



reference frame

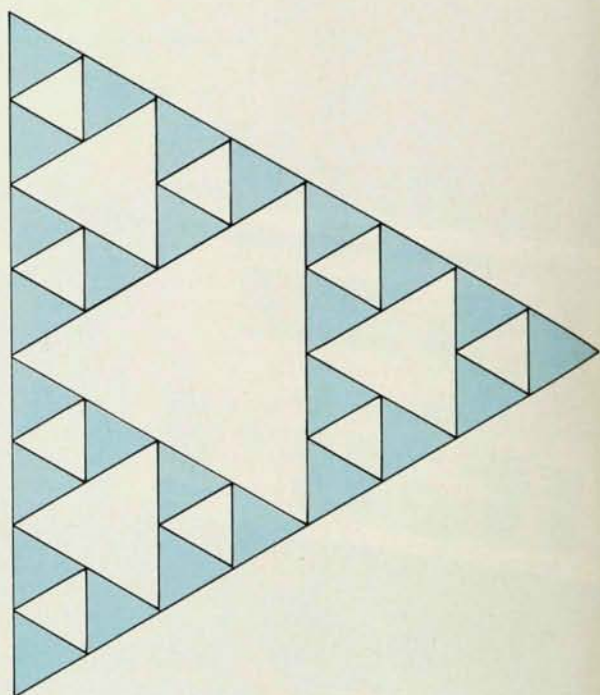
Fractals: Where's the physics?

Leo P. Kadanoff

Why all the fuss about fractals? *Physical Review Letters* complains that every third submission seems to concern fractals in some way or another. Corporate research labs such as Exxon's and IBM's expend perceptible fractions of their entire basic-research budgets on the study of fractal systems. Perhaps a half-dozen conferences during the past year were devoted to the subject. Why?

But first what. What are fractals? Different people use the word *fractal* in different ways, but all agree that fractal objects contain structures nested within one another like Chinese boxes or Russian dolls as, for example, in figure 1. This Sierpinski gasket consists of triangles within triangles and so on to the finest level. The littlest triangles, in color, are hard to draw but easy to describe: They are little copies of the entire object. Some fractal objects are more random in structure. The cover shows a "tree" produced by a computer algorithm called DLA, for diffusion-limited aggregation. The basic DLA algorithm was introduced by Thomas A. Witten and Leonard M. Sander in *Physical Review Letters* 47, 1400 (1981). In DLA, the tree grows unit by unit by the following process: A particle is inserted above the tree with a randomly chosen x -coordinate. The particle then undergoes a random walk in which each step is to a neighboring lattice site; the choice of neighbor is determined by a "throw of dice." The walk continues until the particle reaches a site neighboring the tree. It then stops walking, the tree grows one unit by the addition of that particle and the entire process continues with the insertion of a new particle. Figure 2 shows another example, this time experimental. The basic units are little gold balls, and the process is essentially the same, except the real-world object is three-dimensional while the computer object on the cover is two-dimensional. One final example: Figure 3 shows a top view of a block of Lucite 2 cm thick; monoenergetic MeV electrons

A fractal object. For many other examples of such objects see Benoit Mandelbrot's book *The Fractal Geometry of Nature* (Freeman, New York, 1983).
Figure 1



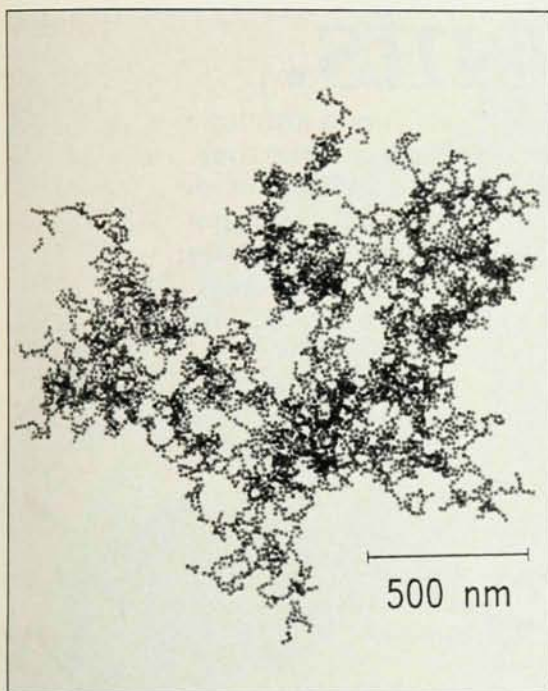
have been shot into it and stopped within a thin layer perhaps 1 cm below the surface. After the injection is completed, a nail is inserted at the side and the electrons shoot out, leaving a trail shaped like a river system behind them.

One reason for our interest in fractal objects is their practical importance. Materials scientists want to produce entirely new structures with entirely new properties, like the sponge of figure 2. When this kind of object is grown to be very large it has huge empty spaces and a density that decreases roughly as $\rho = \rho_0(R/a)^{-1/2}$, where R/a is the ratio of the radius of the object to that of the balls. Dielectric breakdown, shown in figure 3, is of considerable technical importance. As a final example, consider petroleum-bearing rock layers. These typically contain fluid-filled pores of many sizes, which might be effectively understood as a fractal object.

