Random magnetic fields reduce critical dimensionality

Recent work, both experimental and theoretical, on the magnetic-ordering transition in dilute antiferromagnets has posed a new set of questions about the competition between order and disorder in physical systems. While neither theory nor experiment has yet given any conclusive answers, the discussion itself is providing some interesting examples of cross-fertilization between different areas of physics.

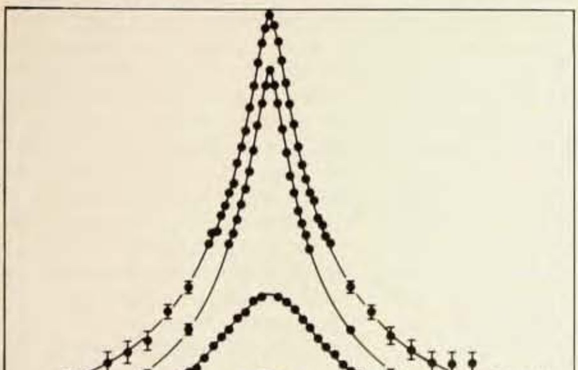
Second-order phase transitions, such as magnetic ordering, are transition from regimes in which long-range order predominates to ones in which thermal disorder predominates. What happens to such a transition when the system itself contains a random parameter?

The Ising model. One particularly simple system for investigating an order–disorder transition consists of a system of quantized spins on a lattice interacting only with their nearest neighbors; the spins have only two states. Ernst Ising investigated this model in 1925 and was able to show that a one-dimensional system exhibits no long-range order at any temperature. Subsequently, in 1936, Rudolf Peierls showed that the two- and three-dimensional Ising systems do show long-range order below a certain temperature. (If the spins have a magnetic moment, then the system is ferromagnetic and the ordering temperature is the Curie temperature. With the opposite sign of the interaction energy, the same system serves as a model of an antiferromagnet.)

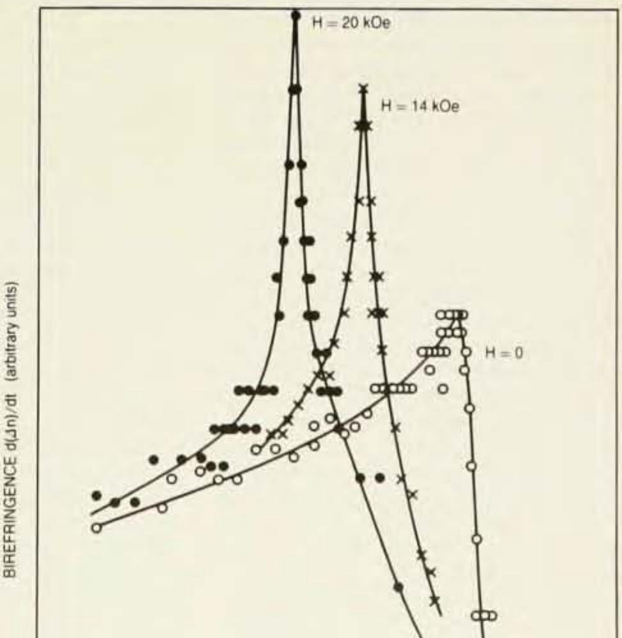
In the modern terminology in which the dimensionality of a system is also considered an analytic variable—suitable, for example, as a basis for power-series expansions about some value—the Ising system has a lower critical dimensionality of 1: For one or fewer dimensions there is no order; for more than one dimension there is order. A uniform magnetic field orders the one-dimensional system, thus lowering the critical dimension to zero. A random magnetic field applied at each lattice point, however, clearly adds a further element of disorder to that introduced by thermal fluctuations and raises the lower critical dimensionality.

Although at first such a system appears experimentally unrealizable, Shmuel Fishman and Amnon Aharony showed[1] that a randomly diluted Ising *anti*ferromagnet subjected to a uniform external magnetic field behaves like an ordinary Ising system with a random field. Such site-diluted magnetic materials can be prepared, and one can then study the magnetic-ordering transition as a function both of the external field and of the dilution of the magnetic species.

Neutron scattering as a function of wavevector in the $[1,\zeta,0]$ direction in the dilute Ising antiferromagnet $Co_{0.35}Zn_{0.65}F_2$ at a magnetic field of 3.5 T. The solid lines are fitted Lorentzian lines above 10 K and Lorentzian plus Lorentzian squared below. At 4 K the line is almost pure Lorentzian squared, with a correlation length of 27 lattice constants. (From reference 4.)



