

Digging deeper into the structure of space-time

Spinors and Space-Time, Volume I: Two-Spinor Calculus and Relativistic Fields

Roger Penrose and Wolfgang Rindler
458 pp. Cambridge U.P., New York, 1984.
\$89.50

Reviewed by Joshua N. Goldberg

The study of asymptotically flat space-times in the vicinity of null infinity was begun around 1960, using two different techniques: direct solution of the Einstein equations (Hermann Bondi and Rainer Sachs) and application of the spin-coefficient formalism (E. Ted Newman and Roger Penrose). By comparing the complexity of these two approaches, one understands immediately why the Newman-Penrose formalism has become the principal tool for studying asymptotically flat space-times. However, the argument for considering spinors as basic geometrical entities that apply "at a deeper level of structure of space-time" goes beyond the simplification of classical field equations. Spinors are complex. As a result, the use of a spinor formalism naturally requires the use of complex analysis even for real fields that live on real manifolds. Underlying such work is the belief that complex numbers and holomorphic functions are an essential part of any description of nature. That is certainly the focus of twistor theory, which makes up a major portion of the forthcoming volume II (*Spinor and Twistor Methods in Space-Time Geometry*, to be published in March 1986). In some ways volume I is a polemic for the use of spinors as the basic geometrical entities on a manifold, in preparation for volume II. The present volume gives a new approach, based on spinors, to analysis on manifolds, but the reader will find very little physics here.

The book begins with the construction of a geometrical interpretation of a spinor as a null flag. The flag pole is defined by the null direction associated with the spinor, while the flag direction

is related to its argument. Once this geometrical description of a spinor is completed, geometry almost disappears from the volume. This is evident from the distribution of the wonderfully clear figures drawn by Penrose: 23 are in the 67 pages of Chapter 1, and 24 in the remaining 356 pages—excluding the appendix, which introduces an interesting diagrammatic method of calculation. The suppression of geometry is a deliberate choice; the authors state explicitly that they want to use algebraic formulations and to define the space-time geometry in terms of the resulting algebraic structures. It is true that the algebraic construction of the null flag is much simpler than the geometrical. Nonetheless it is disconcerting to have the covariant derivative, the curvature tensor and the Lie derivative introduced strictly algebraically, with no reference to their geometrical meaning except for a one-sentence remark about the Lie derivative. Perhaps my unease comes from having learned differential geometry from *Ricci Calculus* by Jan A. Schouten (Springer-Verlag, Berlin, 1954), which describes the Lie derivative in terms of red threads embedded in gelatin.

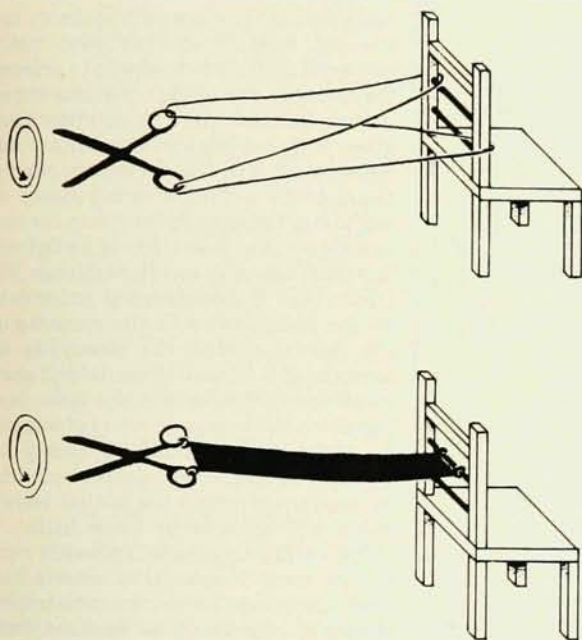
Most of the material in this book has been published before by Penrose with and without collaborators. As indicated in the preface, the genesis of the book is in the notes taken by Rindler on a series of lectures given by Penrose in 1962. Many of us have been waiting for this book since that time. However, not all the material that appears in this volume was available then. Much of the geometrical discussion in the first chapter is new, as is the careful discussion of the vector and spinor modules, which includes a lengthy introduction to the abstract index notation. Because of the emphasis on algebraic methods even the section on spinor analysis contains new relationships and understandings. Not only is the usual translation of tensors into spinor forms given, but also the converse—that is, spinor relations and operations are expressed in terms of tensors. The treatment of spin coefficients uses

non-normalized as well as normalized spin bases. The introduction of spin- and boost-weighted operators allows the authors to reformulate the Newman-Penrose equations using compacted operators. These add to the proliferation of symbols and operations to be learned, but they are used only in the latter part of the book.

