involving random sources, such as randomly distributed monopoles in the early universe or random expectation values for the vacuum in QCD, and that the connection between random fields and a reduction in dimensionality is interesting for all of these. —TVF

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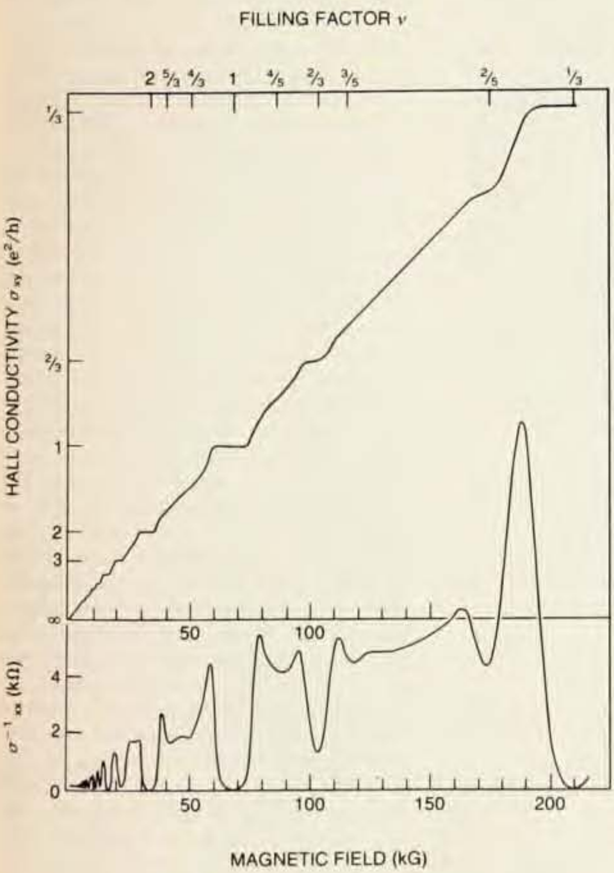
Fractional quantum Hall effect indicates novel quantum liquid

A quite unanticipated extension of the quantized Hall effect appears to have provided us a glimpse of an exotic new state of matter—a two-dimensional quantum liquid of electrons, rendered essentially incompressible by the density quantization of its ground states, whose excitations are quasiparticles of fractional electric charge.

What we must now call the "ordinary" quantized Hall effect was discovered three years ago at the Max Planck Institute in Grenoble by Klaus von Klitzing, Gerhard Dorda and Michael Pepper. They discovered that at very low temperatures the Hall conductivity of a two-dimensional electron gas subjected to an intense magnetic field exhibits quantized steps at integral multiples of e^2/h (PHYSICS TODAY, June 1981, page 17). Some such effect had been predicted five years earlier by Tsuneya Ando (University of Tokyo), but the precision of the result—the quantized conductivity steps were given by ne^2/h to better than one part in a million—caught the theorists completely by surprise.

An even greater surprise was in store for the theorists. In May of last year Daniel Tsui, Horst Störmer and Arthur Gossard at Bell Labs reported[1] that by going to still lower temperatures and higher magnetic fields one finds quantized plateaus of Hall conductivity at *fractional* multiples of e^2/h. Whereas expansions of the Ando theory since 1980 had rendered a quite satisfying account of the precision of von Klitzing's result, no one had expected to see the Hall conductivity quantized at $\frac{1}{3}e^2/h$—the first "fractionally quantized Hall effect" result announced by the Bell Labs group. "It knocked our socks off," says Robert Laughlin (Livermore), who believes he has now come up with an explanation for the new effect.



Fractionally quantized Hall effect is manifested at a temperature of 0.5 K by plateaus in Hall conductance σ_{xy} (top) and dips in the resistivity σ^{-1}_{xx} (bottom) at magnetic field intensities corresponding to electron population densities that are *fractional* multiples of the density of a full Landau level. The plateaus and dips at *integral* values of the filling factor ν, where ν Landau levels are completely filled, manifest the *ordinary* quantum Hall effect. Only at $\nu = \frac{1}{3}$ and $\frac{2}{3}$ are fractional plateaus clearly seen thus far, with an inflection point at $\frac{2}{5}$. The remaining fractional quantum Hall states appear at this temperature only as resistivity minima. At lower temperature, additional plateaus are expected, each with $\sigma_{xy} = \nu e^2/h$.