Specific heat, as measured by the change with temperature of the optical birefringence Δn, in the dilute Ising antiferromagnet $Fe_{0.6}Zn_{0.4}F_2$ at various applied magnetic fields. The lines are guides for the eye only. The persistence of the cusps at nonzero magnetic field indicates that the three-dimensional system still undergoes a second-order phase transition. (From reference 5.)



The fluorides and chlorides of transition metals such as Zn, Mg, Co and Fe provide excellent materials for such studies. Because the divalent ions are chemically very similar but magnetically very different, one can grow uniform crystals in which magnetic and nonmagnetic ions are distributed at random over the lattice sites. Thus, $Co_xZn_{1-x}F_2$ and $Fe_xZn_{1-x}F_2$ are examples of three-dimensional Ising antiferromagnets, and $Rb_2Co_xMg_{1-x}F_4$ is a two-dimensional Ising antiferromagnet. The nonmagnetic zinc or magnesium provide the random dilution.

Experimental work. Over the past several years a number of experimenters has collaborated—in different laboratories and in several groups—on experimental studies of phase transitions in diluted Ising antiferromagnets. They include Robert J. Birgeneau (MIT), David P. Belanger (University of California, Santa Barbara), Roger A. Cowley (Edinburgh), Howard J. Guggenheim (Bell Labs), Mark Hagen (Edinburgh), Hiro Ikeda (Tokyo), Vincent Jaccarino (Santa Barbara), Alan R. King (Santa Barbara), S. K. Satija (Brookhaven), and Gen Shirane (Brookhaven). To examine the ordering of the crystals, they have performed neutron-scattering studies (at Brookhaven) and examined the optical birefringence of the crystals (at Santa Barbara). Neutrons, because they are scattered by the magnetic moments in the crystal, provide a direct probe of the magnetic order. The rate at which the birefringence Δn (the difference in indices of refraction) changes with temperature near a transition is proportional to the magnetic contribution to the specific heat, so that a phase transition shows up as a spike in the slope of the birefringence-vs.-temperature graph, $d(\Delta n)/dT$.

The early results of the neutron-scattering studies showed[2] that the random field destroys the order in both the two- and three-dimensional cases. In fact, Birgeneau told us, the three-dimensional random-field system appeared to exhibit a number of features expected if the effective dimensionality shifted by 2. Meanwhile, the birefringence measurements[3] of three-dimensional systems showed clear spikes in $d(\Delta n)/dT$, even in the presence of an external field, indicating that there is an order–disorder transition even with a random field.

The most recent,[4] as yet unpublished, results of the neutron-scattering work, using $Co_xZn_{1-x}F_2$, indicate a complex state of affairs, Birgeneau told us. For example, the neutron scattering observed in a sample at 1.85 K and zero magnetic field differs depending on whether it was cooled first and the field removed later or was cooled in zero field. It is thus not clear, Birgeneau said, if the usual procedure, which involves cooling the sample in the field, produces the true ground state of the system. Because the scattering in the field-quenched state is constant over periods of several days, however, some kind of equilibrium appears to be reached. The inverse correlation length of the spins as obtained from these neutron-scattering studies, appears to vary as a power of the magnetic field. H. Kogon and David Wallace (Edinburgh) had earlier pointed out[5] that a shift in the effective dimensionality by 2 should manifest itself experimentally as a change in the scattering line shape from Lorentzian to Lorentzian squared. This effect is indeed what the neutron scattering data show, Birgeneau said.

The latest optical birefringence measurements confirm the earlier results[6] from similar experiments. The observed behavior, Jaccarino told us, is consistent with a systematic logarithmic divergence in the specific heat, which is what is predicted and observed for the nonrandom two-dimensional Ising model. The sharp cusp indicates that the random field does not destroy the long-range order at low temperatures, Jaccarino said. Both he and Birgeneau, however, pointed out that the two methods measure different kinds of order, and Birgeneau suggested that the system may at low temperatures exhibit some kind of order other than the antiferromagnetic order seen in pure systems. The Fishman–Aharony calculation indicates that the randon system should exhibit a new kind of order and a lower critical dimension, Jaccarino added. The specific-heat measurements, he said, indicate that the critical exponents change with the application of the random field in just the way predicted by theory.

