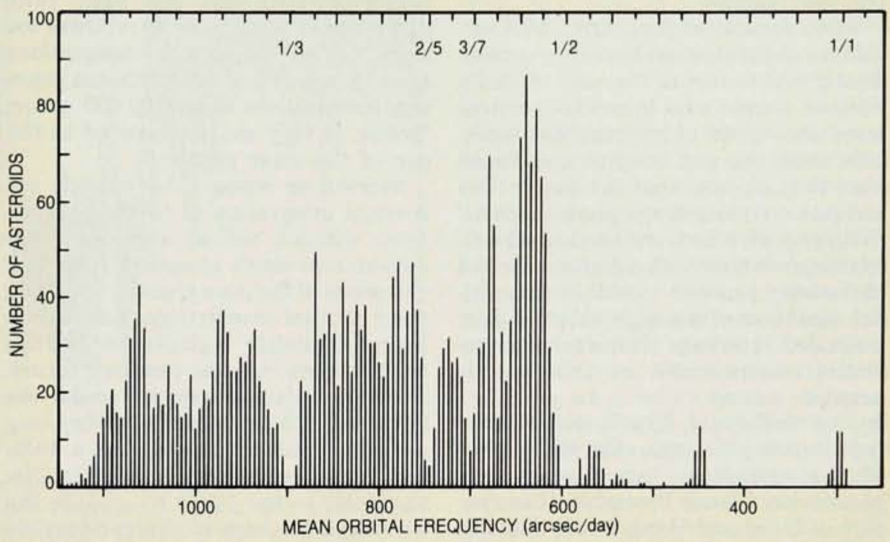
Chaotic orbits and spins in the solar system

It was surely the intricate clockwork regularity of the solar system, as our ancestors observed and pondered it over millenia, that inspired the quest that culminated in the differential equations of Isaac Newton. It is somewhat ironic, therefore, that these same differential equations—usually regarded as epitomizing order and predictabilty—also appear to be generating *chaos* in the solar system. Nowadays, when one speaks of chaos, especially in an essentially conservative, Hamiltonian system like the solar system, one does not mean the unpredictability inherent in intrinsically disorderly phenomena such as thermal noise or innumerable random collisions. The modern study of chaos (PHYSICS TODAY, December 1983, page 46) deals with the onset of wild and unpredictable fluctuations in systems governed by simple, deterministic equations from which one would naively expect nothing but good behavior.

Jack Wisdom, a celestial-mechanics theorist at MIT, appears to have solved two long standing puzzles of the solar system by bringing to bear the new techniques of chaos analysis. The scientific community has known for almost 200 years that stones do indeed fall from the sky. But why so many meteorites have strayed so far from their presumed origin in the asteroid belt has been something of a mystery. A related puzzle is the existence of a sequence of depopulated gaps in the asteroid belt, discovered by the American astronomer Daniel Kirkwood more than a century ago.

At the Washington meeting of the American Physical Society in April, Wisdom presented a symposium devoted to chaos in dynamical systems with three examples "where chaotic behavior is important in the solar system." Two of these appear to resolve the long standing problems of the Kirkwood gaps and the transport of chondritic meteorites to Earth. The third, "one of the simplest physical examples of chaotic behavior," is a prediction. Brightness variation observations of Saturn's misshapen satellite Hyperion, soon to be undertaken by an MIT group, Wisdom predicts, will demonstrate that the spin of this little moon is indeed chaotic. Not all the lesser members of the solar system, he finds, exhibit "the sense of clockwork regularity and predictabilty conveyed by the planets."



Kirkwood gaps are clearly seen in this distribution of orbital frequencies for several thousand cataloged asteroids in the asteroid belt. The fractions above the histogram indicate frequencies that correspond to simple fractions of Jupiter's 11.9 year orbital period. These commensurable periods are clearly depopulated by some sort of resonant interaction with Jupiter. The prominent "3/1 Kirkwood gap," for example, is labeled ⅓ here to indicate that its orbital period is ⅓ that of Jupiter.