The technical interest of fractals is matched by their intellectual interest. Two of the fundamental symmetries of nature are dilation ($\mathbf{r} \rightarrow a\mathbf{r}$) and translation ($\mathbf{r} \rightarrow \mathbf{r} + \mathbf{b}$). We can represent them verbally by talking about a change in our unit of length or in the origin of our coordinate system. Fractal objects are highly nontrivial representations of these symmetries. Thus, for example, an expanded piece of the Sierpinski gasket of figure 1 can be moved in such a way as to make it coincide with the entire gasket—and this operation can be performed in an infinite number of ways. The more random fractals of figures 2 and 3 probably obey similar combined translation-dilation symmetries.

Another example of dilation-symmetric behavior that has been studied in great detail involves the critical phenomena that arise near "second-order" phase transitions. The phenomenology of these transitions has been

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A laboratory-grown fractal produced by David A. Weitz and M. Oliveria (*Physical Review Letters* **52**, 1433 1984). (Photo courtesy of Exxon.)

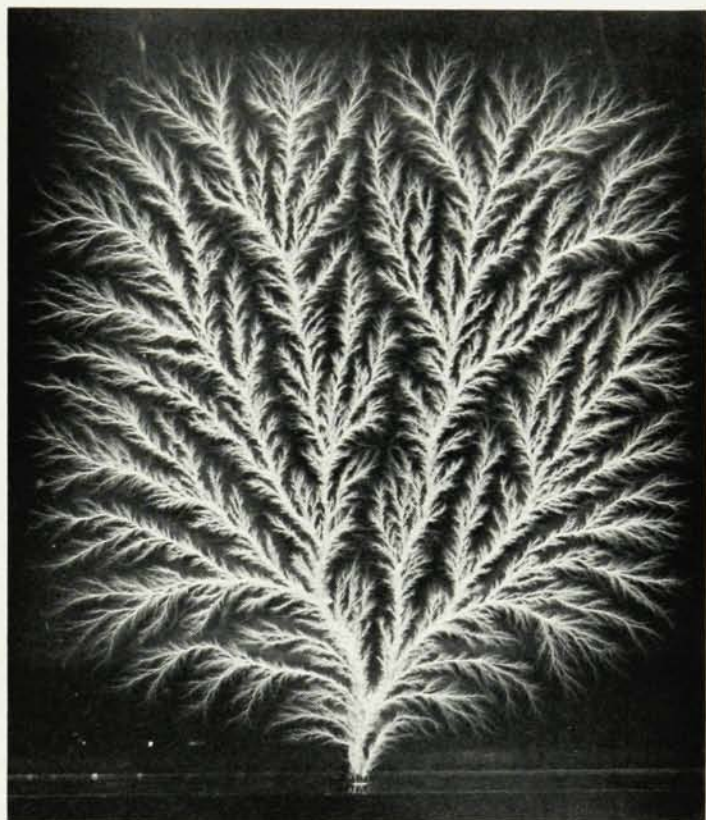
Figure 2

elaborated in considerable detail with the aid of the concept of *universality*, that is, the notion that disparate phase transitions may exhibit quantitatively identical behavior. This concept might be extended to processes that produce fractals. For example, might the fine details of dielectric-breakdown patterns formed in different contexts be identical? Moreover, might real breakdowns in two dimensions (see figure 3) be identical with theoretical models of step-by-step growth, perhaps even with the model simulated on the cover?

One way of answering such questions is to perform appropriate measurements on the objects in question. One quantity commonly measured is the fractal dimension. (This dimension is a

variant of a concept due to the mathematician Felix Hausdorff.) The fractal dimension is defined as $d \ln M(R) / d \ln R$, where $M(R)$ is the mass contained within a distance R from a typical point in the object. If two objects are the same they must at least have the same fractal dimension.

Unfortunately, although this single, rather primitive measurement enables us to distinguish among objects, it never enables us to give a convincing case for their essential identity. Some progress has been made in identifying other qualities, beyond the fractal dimension, that might be universal. However, further progress in this field depends upon establishing a more substantial theoretical base in which geometrical form is deduced from the mechanisms that produce it. Lacking such a base, one cannot define very sharply what *types of questions* might have interesting answers. One might hope, and even expect, that eventually



A "beam tree" produced by dielectric breakdown. Object courtesy of SLAC and Sidney Drell. (Photo by Oscar Kapp.)

Figure 3

a theoretical underpinning—like that of Kenneth Wilson's renormalization approach—will be developed to anchor this subject.

Without that underpinning much of the work on fractals seems somewhat superficial and even slightly pointless. It is easy, too easy, to perform computer simulations upon all kinds of models and to compare the results with each other and with real-world outcomes. But without organizing principles, the field tends to decay into a zoology of interesting specimens and facile classifications. Despite the beauty and elegance of the phenomenological observations upon which the field is based, the physics of fractals is, in many ways, a subject waiting to be born. □