The Hall effect. When a magnetic field is imposed on a current-carrying conductor, the moving charges experience a Lorentz force perpendicular to both the magnetic field and the current. If the current is constrained to flow in the x direction with a magnetic field in the z direction, the conductance of the system is described by a symmetric 2×2 matrix, whose off-diagonal term, σ_{xy}, referred to as the Hall conductance, is the ratio of the current density to the electric field component in the transverse y direction developed by the lateral pile-up of charges due to the Lorentz force.

In the ordinary quantized Hall effect one finds a quantized sequence of plateaus in σ_{xy} at integral multiples of e^2/h as one raises the Fermi energy level of the two-dimensional electron gas in the conducting layer. These plateaus in σ_{xy} are accompanied by the near vanishing of σ^{-1}_{xx}, the usual resistivity in the current direction. That is to say, the current flows almost without resistive loss at each quantum step, with σ^{-1}_{xx} appearing to vanish entirely as the temperature goes to absolute zero.

This old-fashioned version of the quantized Hall effect is now thought to be well understood in terms of the filling of successive Landau levels in the electron gas. A two-dimensional electron gas was realized experimentally by von Klitzing in the inversion layer just below the oxide surface of a silicon MOSFET transistor. The gate electric field constrains the carrier electrons at low temperatures to move only in the x, y plane, while a strong magnetic field is imposed in the z direction.

In response to the magnetic field, the electrons execute tight little cyclotron orbits, splitting the ground state of the electron gas into a sequence of Landau levels. Each electron in the nth Landau level has an energy of $(n + \frac{1}{2})\hbar\omega_c$, where ω_c is the cyclotron frequency corresponding to the magnetic field strength B. The *maximum* (two-dimensional) population density of each Landau level, Be/hc, can be understood by requiring that each electron orbit takes up an area of πr_c^2, where $r_c = (2\hbar c/Be)^{1/2}$ is simply the classical radius of a cyclotron orbit of energy $\hbar\omega_c$. One can also think of the population density of a filled Landau level as being precisely one electron per elementary flux quantum hc/e.

Successive Landau levels are filled by raising the gate voltage and hence the Fermi level and density of the electron gas. When the nth Landau level is just filled at a temperature well below $\hbar\omega_c$, there can be no dissipative scattering of the flowing electrons because the only unoccupied states into which an electron can be scattered require an energy jump of $\hbar\omega_c$. Thus the current flow becomes essentially lossless and the Hall conductivity takes on the simple form ne^2/h, independent of B and other experimental details, until the Fermi level is raised high enough to begin filling the next Landau level.

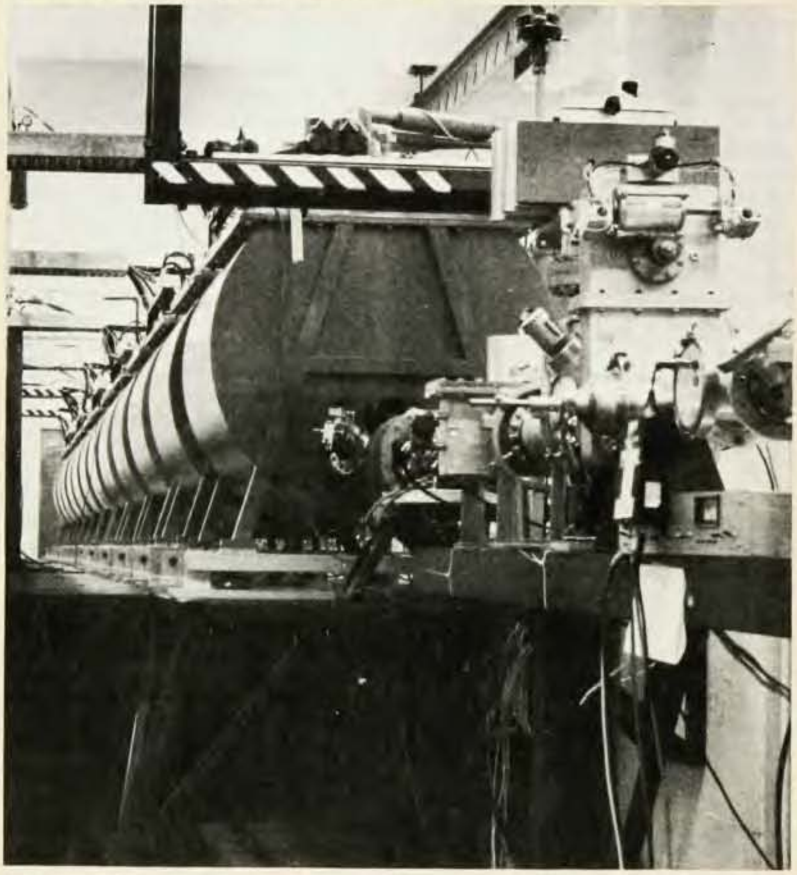
The extraordinarily high precision of this result in spite of localized impurity states has by now been explained to most everyone's satisfaction by Laughlin,[2] Ando,[3] Richard Prange[4] (University of Maryland), David Thouless[5] (University of Washington), Serge Luryi and Rudolf Kazarinov[6] (Bell Labs) and other theorists. These localized states, in fact, turn out to be *necessary* if the conductivity plateaus are to have finite, observable widths as a function of electron density.