The asteroid belt, lying between the orbits of Mars and Jupiter, is home to most of the asteroids in the solar system (PHYSICS TODAY, February, page 32). Several thousand asteroids with diameters in excess of a kilometer have well-determined orbits. The most prominent of these have classical names, and perhaps even moonlets of their own. Plotting the distribution of orbital periods (or semimajor axes) for the cataloged asteroids, one finds prominent gaps at four periods that are simple fractions—⅓, ⅖, ³⁄₇ and ½—of the 11.9-year orbital period of Jupiter. The commensurability of these missing periods with that of Jupiter made it clear to Kirkwood that these gaps had their origin in some sort of resonant phenomenon. Somehow, the regular gravitational tug of the Jovian giant, recurring always at the same phase for these commensurable asteroid orbits, was responsible for their vacancy. But no detailed dynamical explanation of the Kirkwood gaps was satisfactory before Wisdom, four years ago at Caltech, began applying chaos approximation techniques originally developed to study charged-particle orbits in magnetic-confinement fusion studies.

"Dynamical studies of the resonances hadn't found any mechanism for removing asteroids from the gaps," Wisdom told us. "If you make enough simplifying approximations to let you solve the three-body (Sun-asteroid-Jupiter) problem, it doesn't yield gaps." If you make the model general enough so it might be expected to produce gaps, you can't solve it. Attempts to explain the Kirkwood gaps fell into four general catagories:

▶ Statistical hypotheses, going back to the early years of this century, have suggested that the gaps are simply an illusory consequence of the statistics of

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observation. "It's like looking at a pendulum at random brief moments," Wisdom explains. One rarely sees the pendulum near the bottom of its swing, because it spends so much more time near the turning points. Similarly, it has been argued, the resonant gravitational coupling to Jupiter causes the diameters of asteroid orbits near the commensurable gap values to oscillate in pendulum-like fashion over long time periods, passing rather rapidly through the gap centers and spending most of their lives near gap margins. Thus, anyone doing an inventory would see very few asteroids at a given time in the gap regions.

After decades of popularity, Wisdom told us, the statistical hypotheses were finally laid to rest in the mid 1970s by various people who laboriously calculated the orbits of hypothetical asteroids near the gap borders and found that they do not cross the gaps. "It's not just a time-average phenomenon." With enough effort, one can numerically integrate Newton's equations for the three-body problem to obtain the orbital variation of a single asteroid over extended intervals. But deriving statistical distributions in this way is another matter.

▶ The collisional hypothesis became popular five years ago, after the demise of the statistical hypotheses. Propounded by Claude Froeschlé (Observatory of Nice) and Hans Scholl (Heidelberg), among others, it points out that asteroids with nearly commensurable orbits would have somewhat higher eccentricities than the run of the mill. Thus, it was hypothesized, their orbits would be more likely to suffer collisions by crossing the orbits of other, more circularly moving asteroids, eventually depopulating the gaps. But detailed analytic calculations have since shown that collisional effects resulting from these modestly larger eccentricities would not be nearly dramatic enough to account for the observed gaps. George Wetherill (Carnegie Institution, Washington, D. C.) has contended all along that the collisional hypothesis was "never any good." In the steady state, he pointed out, collisions create as many asteroids in the Kirkwood gaps as they destroy.

▶ Gravitational hypotheses, going all the way back to Kirkwood, suggest that asteroids drift away from the commensurable orbits under the influence of Jupiter's gravitational perturbation alone, needing no help from collisions. Analytical studies in the 1960s, applying restrictive approximations to make the problem integrable, did not find asteroids leaving the gaps. Removing these excessively restrictive approximations rendered the problem analytically intractable. "The long-term evolution of dynamical systems is in general a very difficult and unsolved problem," Wisdom points out.

▶ Because these dynamical studies have failed to generate the Kirkwood gaps, people have in recent years suggested "cosmogonic hypotheses." These suggest that, because of the resonant perturbation of Jupiter at the commensurabilities, asteroids could never have formed there in the first place. The Kirkwood gaps, they contend, have been empty since the formation of the solar system five billion years ago. "These very speculative theories seemed to me premature," Wisdom told us, "because the dynamical hypotheses had not been fully disproved. People gave up on them too soon." The numerical integrations done by Scholl and Froeschlé ten years ago covered less than 100 000 years, "which is very short compared to the age of the solar system."

