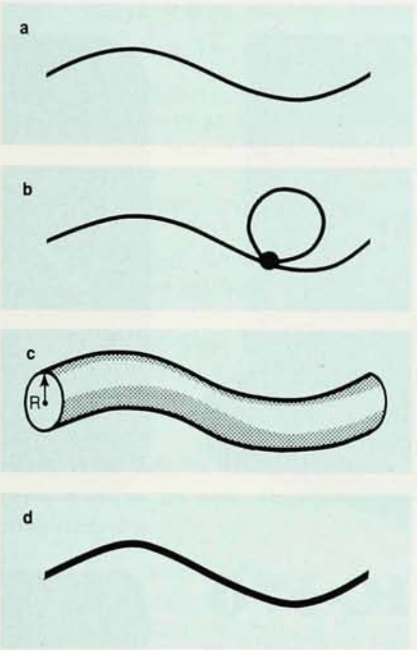
reference frame

The road to four dimensions

Herman Feshbach

Two themes are distinguishable in the history of physics. One theme focuses on the nature of observation: Which questions can we fruitfully ask regarding the behavior of a physical system and which questions make no sense? Isaac Newton's laws of motion, Albert Einstein's special relativity and Niels Bohr and Werner Heisenberg's uncertainty principle fall into this category. The other theme emphasizes the goal of unification—understanding two seemingly independent sets of phenomena in terms of a single theory. The unification of electricity, magnetism and optics by Hans Christian Oersted, Michael Faraday and James Clerk Maxwell is a familiar example. More recently, Steven Weinberg, Abdus Salam and Sheldon Glashow succeeded in unifying the weak and electromagnetic interactions. Today the particle theorists are vigorously pursuing the unification of all the forces of nature—the strong hadronic interaction, the weak, the electromagnetic and the gravitational. Their El Dorado is the golden Lagrangian, from which one can obtain all the interactions as well as all the elementary particles, the building blocks of matter.

One example of such a theory is a generalization of the five-dimensional model formulated by Theodor F. E. Kaluza and Oskar Klein in the 1920s that unified gravitation and electromagnetism. (Interestingly, Einstein delayed publication of Kaluza's paper by two years.) Four of the dimensions describe space–time, while the fifth is required to obtain the electromagnetic fields. Because now we wish to accommodate the strong and weak interactions as well, a larger number of dimensions is required, 11 in all. The internal symmetries one observes in space–time are to be explained in terms of the symmetries of the seven-dimensional internal space that has been added on to space–time. The use of extra dimensions for this purpose is not especially unusual. What is novel is the requirement that the Kaluza–Klein Lagrangian be the extension of Einstein's gravitational Lagrangian from 4 to 11 dimensions. The Kaluza–Klein Lagrangian deals only with the gauge bosons, such as the photon and the graviton, the quantum of the gravitational field. The complete Lagrangian will contain in addition an 11-dimensional matter Lagrangian for the fermions. To obtain the resultant space–time theory and to determine its symmetries one uses a procedure referred to as "dimensional reduction."



Extension of one-dimensional space to include an "internal" degree of freedom. At each point of the space (**a**) one constructs a compact one-dimensional space (**b**). The characteristic size, R, of the resulting two-dimensional space (**c**) determines the mass scale in the compactified one-dimensional space (**d**).

A straightforward method of dimensional reduction involves the expansion of the fields in terms of a set of orthogonal functions complete in the internal space; the coefficients are functions of the four space–time coordinates. One introduces this expansion into the Lagrangian and integrates over the internal space coordinates, which yields an effective Lagrangian in space–time. To illustrate, consider the familiar case of the dimensional reduction of a three-dimensional space to a one-dimensional space. One way is to introduce spherical coordinates (r,θ,ϕ) and to expand in terms of the spherical harmonics $Y_{lm}(\theta,\phi)$. The expansion coefficients are functions of r. Introducing this expansion into the Lagrangian and integrating over θ and ϕ yields an effective Lagrangian that depends only on r and contains an effective potential energy $l(l+1)/r^2$ (the centrifugal potential). This potential energy reflects the elimination of two of the dimensions and—because l is an integer—also reflects the spherical symmetry implied by the use of spherical harmonics. The eliminated space described by θ and ϕ is said to be compact because, roughly speaking, the variation of each variable is bounded. The choice of spherical harmonics implies the rotational symmetry of the internal space; it is reflected in the remaining one-dimensional space through the invariance of r with respect to rotation. One can introduce other symmetries by using other coordinate systems, and thus other orthogonal series, to effect the dimensional reduction. In my three-dimensional example these could be relatively simple, such as circular and elliptical cylindrical coordinate systems, or quite complex, as in the case of toroidal coordinates.

