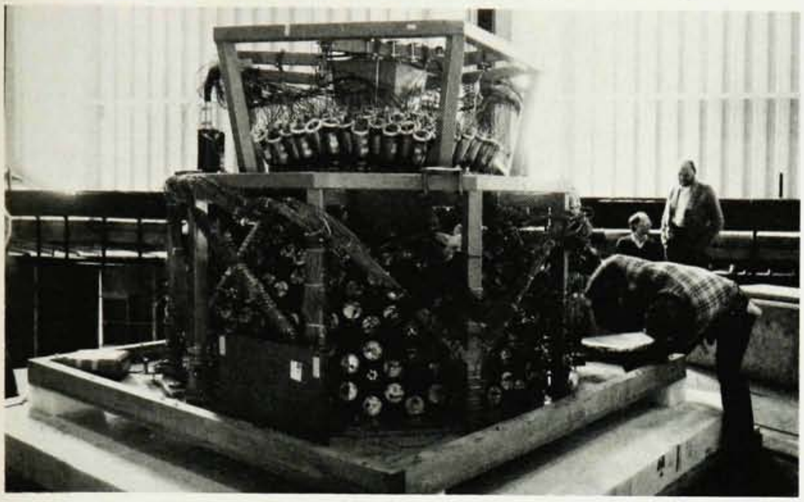
For a neutrino of a given energy, the resonant electron density is given by

$$N_e = (m_2^2 - m_1^2)\cos 2\theta / 2\sqrt{2}GE$$

If a neutrino traversing a region of continuously decreasing density passes through this resonant density, the effective mixing angle (assuming that θ, the vacuum mixing angle, is small) will go from nearly π to nearly 0. Thus an entering electron neutrino emerges from the region of its resonant density as a muon neutrino.

Unlike ordinary, nonresonant oscillation, this is an irreversible conversion, brought about by the fact that the vacuum oscillation and the matter effect are in phase when $l = l_0\cos 2\theta$. A necessary condition for this resonant metamorphosis is that the region of

Plastic Ball flies to CERN—and higher energies



At the Lawrence Berkeley Lab's Bevalac in February, the Plastic Ball nuclear particle detector was being carefully crated for its jumbo-jet flight to CERN—and higher heavy-ion energies. For the past four years, this hollow, faceted ball of 814 plastic/CaF_2 scintillation modules has been monitoring the fragments coming off high-energy heavy-ion collisions at the Bevalac, which can accelerate heavy-ion beams to 2 GeV/nucleon. From the outside, we see mostly the cabled ends of photomultiplier tubes. The Plastic Ball, built at LBL in collaboration with the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, was the first nuclear-physics detector capable of simultaneously identifying and measuring essentially all of the charged particles produced in a high-energy heavy-ion collision. Among the Plastic Ball's successes has been the first observation of "collective flow" in compressed nuclear matter.

In its new incarnation at the CERN Super Proton Synchrotron, the Plastic Ball will be looking at the debris from 200-GeV/nucleon collisions. SPS is primarily a proton–antiproton storage-ring collider nowadays, but for several weeks in November and again next May it will be devoted to heavy-ion physics. With a heavy-ion source (built at GSI) and a new rf-quadrupole injection system (built at LBL), SPS will be able to accelerate nuclei as heavy as calcium for fixed-target experiments. The fond hope, in these extraordinarily high-energy collisions between heavy ions, is to carry the compression of nuclear matter so far that one will see the transition to a quark–gluon plasma. Starting at about the same time next fall will be the heavy-ion program at the Brookhaven AGS accelerator, which will provide fixed-target experiments with 15-GeV/nucleon beams.

resonant density be wide enough that the process is adiabatic. That is to say, the Hamiltonian experienced by the neutrino must vary slowly enough that the neutrino remains in an eigenstate, adjusting its mass smoothly to the local density as it goes. This adiabatic condition requires that the oscillation length l_m in the resonant density region be much smaller than its width.

Bethe's rederivation of the Mikheyev–Smirnov resonance stresses its kinship to adiabatic level-crossing transition, a phenomenon well known in atomic physics. In the absence of mixing, Bethe points out, the square of the effective ν_e mass in the presence of the charged-current potential in matter would grow linearly with density, while the effective ν_μ mass would remain independent of density. Where the two m^2 curves would cross is precisely the resonant density of Mikheyev and Smirnov (see figure on page 17). But when mixing is introduced, coupling the two neutrino flavors, the actual m^2 eigenvalue curves move smoothly from one straight line to the other near the crossing point; the eigenvalue curves never actually cross. If the density change is adiabatically slow, the neutrino will stay on the same eigenvalue curve, its state vector turning slowly from the $|\nu_e\rangle$ direction toward $|\nu_\mu\rangle$ as it traverses the resonant density.