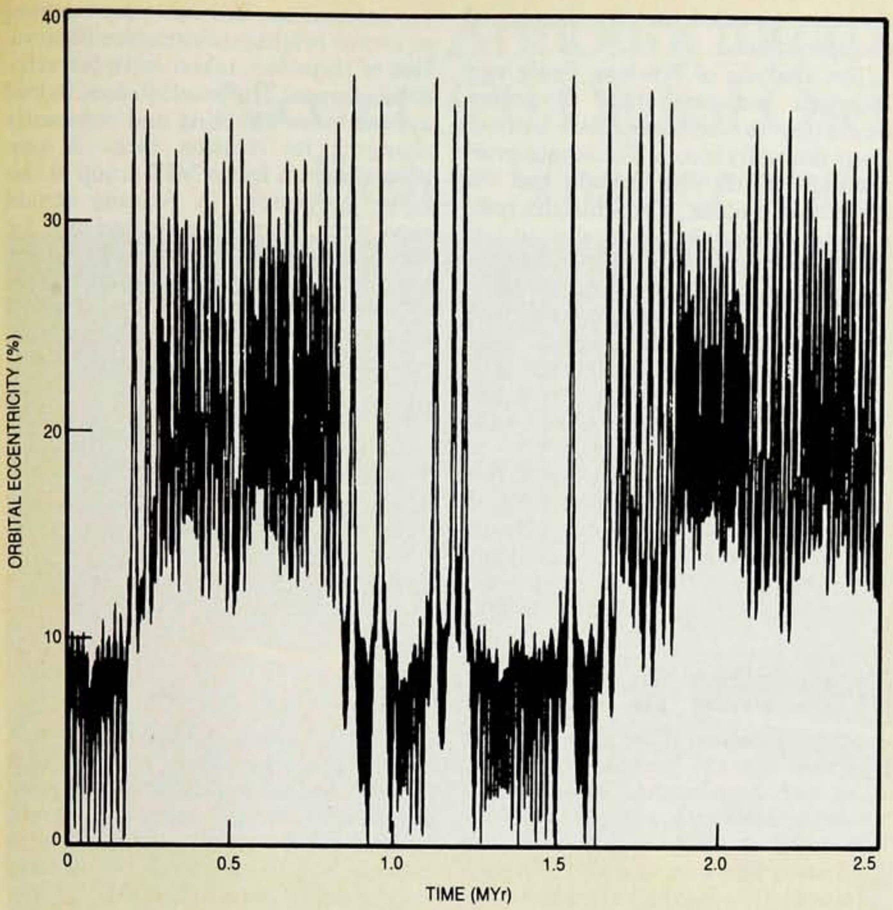
Shortcut to chaos. The explicit numerical integration of Newton's equations without radical approximations consume so much computer time that the orbits of the nine planets, including their mutual interactions, have never been explicitly calculated beyond 5×10^5 years into the past and future. Seeking a faster way to study the behavior of the asteroids over very long periods, Wisdom came across a technique developed by Boris Chirikov (Novosibirsk) in the 1970s to analyze the transition to chaos of charged-particle orbits in magnetically confined fusion plasmas. Adapting Chirikov's method to the problem of an asteroid in resonance with Jupiter, Wisdom in 1981 was able to do computer analyses[1] of the long-term behavior of the asteroid orbit *a thousand times faster* than the standard integration techniques.

Wisdom's method takes advantage of the fact that the time evolution of the asteroid orbit involves a number of very different frequency scales. The orbital period of an asteroid in the 3/1 Kirkwood gap (the one he primarily studied) is a little less than four years. The pendulum-like vibration of its major axis due to resonance with Jupiter has a period of a few hundred years. Still longer is the precession of the asteroid's perihelion—with a period of tens of thousands of years. In the study of transitions to chaos, one tries to replace the full differential equation describing the behavior of the system by an algebraic mapping that carries the system over a sequence of discrete time intervals. Chirikov had developed a rather general approximation technique for making such mappings. Wisdom applied the technique by "strobing out" the highest—least interesting—frequencies in the asteroid problem. That is to say, he looked at the system only at stroboscopic intervals corresponding to the orbital period of Jupiter. If there were no longer-term variations in the problem, the mapping point would remain fixed in phase space from one strobe time to the next. The movement of the map point describes only variations slower than the annual revolution—rather like examining the departure of a phonograph turntable from the nominal setting with a strobe disk under 60-Hz illumination. Wisdom gains a thousandfold increase in computing speed because the mapping algorithm obviates the need to integrate the differential equations over many smaller time intervals within the 12-year strobe step; and because the mapping rule is algebraic rather than differential, one has the full digital accuracy of the computer.