The fractional quantum Hall effect was discovered quite by accident last year in the course of a search for a quite different phenomenon—Wigner crystallization. Eugene Wigner had predicted in 1934 that a two-dimensional electron gas would crystallize at sufficiently low density and temperature. Indeed this effect has recently been observed for electrons on liquid helium surfaces. The two-dimensional electron gases confined at transistor interfaces, however, are too dense (more than 10^{11} electrons per cm^2) for Wigner crystallization at accessible temperatures.

In the presence of a strong magnetic field, however, the situation was thought to be different. If the resulting cyclotron orbits localize the conduction electrons sufficiently—that is to say, if the Larmor radius r_c is much smaller than the mean spacing between electrons—Wigner crystallization becomes energetically plausible even at relatively high electron densities. The Wigner crystal would involve only the conduction electrons. With a periodicity on the order of 100 Å, it has nothing to do with the underlying atomic lattice.

Tsui (now at Princeton) and his Bell Labs colleagues therefore set out to look for Wigner crystallization two years ago in the same type of GaAs/AlGaAs heterostructures with which they had observed the ordinary quantum Hall effect shortly after von Klitzing's discovery. In these high-mobility semiconductor heterostructures (see PHYSICS TODAY, April 1979, page 20) the two-dimensional electron gas is confined near the interface in the GaAs layer by the Coulomb attraction of the dopant donors in the AlGaAs layer and the drop in the conduction band edge across the interface. Because the electron density is essentially fixed by this band-edge drop, in contrast to the variable gate voltage of von Klitzing's silicon MOSFET, the Bell Labs group varied the filling of the Landau levels by varying the imposed magnetic field. As B increases, the cyclotron orbits become tighter and hence the maximum allowed popula-

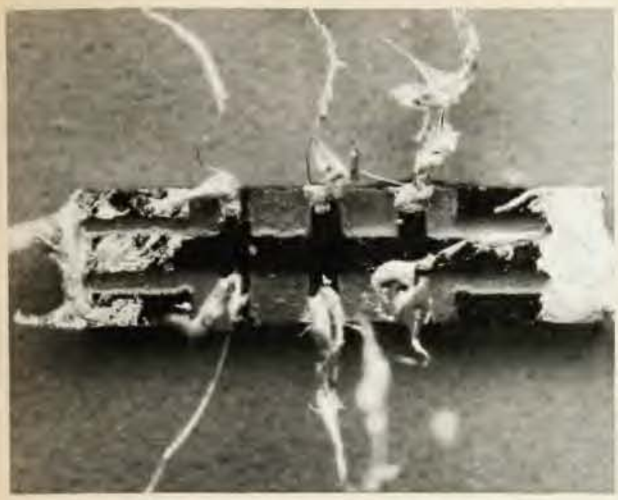
Superconducting heavy-ion linac at Stony Brook



The world's first university-based superconducting heavy-ion linac was dedicated in April at the Stony Brook campus of the State University of New York. By adding a ten-meter-long superconducting booster to Stony Brook's fifteen-year-old tandem Van de Graaff accelerator at a cost of $4.6 million, the Stony Brook group has produced a 20-megavolt linear accelerator capable of accelerating ions with mass numbers from 16 to about 100. The maximum energy per nucleon, 8.3 MeV, is attained for nickel, in the middle of this mass range. The only other comparable superconducting linac currently in operation is at Argonne National Laboratory.