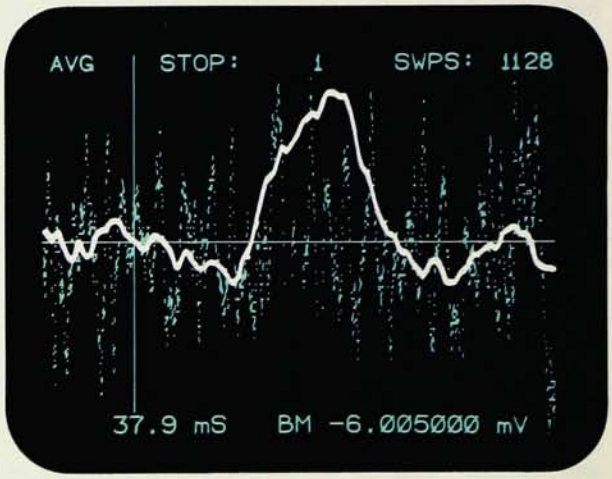
The lessons to be learned from this example are:

▶ The symmetries of the internal space select the orthogonal set of func-

---

Herman Feshbach is a nuclear theorist who was chairman of the MIT physics department for ten years.

Signal Averaging Plus.



Averaged signal emerges from noise on a visual evoked potential.

NIC-370

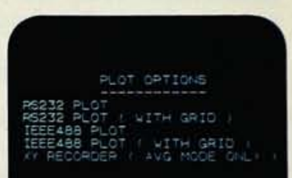
With sweep repetition rates as high as 150 per second, the dual input NIC-370 signal averager can outperform most computer systems. Sampling at speeds up to 1 MHz with 10-bit resolution, it averages into 24-bit memories to rapidly enhance your signal-to-noise ratio. Use the NIC-370 to average repetitive transients with selectable pre-trigger delay. Or as a digital scope for one-shot phenomena. In addition, ease your experimental interface with built-in aids such as artifact rejection and selectable sweep length from 32 to 2,000 points.

Plus Histogramming. Acquire and study repetitive pulse trains using dual-input multichannel scaling (pulse counting). Even compare analog data with related pulse activity using the unique averaging/pulse counting mode. Both with up to 2,000 channels of resolution per input. Or study the statistical properties of on-going pulse trains with frequency or interval histograms presented in a standard bar graph format.

Plus Analysis. Expand and examine your data in detail. Use the cursor and numerics to measure the exact coordinates of any feature. Manipulate the data with pushbutton programs or send it to your computer via GPIB or RS232 interfaces. Complete your report with a hardcopy plot using the XY recorder or digital plotter outputs. As an option, add a bubble memory cassette system, storing up to 21 records on each cassette ready for recall at any time.

Plus Simplicity. Don't miss important data because of set-up error. The NIC-370 is easy to use and backed by over twenty years of experience in signal acquisition. Find out how the NIC-370 can help you with your signal analysis problems. Call 800-356-3090 or 608-273-5008.

Interval Histogram profiles the statistical variation in time between pulses.



The NIC-370 sends displayed information to plotters for hard copy records.

Nicolet

"Instruments of Discovery"

Circle No. 8 for more information

Please contact me to discuss my application and arrange for a demonstration.

Name ______ Title ______

Company ______ Phone ______

Address ______

City ______ State ______ Zip ______

Send to:
Nicolet Test Instruments Division
P.O. Box 4288 • 5225-2 Verona Road
Madison, Wisconsin 53711-0288

NIC-370

tions to be used.

▶ The symmetries are reflected in the space- and time-dependent expansion coefficients.

▶ Many symmetries are possible for the internal space, but they are limited by the constraint that on dimensional reduction one should obtain the symmetry of the standard model $SU(3)\times SU(2)\times U(1)$, and, of course, the Einstein theory of gravitation.

Kaluza–Klein theories make the internal space compact so that the extra dimensions are not directly observable and one obtains the Yang–Mills equations—the analog of Maxwell's equations for quantum chromodynamics. One can make the internal space compact by introducing angle variables, but in general at least one of the seven-dimensional variables, say r, can be unbounded. To restrict it, one introduces a cutoff by taking the dependence on r to be periodic with a characteristic length R, so that the magnitude of r is effectively less than $2\pi R$. Introducing a length is equivalent to introducing a mass scale given by $\hbar/Rc$. As a second step in the dimensional reduction, R is taken to be very small, on the order of the Planck length, 10^{-20} fm. The mass scale is then 10^{19} GeV/c^2 so that the system is of small dimension in the internal space and the corresponding masses are unobservable except for one possibility, namely if the mass is zero. If the mass is zero, there is no dependence of the internal-space metric on the internal-space coordinates and the zero-mass particles can become the gauge particles, such as the photon in the original Kaluza–Klein theory. For the internal space of seven dimensions one obtains the Yang–Mills equations, justifying the small value of R.

The Kaluza–Klein theory has only one parameter, the Newtonian gravitational constant; the parameter R does not appear in the final equations. The compactification of the internal space and the scale chosen for R can be justified on the basis of the dynamical equations following from the 11-dimensional Lagrangians if they are supersymmetric. But that is a long story, which I will not recount here. I have also not discussed why the internal space is chosen to be seven dimensional.

This remarkable theory is flawed. In particular it cannot describe the asymmetry required by the violation of parity conservation in the weak interactions. This difficulty is not present in the ten-dimensional superstring model, now the leading candidate for unification. Although technically not a Kaluza–Klein theory, a superstring theory makes a similar use of extra dimensions in space–time. □