The very first test asteroid in the 3/1 Kirkwood gap to which Wisdom applied the mapping he had recently derived did nothing very spectacular for about a quarter of a million years (roughly ten precession periods of its perihelion). As the computer ground out the centuries, the orbital eccentricity (the Sun's distance from the geometric center of the elliptical orbit divided by the semimajor axis) oscillated quietly and regularly about a very normal value of less than 10%. But then Wisdom's patience was rewarded with a rather startling and wholly unanticipated display. After behaving itself for 2×10^5 years, the eccentricity variation became noticably more irregular and then "suddenly" shot up chaotically to fluctuations reaching 35%.

The 3/1 Kirkwood gap is about $2\frac{1}{2}$ times as far from the Sun as we are (2.5 AU). With an eccentricity exceeding 30%, the perihelion of an asteroid in the Kirkwood gap would cross the orbit of Mars (1.5 AU)! Thus Wisdom had at last found a plausible explanation for the Kirkwood gaps. Given enough time, it seems, an asteroid with a period commensurable with that of Jupiter will experience a chaotic burst of eccentricities high enough to put it in an orbit where it is likely, sooner or later, to have a close encounter with Mars. Wisdom repeated the mapping calculation with several hundred test asteroids with varied initial conditions within the 3/1 Kirkwood gap. A general pattern emerged: Long periods—on the order of 10^5 years—with regular behavior at low eccentricity were interspersed at unpredictable intervals of wild, chaotic fluctuations, with eccentricities going up to about $\frac{1}{3}$.

Meteorites. This result was quite astonishing. It was, in a sense, the collisional hypothesis come back with a vengeance. It had been known from the earlier calculations that commensurable orbits would develop somewhat higher eccentricities than the rest, but no one had anticipated anything like ellipses distorted enough to cross the



Typical chaotic variation of the eccentricity of a test asteroid in the 3/1 Kirkwood gap, as computed , in planar approximation, by Jack Wisdom's mapping technique. Over a 2.5-million-year interval, one gets chaotic alternations between epochs of low and high eccentricity. Doing the full three-dimensional problem, including nonplanar efects, one gets still higher eccentricities, large enough to cross Earth's orbit.

orbit of Mars. The Kirkwood gaps now had a plausible origin. Close encounters (not necessarily collisions) with Mars would eventually perturb these high-eccentricity orbits out of the commensurable bands. But the meteorite problem remained unsolved. To get Earth-crossing asteroid orbits—and hence the observed abundance of rocks falling from the sky—one needed eccentricities of at least 57%.

Refining and extending the original mappings to include nonplanar effects—the slow variation of the plane of Jupiter's orbit relative to the asteroid belt—Wisdom soon found that the mappings were indeed yielding chaotic bursts, with fluctuations extending up to eccentricities of 60%. But "even I don't believe the mapping when it gets that high," Wisdom confesses. "The mapping involves too many approximations that implicitly assume the eccentricity is small."

The only way to verify these supposedly Earth-crossing orbits, and indeed the chaotic behavior altogether, was to look for them in long-term integrations of the explicit differential equations. But that takes a lot of computer time—and money. "It was hard to get enough computer time," Wisdom recalls, "because people didn't really believe it." It was widely thought that these surprising chaotic outbursts were just artifacts of the mapping.

At the Royal Greenwich Observatory in 1982, Wisdom was able to verify[2] the chaotic episodes with eccentricities large enough to cross the orbit of Mars by numerical integration of Newton's equations for the simplified planar case. But the full three-dimensional treatment, which ought to yield Earth-crossing eccentricities by taking deviations from coplanarity into account, requires considerably more computer time. The appearance of these very high eccentricities eluded him at Santa Barbara. He was, however, able to verify that the outbursts met the "Lyapunov-exponent" criterion for true chaos: A dynamical system is said to be chaotic if two entities (in this case asteroids) that begin life with almost identical conditions diverge exponentially with the passage of time. In that case the Lyapunov characteristic exponent is greater than zero. This implies a dependence on initial conditions so hypersensitive that it thoroughly destroys predictability, despite the simple, deterministic equations that govern the system. After a few hundred thousand years, two asteroids that were initially traveling together in one of the Kirkwood gaps will be completely uncorrelated.