Peter Paul and Gene Sprouse are the co-directors of the new Stony Brook facility. Nuclear-physics research at the Stony Brook linac will include detailed studies of heavy-ion reactions, high-spin nuclei and nuclei with neutron/proton ratios far from stability.

The booster consists of 40 computer-controlled resonator cavities contained within 12 cylindrical cryostat modules, which maintain the superconducting resonators (copper lined with superconducting lead) at liquid-helium temperature. The facility went into full operation shortly after its dedication. Similar accelerators are in various stages of planning or construction at Florida State University, Oxford, the Weizmann Institute, the Australian National University and the University of Washington.



The first observation of the fractionally quantized Hall effect was made by the Bell Labs group with this high-mobility GaAs/AlGaAs heterostructure, 7 mm long and 2 mm wide, at a temperature of about 0.5 K. The dark area contains the two-dimensional electron gas, confined in the interface between GaAs and AlGaAs layers. Current flows in the long direction while Hall voltage is measured across the structure.

tion of the Landau levels increases. When only the lowest Landau level is occupied, one describes its population density by the "filling factor" $\nu = \rho hc/Be$, the *actual* electron density ρ divided by the density of the *filled* Landau level. Thus as B is raised at fixed electron density and Fermi level, the capacity of the lowest Landau level increases, gradually reducing the filling factor toward zero.

With a magnetic field around 80 kG at a temperature of 4 K, the Bell Labs group had confirmed von Klitzing's result by observing successive conductance plateaus at filling values of $\nu = 3$, 2 and 1 as they rasied the magnetic field—that is to say, as successive higher Landau levels were emptied. Hoping to see Wigner crystallization as they depopulated the lowest Landau level below $\nu = 1$, they took their apparatus in the fall of 1981 to the Francis Bitter Magnet Lab at MIT, where they could subject it to 280 kG at a temperature of about 0.5 K.

As the filling factor was reduced below $\nu = 1$ by increasing B, they were astonished to find a *new* Hall conductance plateau at $\nu = 1/3$, with $\sigma_{xy} = 1/3\, e^2/h$ to better than one part in 10^4. Investigating this new fractional Hall quantization further in the succeeding months, they discovered additional conductance plateaus or resistivity minima[7] at $\nu = 2/3, 4/3, 5/3, 2/5, 3/5, 4/5$ and $2/7$; at each of these fractional filling values the Hall conductance takes on the value $\sigma_{xy} = \nu e^2/h$, with a simultaneous near vanishing at σ^{-1}_{xx}. None of these fractional plateaus made any sense in the context of Wigner crystallization or the ordinary quantized Hall effect.

Two experimental differences between the normal and fractional quantized Hall effects appear to be important: Whereas the fractional effect is seen only at temperatures below 1 K, the ordinary effect manifests itself at significantly higher temperatures. Furthermore, the fractional quantization was seen only in the very cleanest heterostructures grown by the Bell Labs group. It appears to require much more perfect interfaces than does the ordinary effect; for this reason, apparently, it has not been seen at all in silicon MOSFETS.

The first theoretical assaults on the new effect attempted to explain it in terms of Wigner crystallization. But this soon proved to be an unprofitable approach. One would need to understand why crystallization occurs only at specific quantized fractions of the maximum Landau densities. Hartree–Fock approximations, which calculate the wave function of each electron iteratively in the mean potential field of all the others while ignoring two-particle correlations, ought to give a good description of a crystalline electron ground state. Last fall, Daijiro Yoshioka and Patrick Lee at Bell Labs showed[8] that neither the first-order Hartree–Fock calculation nor its second-order corrections yielded any special effects at $\nu = 1/3$. They also concluded that the phenomenon could not be explained in terms of mixing of the ground state with higher Landau levels.

The calculations strengthened the prior expectation that a Wigner crystal could have a continuum of possible lattice constants; it would have no preference for a particular quantized electron density. Furthermore, the almost lossless current flow at the fractional Hall plateaus would be hard to understand in the context of a Wigner crystal. Such a moving two-dimensional solid array of electrons would almost certainly get hung up in the inevitable localized imperfections of the interface, generating peculiar transport properties inconsistent with the observed near vanishing of σ^{-1}_{xx}.