Theoretical work. The first investigation of random-field effect on an Ising model is a 1975 paper[7] by Yoseph Imry (IBM) and Shang-Keng Ma (University of California, San Diego). By arguing that the energy gained by forming small, randomly oriented domains in the random field must compete with the energy lost at the boundaries of the domains (because the spins are not aligned across the boundary), Imry and Ma concluded that one- and two-dimensional systems should become disordered in a random field, while the three-dimensional system remains ordered. The lower critical dimension is thus 2.

There have been several attempts to improve the heuristic argument of Imry and Ma. Erling Pytte, David Mukamel and Imry (all at IBM) argued[8] that a roughening of the domain walls adds to the disorder of the system, so that the lower critical dimension ought to be 3, not 2. Geoffrey Grinstein (also at IBM) and Ma used a renormalization-group calculation to show[9] that the domain walls are smooth enough not to affect the critical dimension given by the Imry-Ma argument. Jacques Villain (Grenoble) has independently arrived at the same result. Grinstein told us that the situation is not settled, and that although he believes that the lower critical dimension is 2, that is a belief and not a theorem.

Another approach,[10] based on work by Grinstein and by Aharony, Imry and Ma, uses renormalization-group arguments and perturbation expansions about the upper critical dimensionality of 6, where the series converges. Georgio Parisi (Frascati) and Nicolas Sourlas (École Normale Supérieure) used an elegant argument, based on supersymmetry, to show[11] that order-by-order the terms of the d-dimensional random field problem are equivalent to the zero-field terms in $d-2$ dimensions. (A supersymmetry in elementary-particle physics refers to a symmetry under which bosons and fermions can be equivalent.) According to this argument, then, the lower critical dimensionality should be 3. Grinstein told us that the perturbation theory works well near 6 dimensions, but that the convergence of the series for fewer than 4 dimensions (and hence the validity of the result) is questionable.

Last year John Cardy (University of California, Santa Barbara) generalized[12] the argument of Parisi and Sourlas to avoid explicit use of the perturbation theory and arrived at the same result. Antti Niemi independently and somewhat later came up with the same conclusions.[12] Cardy told us that the domain walls responsible for discovering the finite-size domains in the ordinary one-dimensional Ising model carry over (under the supersymmetry argument) to a new kind of localized domain wall in the three-dimensional random-field model. However, Cardy said, the equivalence of the random-field Hamiltonian and the supersymmetric Hamiltonian cannot be shown independently of perturbation theory, so that the fundamental question about the validity of the three-dimensional result remains unanswered.

Prospects. Because the arguments from supersymmetry are very interesting in elementary particle physics, a resolution of the theoretical and experimental situation for the Ising model may in turn affect work in quantum field theory, Cardy told us. An analogous situation may hold, with the ordinary four-dimensional quantum field theory being equivalent to a six-dimensional classical theory with random sources, Cardy said. Roman Jackiw, a particle theorist at MIT and Niemi's thesis adviser, says that there are many potentially interesting ideas involving random sources, such as randomly distributed monopoles in the early universe or random expectation values for the vacuum in QCD, and that the connection between random fields and a reduction in dimensionality is interesting for all of these. —TVF

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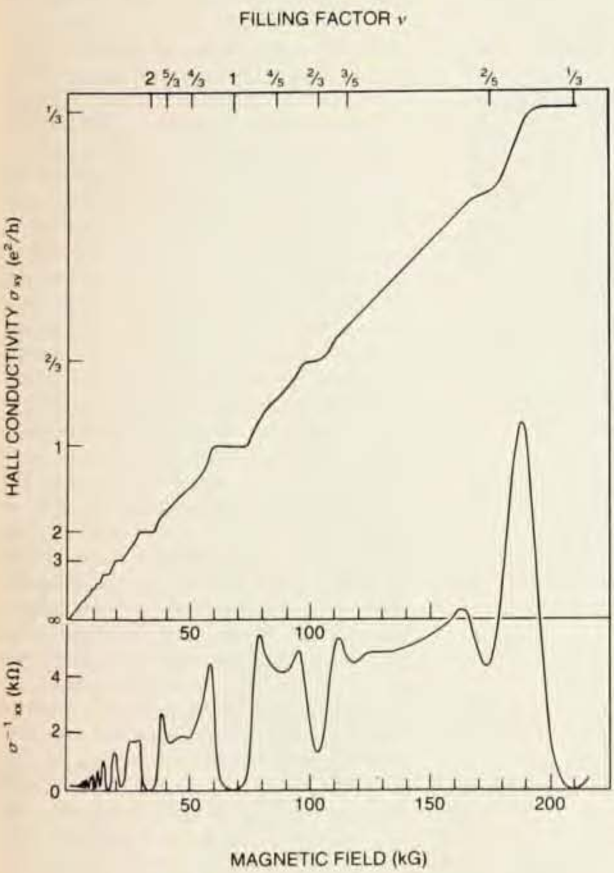
Fractional quantum Hall effect indicates novel quantum liquid