Having moved to MIT last year, Wisdom finally got enough computer time to verify[3] that, in the nonplanar case, the full numerical integration does indeed yield Earth-crossing eccentricities exceeding 57%. (Integrating the motion of a single test asteroid for the necessary 10^6 years, Wisdom told us, takes about 150 hours on a VAX 780 computer.) The case for these Earth-crossing chaotic excursions as the principal source of meteorites is strengthened by a complementary study carried out last year by Wetherill in response to Wisdom's mapping results. In 1979, Wetherill and Douglas Ravel had used photographic data from the 16 stations covering the million square kilometers of the "prairie network" to calculate the Earth-impinging orbits of ordinary chondrites, the most abundant of the stony meteorites. Assuming that Wisdom's chaotically perturbed asteroid orbits were indeed reaching to within 1 AU of the Sun, Wetherill set out to calculate what would be the observed distribution of ordinary chondrites impinging on the earth. He found that meteorites of such origin would indeed reproduce what we see. In particular, his calculations reproduced the well-known "afternoon effect". Twice as many ordinary chondrites fall in the afternoon as in the morning. This is just what one would expect if meteorites are in general fast-moving objects catching up with the revolving Earth rather than slower debris we run into. Wetherill also calculated that lower-eccentricity, Mars-crossing asteroids would not do the trick. Some of these would indeed be propelled to Earth-crossing orbits by the slingshot effect of Mars' gravity. But this mechanism, he calculated, is far too slow to account for the observed abundance of meteorites. "The most important thing about Wisdom's result," Wetherill told us, "is that it reproduces the observed orbits and abundance of the ordinary chondrites." The rarer achondritic meteorites, presumed to come from the inner margin of the asteroid belt, had presented less of a puzzle.

Hyperion. Wisdom, working with Stanton Peale (Santa Barbara) and François Mignard (CERGA, Grasse, France), believes he had uncovered yet another example of chaotic behavior in the solar system. Photographs from the 1981 encounter of Vogager II with Saturn had shown that Hyperion, one of the planet's lesser moons, was extremely aspherical: its short diameter, about 200 km, was only half its long diameter. The Voyager II photos did not, however, determine the spin state of this peculiar little satellite.

Given the very aspherical shape and highly eccentric orbit of Hyperion, Wisdom and his colleagues set out to predict its spin state. A satellite of such ungainly form is unusually susceptible to torques. Once again he availed himself of a technique developed by Chirikov to ferret out chaotic behavior in fusion plasmas—the Chirikov resonance-overlap criterion.

In the case of Hyperion, one is concerned with spin-orbit resonances. Mercury, for example, is in a pendulum-like $^3/_2$ spin-orbit resonance about the Sun; it rotates on its axis precisely three times for every two annual revolutions. A more common spin-orbit resonance is the "synchronous" spin state exhibited by our own moon. Spun down over eons by dissipative tidal effects, the Moon spins once every time it circles the Earth, always showing us the same face.

The analysis of Wisdom, Peale and Mignard indicated that Hyperion would try to oscillate about both of these mutually incompatible spin-orbit resonant states: the $^3/_2$ state and the synchronous state. The Chirikov resonance-overlap criterion applies to just such a situation. If a dynamical system is trying to make a compromise between inconsistent resonances, one gets chaos. Such chaos-analysis techniques are shortcuts. In the end they had to subject the system to a full analysis of the Newton–Euler equations.