Such considerations led Laughlin, Yoshioka (now at Tokyo University), Lee (now at MIT), and Bertrand Halperin (Harvard) to investigate the possibility that the two-dimensional electron ground states responsible for the fractional quantized Hall effects are neither Wigner crystals nor any other periodic solid phase, but rather quantum-liquid states. Earlier Hartree–Fock calculations had indicated that a liquid state would have higher energy than a Wigner crystal, suggesting that the crystal would be the natural ground state. But the Hartree–Fock approach almost certainly *overestimates* the energy of liquid electron states by ignoring correlations. This is not a serious problem for Wigner crystals, where electrons are kept relatively immobile at fixed intervals. But ignoring correlations in the more anarchic liquid state lets the electrons get too close to each other too often, raising the potential energy unrealistically.

Yoshioka, Halperin and Lee have performed a numerical calculation,[9] diagonalizing the Coulomb Hamiltonian of a small number of electrons (four to six) in the lowest Landau level, confined in a two-dimensional rectangular box with periodic boundary conditions and a uniform positively charged background. They find that the ground state of this model system has significantly lower energy than a Wigner crystal, and that the two-electron correlations look much more like a liquid than a crystal. Plotted as a function of density, the energy per electron of this calculated ground state exhibits a modest downward cusp at precisely $1/3$ the density of the filled Landau level. "We've provided a calculation, *not* an explanation," Halperin told us. "We didn't have to be particularly clever to solve this small model system explicitly. But it does lend support to the notion that we're seeing something quite different from a crystalline state."

Laughlin has gone one step further. He has actually written down[10] an explicit multiparticle wavefunction that appears to account for most of the peculiar properties observed in the fractionally quantized Hall effect. It confronts us with a sequence of quantum-liquid group states "quite unlike any other condensed matter system we know of." The central feature of Laughlin's wave function is a factor $\Pi_{j<k}(z_j - z_k)^m$, a product over all pairs of electrons, where z_j is the position coordinate of the jth electron, regarded as a *complex* number. This unusual treatment of a physical interface as if it were the complex plane has the consequence that the wavefunction changes phase by $2\pi m$ as one electron is made to complete a loop around its neighbor. Thus m must be an integer; Fermi statistics further requires that this integer be odd. The case $m = 3$, for example, accounts for the Hall plateau at $\nu = 1/3$.

The two-particle correlation is found by squaring the wave function, giving for $m = 3$ a correlation probability that goes to zero as the *sixth* power of the separation between electrons. This extraordinarily stringent tendency of the electrons to avoid one another would explain the very low Coulomb energy of this state, much lower than the rival crystal state. Pinning the nodes of the wavefunction to the electron positions also accounts for its stiffness, that is to say, its incompressibility.

A fortunate feature of the wavefunction, Laughlin told us, is that its absolute square corresponds to a classical problem that has been calculated in great detail. This classical analog is a "two-dimensional" plasma of infinitely long rods (transverse to the plane) of negative charge separated by logarithmic Coulomb repulsion in a uniform positive neutralizing background. "I was able to look up lots of properties that plasma physicists had already calculated for this *unphysical* analog system." The energy per electron turns out to agree remarkably well with that calculated for the simple model system of Yoshioka, Lee and Halperin, lending strong support to the quantum-liquid hypothesis.

