reference frame

From broken symmetry to chaos

Herman Feshbach

The low-lying energy levels of quantum systems often exhibit a high degree of symmetry: crystals, the rotational levels of a deformed nucleus, the most deeply bound states of hadrons. However, all of these systems violate the exact symmetries of their Hamiltonians, exhibiting "broken symmetry." Crystals and nuclei do not meet the requirements following from the translational and rotational invariance of their Hamiltonians, and hadron states violate the invariances associated with the internal degrees of freedom of strangeness and isospin. In each of these systems, the symmetry is broken by focusing observation on a few of the degrees of freedom, so that the symmetry the system does exhibit is of reduced dimensionality. As a group theorist would say, the group associated with the broken symmetry is a subgroup of the one associated with the exact symmetry.

Closely related to symmetry is "order," which describes the correlations among constituents of the system. In a crystal the order is nearly perfect and the correlations have a long range. Order in ferromagnetism is measured by the magnetization; the correlation scale is given by the size of the magnetic domains. In nuclei the collective degrees of freedom, which describe the coherent participation of many nucleons, are a manifestation of order. Order is measured by the quadrupole moment in deformed nuclei.

If one adds excitation energy, order tends to be reduced, until eventually, a change in phase occurs: The crystal melts, the liquid vaporizes to form a gas, the spin orientation of the magnetic domains is randomized, the nucleus may fission or—with enough energy added—disintegrate into a gas of nucleons. In each case, the highly ordered phase is replaced by a less ordered one. At the same time, the broken symmetry is removed and exact symmetry approached. Despite their less ordered state, both the gas and the liquid do satisfy rotational and translational invariance, on the average. For example, translation invariance applies to the average density of a liquid, but at any given instant the liquid contains inhomogeneities. Several changes in phase may occur, each one making the system less ordered but bringing it (on the average) closer to exact symmetry.

Disorder shows up well before the phase changes occur. In nuclei, as the excitation energy increases, the density of levels grows rapidly, so that the number of modes of motion becomes very large. Slow-neutron scattering by nuclei gives very sharp resonances, and one can use these resonances to measure the energies and widths of nuclear levels. The probability distribution $p(s)$ for the spacing s between the energies of neighboring levels is the "Wigner distribution":

$$p(s)\,\mathrm{d}s = (\pi/2)(s/D^2)\,\mathrm{e}^{-(\pi/4)(s/D)^2}\,\mathrm{d}s$$

where D is the average spacing. The distribution of the widths is the statistical χ^2 distribution for one degree of freedom. These are remarkable results because these distributions can be obtained by assuming that the energy levels are eigenvalues of a Hamiltonian with random matrix elements and that the widths are proportional to the square of the components of the eigenvector of that Hamiltonian. The distribution of the matrix elements is assumed to be invariant under a change in the basis wavefunctions. These elements—the widths and energy-level spacing—are effectively random; the specific properties of the system enter these distributions only through the average values of the width and spacing. These properties of randomness hold not only for nuclei, but for atoms as well, for bound states as well as unbound ones. This random behavior is of great practical value because it allows us to use statistical concepts to predict average reaction cross sections, expressing the results in terms of average quantities obtained from simplified versions of the many-body problem.

Some authors have speculated that this apparently random distribution of nuclear levels is an example of quantum chaos. Although the systems we are considering are described by the Schrödinger equation, at sufficiently high excitation they become effectively random, a phenomenon that reminds one of the chaos of classical dynamics. As in classical dynamics, ordered states do not disappear completely. For nuclei, the excitation energy added by an incident projectile can go into a collective mode, producing a resonance in the cross section that rises above a background originating in the random behavior of the system.

For crystals, the mechanisms leading to disorder are not clear. Some theorists have considered a two-di-

continued on page 9

In February we introduced Reference Frame, a column of opinion to be written by a number of regular contributors, each an eminent physicist. This month's columnist, Herman Feshbach, is a nuclear theorist whose research in recent years has focused on nuclear reactions. He and the late Philip Morse wrote the classic text *Methods of Theoretical Physics*. Feshbach was chairman of the MIT physics department for 10 years, President of The American Physical Society in 1980–81 and chairman of the Nuclear Science Advisory Committee in 1981–84.

0031-9228 / 86 / 0400 07- 02 / $01.00

reference frame

continued from page 7

mensional model consisting of spins with nearest-neighbor interactions. In that "planar Heisenberg" model, excitation energy leads to spin-vortex formation. At low temperatures, a vortex will overlap an antivortex. The net vorticity is then close to zero. As the temperature increases, these bound vortex pairs separate and finally become free, forming a "gas" of vortices, which carry the disorder. At high temperatures, correlation between spins becomes short ranged, unlike the long-range correlation at low temperature. One thinks of the vortices as defects that become free to move at sufficiently high temperature. For three-dimensional systems, one can speculate that the defects are dislocations and that these will multiply with increasing temperature, with a corresponding increase in disorder.

Manifestations of this disorder should grow more and more observable as the energy excitation increases before the transition to the defect fluid. Will these reflect a chaotic behavior analogous to that exhibited by atoms and nuclei? Several models have shown signs of such chaotic behavior. For example, in a model with competing ferromagnetic and antiferromagnetic interactions, a phase of the system is formed in which strong and weak spin correlations appear in a chaotic sequence as one probes the system at successively larger distances.

Recently developed theories of the history of the universe presume an initial high-temperature regime in which the exact symmetries are all satisfied. As cooling proceeds, these symmetries are successively broken and separate interactions or particles become identifiable. The first to make its appearance is gravity; next the hadron symmetry is broken, and baryons emerge. Finally, the electroweak symmetry is broken giving rise to separate electromagnetic and weak interactions. Order is carried by the symmetry-breaking Higgs fields. Initially these fields are random, but as the universe cools they become successively more ordered, thereby inducing the symmetry breaking. The role of chaos in these transitions has not been studied.

Quantum chaos, its characterization and its consequences—particularly as a change of phase is approached—form a fascinating area of research, of fundamental importance. Under what circumstances do many-body quantum systems exhibit chaotic behavior, and what are the observable consequences? These questions are as yet unanswered. □