Mikheyev and Smirnov apply the theory to the solar-neutrino problem by formulating and numerically solving the differential equations for a neutrino traveling outward from the high-density region at the center of the Sun. Bethe undertook to get essentially the same result by a simple hand calculation "that would not require reproducing their computer programs to confirm it. Any physicist can immediately understand and confirm my argument," he told us.

Bethe argues that all the lost neutrinos come from B^8 decay. Neutrinos from all other solar sources have energies less than 2.8 MeV, too small to fulfill the resonance condition at any density in the Sun. The B^8 neutrino spectrum extends out to 14 MeV. All these boron neutrinos are presumed to originate in a small central region of the Sun with a mean density of about 130 g/cm^3. Thus, Bethe points out, there will be a critical energy (depending on the as yet unknown neutrino mass splitting and mixing angle) above which all neutrinos coming from this small core will pass through a region of resonant density, emerging as undetectable muon neutrinos. The higher the neutrino energy, the lower (and further from the center) is the density at which resonance occurs.

Attributing all the lost neutrinos to the B^8 and taking straightforward analytic expressions for the B^8 decay spectrum and the energy dependence of the Brookhaven detector's response, Bethe obtains a simple integral that immediately gives a critical energy of about 8 MeV, well above the upper limit for all the other solar neutrino spectra. This critical energy plus the central solar density of 130 g/cm^3, taken from Bahcall's standard solar model, immediately yields a value of 1×10^{-4} eV^2 for $(m_2^2 - m_1^2)\cos2\theta$. Making the reasonable assumption that the mixing angle is small and ν_e is much lighter than ν_μ, Bethe gets an estimate of 0.01 eV for the mass of the muon neutrino.

It will be very difficult to confirm so small a mass splitting by laboratory experiment. "Observation of solar neutrinos with a gallium detector," Bethe suggests, "would be the best confirmation—or disproof—of our theory." Detectors exploiting the conversion of Ga^{71} to Ge^{71} by neutrino capture have an energy threshold of only 0.23 MeV, permitting one to see much of the proton–proton neutrino spectrum. With resonant neutrino conversion in the Sun removing only neutrinos more energetic than 8 MeV (exclusively from B^8 decay), the fractional suppression would be much smaller in a detector sensitive to lower-energy neutrinos. Whereas the chlorine detector seems to be losing ⅔ of its signal to neutrino conversion, the Mikheyev–Smirnov theory expects that a gallium detector would come up only about 10% short. Subtracting the resonant-conversion loss of 88% of the B^8 signal from Bahcall's standard-model calculation for gallium detectors, Bethe predicts that the gallium detectors will see about 110 SNU if it's really resonant conversion that's causing the ν_e shortfall.

Not only will this be a crucial test of the Mikheyev–Smirnov conversion mechanism; it will also shift the responsibility for testing the standard solar model from the very peripheral B^8 decay to the p–p and C–N–O sequences that are firmly believed to be the essence of the solar engine. The neutrino-output prediction for these processes is far less sensitive to small uncertainties in the solar parameters than is the minuscule solar population of B^8. Until now one has had to make do primarily with B^8 neutrinos because they are the only ones energetic enough to be seen with high capture cross section in the South Dakota chlorine detector.

"After Bethe's paper we really have to do a gallium experiment," Davis asserts. In February DOE finally turned down a Brookhaven–Los Alamos proposal to build a 30-ton gallium detector for $15 million. Because gallium is sensitive to more of the solar neutrino spectrum, you need less of it, but gallium is a lot more expensive than dry-cleaning fluid. "We went to the wrong bank," Davis told us. The funding of such a detector falls under the heading of "nuclear physics" at DOE, "and the nuclear physicists have other priorities."

The group at the Max Planck Institute for Nuclear Physics in Heidelberg that had been working on the gallium proposal with Brookhaven has now formed a German–French–Italian consortium that recently won approval of its plan to built a 30-ton gallium detector in the Gran Sasso tunnel in the Apennines. The Soviets are building a 60-ton gallium detector.

It turns out that Bethe's solution is one of two that satisfy the resonance condition and the adiabatic condition. Bethe is pleased with the back-of-the-envelope character of his calculation. But, he admits, one does pay a price for its simplicity. It fails to take note of a second solution adumbrated in the Russian paper and recently calculated[5] in great detail by Rosen and James Gelb at Los Alamos. They point out that the second solution, corresponding to $\nu_e \to \nu_\mu$ conversion nearer the surface of the Sun, predicts a signal significantly less than 110 SNU for the gallium detectors. In this second solution, in contrast to Bethe's, it's the high-energy neutrinos one sees, the low-energy neutrinos having been lost by conversion. This solution yields a range of values for $(m_2^2 - m_1^2)$, from Bethe's 10^{-4} eV^2 down to 10^{-7} eV^2, with the mixing angle increasing as the mass splitting gets smaller (see figure on page 18). The Bethe solution, by contrast, specifies only a minimum mixing angle. "There are now two possible solutions," Bethe concedes, "but *only* two. The gallium detectors can decide between them."