A quite unanticipated extension of the quantized Hall effect appears to have provided us a glimpse of an exotic new state of matter—a two-dimensional quantum liquid of electrons, rendered essentially incompressible by the density quantization of its ground states, whose excitations are quasiparticles of fractional electric charge.

What we must now call the "ordinary" quantized Hall effect was discovered three years ago at the Max Planck Institute in Grenoble by Klaus von Klitzing, Gerhard Dorda and Michael Pepper. They discovered that at very low temperatures the Hall conductivity of a two-dimensional electron gas subjected to an intense magnetic field exhibits quantized steps at integral multiples of e^2/h (PHYSICS TODAY, June 1981, page 17). Some such effect had been predicted five years earlier by Tsuneya Ando (University of Tokyo), but the precision of the result—the quantized conductivity steps were given by ne^2/h to better than one part in a million—caught the theorists completely by surprise.

An even greater surprise was in store for the theorists. In May of last year Daniel Tsui, Horst Störmer and Arthur Gossard at Bell Labs reported[1] that by going to still lower temperatures and higher magnetic fields one finds quantized plateaus of Hall conductivity at *fractional* multiples of e^2/h. Whereas expansions of the Ando theory since 1980 had rendered a quite satisfying account of the precision of von Klitzing's result, no one had expected to see the Hall conductivity quantized at $\frac{1}{3}e^2/h$—the first "fractionally quantized Hall effect" result announced by the Bell Labs group. "It knocked our socks off," says Robert Laughlin (Livermore), who believes he has now come up with an explanation for the new effect.



Fractionally quantized Hall effect is manifested at a temperature of 0.5 K by plateaus in Hall conductance σ_{xy} (top) and dips in the resistivity $\sigma^{-1}{}_{xx}$ (bottom) at magnetic field intensities corresponding to electron population densities that are *fractional* multiples of the density of a full Landau level. The plateaus and dips at *integral* values of the filling factor ν, where ν Landau levels are completely filled, manifest the *ordinary* quantum Hall effect. Only at $\nu = \frac{1}{3}$ and $\frac{2}{3}$ are fractional plateaus clearly seen thus far, with an inflection point at $\frac{2}{5}$. The remaining fractional quantum Hall states appear at this temperature only as resistivity minima. At lower temperature, additional plateaus are expected, each with $\sigma_{xy} = \nu e^2/h$.

The Hall effect. When a magnetic field is imposed on a current-carrying conductor, the moving charges experience a Lorentz force perpendicular to both the magnetic field and the current. If the current is constrained to flow in the x direction with a magnetic field in the z direction, the conductance of the system is described by a symmetric 2×2 matrix, whose off-diagonal term, σ_{xy}, referred to as the Hall conductance, is the ratio of the current density to the electric field component in the transverse y direction developed by the lateral pile-up of charges due to the Lorentz force.

In the ordinary quantized Hall effect one finds a quantized sequence of plateaus in σ_{xy} at integral multiples of e^2/h as one raises the Fermi energy level of the two-dimensional electron gas in the conducting layer. These plateaus in σ_{xy} are accompanied by the near vanishing of $\sigma^{-1}{}_{xx}$, the usual resistivity in the current direction. That is to say, the current flows almost without resistive loss at each quantum step, with $\sigma^{-1}{}_{xx}$ appearing to vanish entirely as the temperature goes to absolute zero.

This old-fashioned version of the quantized Hall effect is now thought to be well understood in terms of the filling of successive Landau levels in the electron gas. A two-dimensional electron gas was realized experimentally by von Klitzing in the inversion layer just below the oxide surface of a silicon MOSFET transistor. The gate electric field constrains the carrier electrons at low temperatures to move only in the x, y plane, while a strong magnetic field is imposed in the z direction.