The chapter on classical fields contains a brief but clear description of vector bundles, which are then applied to Yang-Mills fields. But the most interesting part of this long chapter comes at the end, with the discussion of exact sets of fields and the null-surface initial-value problem. The field equations of an exact set determine, at a point, all the derivatives of the set of spinor fields in terms of the completely symmetrized derivatives alone. Solutions for massless spinor fields are constructed as two-surface integrals over the null datum. For interacting fields (not necessarily massless) that form an exact set, this leads to the construction of a formal solution in terms of a series of integrals suggestive of Feynman integrals.

Although spinor techniques have been exceedingly useful in the study of the Einstein equations, this book looks forward to more profound applications of two-component spinors in physics. One has already seen the importance of spinors in the proof of the positivity of energy for isolated systems and in the generalizations of the Einstein theory using supersymmetry, which couples spinor and tensor fields. In addition there is the work by Penrose and his collaborators on twistor theory, which is motivated by the isomorphism between the conformal group and $SU(2,2)$. A major portion of volume II is devoted to this work, which has yielded significant contributions to the study of instantons in Yang-Mills theories. Apart from these, there are indications in recent work by Abhay Ashtekar that a viable quantization of the Einstein theory can be constructed using spinors rather than the metric tensor itself. Therefore it appears that the use of two-component spinors will be important in future developments in physics

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Dirac's scissors problem. One can "undo" a 4π (but not a 2π) rotation of the scissors around its axis of symmetry—that is, one can untangle the string without moving the chair or rotating the scissors. This can also be demonstrated by replacing the string with a belt—a twist of 4π is undone by looping the belt once over the scissors. This figure, drawn by Roger Penrose, is reprinted from the book under review.

and that this book and, presumably, the succeeding volume will be foundation texts for a generation of physicists, as were the books by Luther P. Eisenhart and Schouten in their time. However, for those who would like a more physical introduction to the subject before studying this text, let me suggest the article by Penrose, "Structure of space-time," in *Battelle Rencontre, 1967 Lectures in Mathematical Physics*, edited by C. M. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).

Finally, a comment on the appearance and formal structure of this volume is in order. Given the variety of symbols and the large number of indices, along with the need to identify indices with primes, carets or dots as well as to distinguish italic from bold-face upright indices, the use of a larger typeface would have been helpful. A larger face would also have helped the editors to pick up the many typographical errors and broken letters. However, the organization of the book has been carefully thought out and the

material is very well presented. In addition to the table of contents, the book has an excellent index, list of symbols and references section. It would have been useful had the authors also included the table of contents for volume II as well as perhaps its index.

An Idiot's Fugitive Essays on Science

Clifford Truesdell

654 pp. Springer-Verlag, New York, 1984.
\$58.00

Idiot, *idiōtēs*, a private citizen, a non-member of everything—thus Clifford Truesdell specifies his position in the world. In this collection he has brought together reviews, addresses and historical essays from the past 35 years, some revised and most of them the fruit of long experience and much reading. Two themes run through the pages: that good scientific theory is based on rigorous mathematics and that the ancient theory of continuum mechan-

ics (to which Truesdell has made his own substantial contribution) is good scientific theory. Because so many of the basic phenomena of flow, bending and plastic deformation have been studied from the beginning of modern mathematics, and because later discoveries have not changed the outlines of the subject, he can look back over a historical panorama that has no parallel in science except possibly for celestial mechanics. But that is a subject in which masses are often idealized as points, and it holds no interest for this practitioner of continuum mechanics, for whom the intrusion of the atomic hypothesis and of theories that emphasize it holds the same charm as the arrival of a busload of yelling schoolchildren in a sunswept pastoral landscape.

Possibly the most rewarding part of the book is an 87-page section, consisting of book reviews and a long biographical essay, on Leonhard Euler. Not only do we gain some idea of Euler's protean involvement in fields such as naval architecture, chemistry, optics, electricity, terrestrial magnetism, geography, mortality tables and the culture of mulberries and corn, we also see him in the intellectual milieu of the 18th century, in St Petersburg and during a 25-year interval at the court of Frederick the Great. There he was the statesman of science, answering every letter he received with encouragement, advice and helpful criticism, towering above the fields and swamps in which troops of academicians fought their squalid little campaigns of jealousy and revenge (a favorite theme of Truesdell's). But Truesdell focuses on Euler principally as the mathematician who invented the derivative as the limit of a quotient, who critically studied the convergence and divergence of series, who gave the modern definition of a function almost a century before Peter G. L. Dirichlet, who invented countless special methods still in use and who participated at every stage of the early development of the mechanics of rigid, elastic and fluid bodies. Truesdell writes that "only blind prejudice or special pleading could deny to Euler the rank of solely