Rosen and Gelb calculate not only the integrated neutrino signals expected for the two solutions, but also the shapes of the neutrino spectra. Rosen argues that the next generation of detectors (after the gallium detectors currently under development) should have the capacity to measure these spectral shapes. Simply seeing 110 SNU in a gallium detector would be consistent with Bethe's solution, but, Rosen points out, it would also be consistent with mechanisms other than neutrino oscillation. It might be that "wimps" (weakly interacting massive particles hypothesized to deal with the Davis data as well as the cosmological dark-matter problem), or some other aid to thermal transport, are suppressing the production of B^8 by lowering the central temperature of the Sun. A very recent calculation by John Faulkner (University of California, Santa Cruz) and Douglas Gough (UK Institute of Astronomy) argues[6] that wimps would kill two birds with one stone: They would account not only for the missing B^8 neutrinos but also for solar pulsations, which have been another sore point for the standard solar model. Unambiguous evidence for Mikheyev–Smirnov oscillations, as against wimps, would come from seeing how the 110 SNU worth of neutrinos are distributed in energy.

What if the gallium detectors see not 110 SNU, but a much smaller signal—something like 10 SNU? In that case, Wick Haxton (University of Washington) points out, there's no ambiguity. None of the nonstandard solar models could suppress neutrino production in the Sun to such an extent. So small a signal, he argues, would clearly correspond to the second solution for the Mikheyev–Smirnov neutrino-oscillation mechanism. Haxton has recently shown[7] that this second solution can also be understood in terms of Bethe's adiabatic-approximation picture. The second solution corresponds to the survival of high-energy neutrinos because they fail to satisfy the adiabatic condition, while the lower-energy neutrinos are adiabatically transmuted.

We've been talking as if there were only two neutrino flavors, largely ignoring ν_τ, the neutrino that goes with the τ lepton, 17 times as heavy as the muon. If one simply assumes, as Bethe does, that the disappearance of solar neutrinos is due to $\nu_e \to \nu_\mu$, and if one takes his value for the ν_μ mass, then the seesaw model of Gell-Mann and company yields a ν_τ mass estimate that ranges from about 2.5 eV to 100 eV. Such a ν_τ would play no significant role in the Sun, but its vacuum oscillation length for $\nu_\mu \rightleftarrows \nu_\tau$ oscillations would be only a few hundred meters. This might be a feasible length for laboratory experiments, Bethe suggests. The vacuum oscillation length for $\nu_e \rightleftarrows \nu_\mu$, on the other hand, is on the order of a thousand miles if one believes the Bethe mass splitting. For such oscillations one has little choice other than to look at solar neutrinos.

The seesaw model, in essence, asserts that the typical quark mass of each "generation" of elementary particles is the geometric mean between the corresponding neutrino mass and some very large universal scaling mass. Paul Langacker (University of Pennsylvania) points out that if one *really* takes the seesaw model seriously, invoking the grand-unification mass (10^{14} GeV) as its scaling mass, all three neutrinos would be ten thousand times lighter than Bethe's estimates, and $\nu_e \to \nu_\tau$ would in fact be the principal mechanism for the disappearance of the solar neutrinos.

The seesaw model accords with the general prejudice that if the neutrinos have different, non-vanishing masses, the electron neutrino should be the lightest. The Mikheyev–Smirnov paper, following a sign error that crept into the ν_e–e interaction term in one of the Wolfenstein papers, concluded that ν_e should be heavier than ν_μ. This put several people off the Mikheyev–Smirnov theory, "but not me," Bethe told us. In the end, Langacker put things right by finding the sign error and pointing it out to Bethe.

—Bertram Schwarzschild

References

1. H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).
2. S. P. Mikheyev, A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985), to be translated in Sov. J. Nucl. Phys.
3. J. N. Bahcall *et al.*, Astrophys. J. **292**, L79 (1985).
4. J. K. Rowley, B. T. Cleveland, R. Davis Jr, in *Conf. on Solar Neutrinos and Neutrino Astronomy 1984*, AIP Conf. Proc. No. 126, M. L. Cherry, W. A. Fowler, K. Lande, eds., AIP, New York (1985).
5. J. Gelb, S. P. Rosen, submitted to Phys. Rev. D (1986).
6. J. Faulkner, D. Gough, Nature **321**, 226 (1986).
7. W. C. Haxton, "Adiabatic conversion of solar neutrinos," University of Washington preprint (1986). □