Wisdom and his colleagues predicted that both the spin axis direction and the rotation rate of Hyperion would fluctuate chaotically—the axis tumbling wildly and the spin varying unpredictably from zero to two rotations per orbit. Since then, they have looked at recent brightness-variation observation of Hyperion, taken with terrestrial telescopes. The satellite does indeed appear to be tumbling and constantly changing its rotation rate. A new observing run by an MIT group at the Low Observatory in Arizona should soon yield more definite evidence for the chaotic spin of Hyperion. If the presumption is confirmed, they tell us, "Hyperion will be the first example of chaotic behavior among the permanent members of the solar system." —BMS

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Cornell electron–positron ring to be upgraded

In June, NSF approved a three-year, $36-million project to upgrade Cornell's CESR electron–position storage ring. The approval came as good news for Karl Berkelman, who was to become director of the Cornell Newman Laboratory a few weeks later, succeeding Boyce McDaniel, who retires from the directorship he has held since he succeeded Robert Wilson in 1967.

The half-mile-circumference CESR ring,which began operation in 1979, was designed for maximum beam energies of 8 GeV, but the machine's physics program has been largely concentrated around 5 GeV, where colliding electrons and positrons can form the four upsilon mesons, with masses ranging from 9.4 to 10.6 GeV, which have yielded a rich harvest of heavy-quark spectroscopy. These upsilon resonances are thought to be bound states of the bottom quark and its antiquark. The heaviest of these is actually above threshold for the production of "bare-bottom" mesons (PHYSICS TODAY, April 1983, page 20), making CESR the world's premiere source of mesons carrying the fifth, and heaviest, of the well-identified quarks.

The CESR upgrade project has three principal components: increasing the storage ring's luminosity, upgrading the Cleo detector system to match the resulting higher reaction rates, and improving the facility's computer capabilities. Running with seven particle bunches in each of the countercirculating beams last fall, CESR achieved luminosities of up to 1.4 events per picobarn per day. But the demands these very high beam currents placed on the machine's rf system resulted early this year in a failure of the main power cavity window. Since then, the machine has been running with fewer bunches; but the problems resulting from such high-luminosity operations are being dealt with, and the goal of the improvement program is to push the luminosity to twice the level already attained. This is to be accomplished by improving the injection system and installing rare-earth–cobalt permanent quadrupole magnets close to the interaction points to squeeze the colliding beams down to smaller cross sections. These improvements are scheduled to be completed in time for the completion of the upgraded Cleo detector—styled Cleo II.

The Cleo II phased upgrade, when it is completed in 1987, will have replaced all the original components of the Cleo detector, which is operated by a Carnegie–Mellon, Cornell, Harvard, Ohio State, Rochester, Rutgers, SUNY Albany, Syracuse, Vanderbilt collaboration. A high-resolution vertex drift chamber was installed last year. Later this year the main charged-particle tracking chamber inside the detector's solenoid magnet will be replaced by a new 51-layer cylindrical drift chamber, 2 meters long and 2 meters in diameter.

The second phase calls for the replacement of the present solenoid coil, whose magnetic field provides the curvatures by which charged-particle momenta are determined, by a three-meter-diameter superconducting magnet. Inside the solenoid will be a new scintillator time-of-flight measuring system and a cesium-iodide shower-counter array. This thirty-ton array of 8000 CsI crystals, providing extraordinarily high resolution for the measurement of electrons and photons, will be the most expensive component of Cleo II. Surrounding the solenoid will be a new magnet iron (which, in addition to serving as the magnet's return yoke, stops all charged particles except muons) and a new array of muon wire chambers. The new detector will have substantially improved spatial granularity, momentum resolution and particle-identification capacity for charged hadrons. Furthermore, its energy resolution for electrons and photons "will far exceed that of any previous general-purpose detector used in e^+e^- physics," Berkelman told us.

The greater event rate and complexity resulting from the increased machine luminosity and the detector expansion will require a fivefold increase in computing capacity at CESR. The building expansion that will make room for this additional computing power will also serve the needs of CHESS, the synchrotron radiation facility that uses CESR's x-ray synchrotron emission for biological and materials research.

McDaniel will continue to serve as chairman of the Board of Overseers for the Superconducting Super Collider. He will also continue teaching at Cornell, whose faculty he joined after leaving the Manhattan Project at the end of the war.

Berkelman took his Ph.D at Cornell in 1959 and joined the faculty in 1961. He has made major contributions to the development of the Cleo detector and its use in the elucidation of heavy-quark physics. Cleo is one of two detector systems at CESR. The other detector, CUSB, operated by a Columbia–Stony Brook collaboration, is a less general system, specialized for high-resolution measurements of electrons and photons. —BMS□