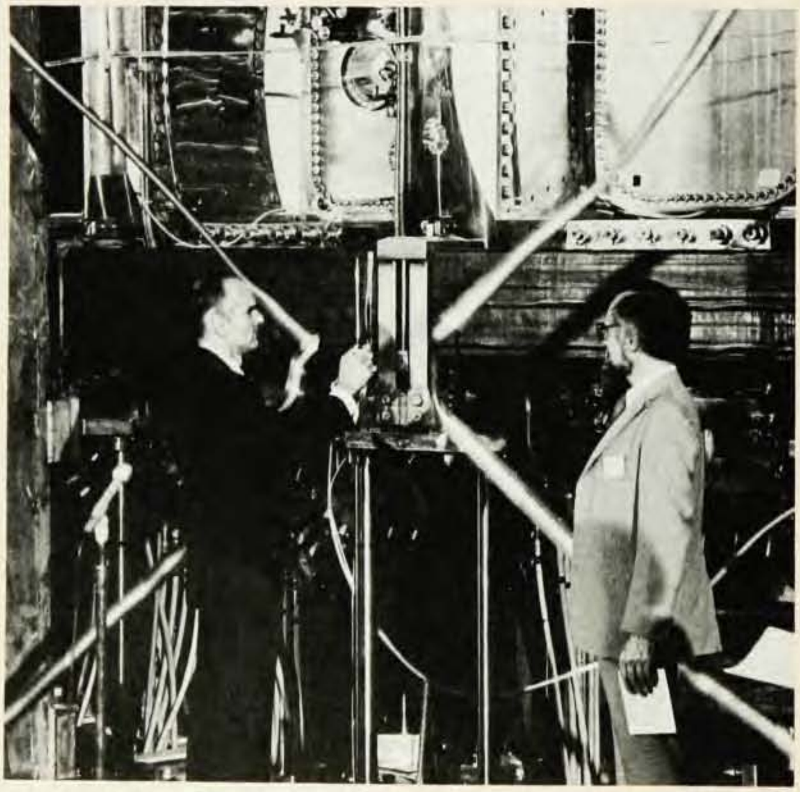
The analytic form of the wavefunction makes sense only at odd integral values of m. Thus it describes an infinite sequence of quantum-liquid ground states at the precise fractional densities $\nu = 1/3, 1/5, 1/7, \ldots$. For values of m larger than about 70, Laughlin's ground states are essentially Wigner crystal states. Arguing that the system treats electrons and hole symmetrically, one gets another sequence of fractional Hall plateaus at $\nu = 2/3$, $4/5$, $6/7 \ldots$. This does not, however, explain the plateaus observed at other fractional densities such as $2/5$. "These still present something of a problem," Laughlin admits. Halperin has recently proposed[11] a generalization of Laughlin's wavefunction that he hopes will resolve this question.

Unlike other quantum liquids, superfluid helium for example, the fractional Hall states are essentially incompressible. To raise their densities above the fixed rational fractions of the filled Landau density, Laughlin has shown, one must overcome an energy gap corresponding to about 4 K. This energy gap corresponds to the cost of generating the elementary excitations of the quantum liquid. These excitations are perhaps the most exotic consequence of Laughlin's theory. They are localized quasiparticles of fractional electric charge. The excitations of the $\nu = 1/3$ ground state, for example, are quasiparticles of charge $1/3$. Although fractional charges have previously been suggested for solitons and other periodic arrays (PHYSICS TODAY, July 1981, page 19), they have never before appeared in liquid systems. The energy cost of generating these quasiparticles explains the tendency of the liquid to remain stable at fixed density. The size of the energy gap explains why the fractional quantum Hall effect is not seen at higher temperatures.

The quantum-liquid states are incompressible, Laughlin explains, "because changing their volume is tantamount to injecting quasiparticles." The response of the system to compressive stresses is analogous to the response of a type-II superconductor to the application of a magnetic field. The system first generates Hall currents, and then at a critical stress it collapses by a single area quantum $\pi r_c^{\,2}$, nucleating a local charge accumulation of $-\nu = -1/m$. This quasiparticle, like a flux line, is surrounded by a vortex of rotating Hall current.

In constrast to superfluid helium and superconductivity, the fractional Hall states are not coherent macroscopic quantum states. They exhibit no phase transition at finite temperatures; the vanishing of σ^{-1}_{xx} and the precise quantization of σ_{xy} occur only at absolute zero. But they are quantum liquids of a very novel sort. —BMS

TFTR dedicated at Princeton



With a ceremonial ribbon cutting, Energy Secretary Donald Hodel (left) dedicated the Tokamak Fusion Test Reactor in May at the Princeton Plasma Physics Laboratory, with PPPL director Harold Furth looking on proudly. TFTR is the first of a new generation of large tokamaks that are expected to achieve plasma conditions sufficient for the demonstration of "scientific breakeven" (see PHYSICS TODAY, March 1983, page 17). The first breakeven experiments at TFTR with a deuterium-tritium plasma are scheduled for 1986. The TFTR project is headed by Don Grove. Grove and Paul Reardon directed the construction of the $314 million tokamak.

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