The structure of the nucleon

The nucleon may be composed of a cloud of mesons surrounding and squeezing a smaller core of quarks.

Gerald E. Brown and Mannque Rho

We have known for years that the nucleon must have a finite size. In the 1950s, with the development of quantitative calculational techniques in quantum electrodynamics, there were many attempts to describe the size of the nucleon, but none was successful. The advent of quark physics and the demonstration through high-energy deep inelastic scattering of electrons by nucleons that there are three objects in the central nucleon core, and that these objects behave at high energies as if they are free and massless, gave impetus to a new description of nucleonic structure. Ever since Hendrik A. Lorentz's work on the theory of the electron, we have been trying to give elementary particles finite sizes to make their self energy finite. Whereas Lorentz introduced rods to hold his extended electron together (the method did not work), we now believe that the vacuum exerts a pressure on the "bubbles," or "bags," that we have to make to allow quarks to exist, and that this pressure keeps the bubbles from expanding. As the drawings in figure 1 and on the cover of this issue indicate, we can think of the nucleon as three colored quarks in such a bubble, surrounded by a cloud of mesons.

Our current thinking includes a number of points that simplify this description:

▶ The excitation spectrum of the nucleon is much simpler than that of a three-body system of nucleons because, as quarks cannot separate to infinite distances, the nucleon has no continuum, while the separable three-body system is greatly complicated by a continuum.

▶ The residual forces that split nucleonic states have complete analogs in atomic physics, with the color coupling constant α_s replacing the fine-structure constant α.

▶ The quark picture, especially with the notion of "asymptotic freedom," may give us a mechanism for handling the nucleon–nucleon force at short distances.

In this article we will describe the implications that our modern picture has for nucleonic structure and for the behavior of the nucleon in the nucleus. We will find that the "bag" model of the nucleon, in which quarks are confined by fiat, is an attractive theory that allows us to carry out many calculations. And we will see how symmetry considerations and conservation laws suggest that a pion cloud surrounds the confinement region, and how this cloud holds the size of the nucleon's central core of quarks to about 0.5 fm. The existence of this core requires us to modify the conventional boson-exchange model of nucleon interactions at short distances. How this is to be done is one of the leading concerns of nuclear physics.

How large is the nucleon?

One might think that the best indicator of the size of the nucleon would be its electric form factor, which describes the spatial distribution of charge in the nucleon and is determined by elastic electron scattering. With this approach we have determined accurately the sizes of atoms and of nuclei. In one of the current pictures that we will describe, however, the nucleon has two regions. There is a core region in which nearly massless quarks move almost freely, with only weak interactions between them—a situation known as asymptotic freedom (see the glossary on page 32)—and there is a larger external region, in which pions and other mesons exist, which we will call the meson cloud.

A number of fundamental approaches, such as lattice gauge theories (see PHYSICS TODAY, July 1982, page 19), are now beginning to sort out how and where the quarks, gluons and other entities figure in forming the domain, called the "bag," in which quarks are confined. At present these calculations can make no definite statement as to how the nucleon gets delineated into the two regions of inner quark core and outer meson cloud. But there are strong indications that a nearly perfect symmetry, called "chiral symmetry," plays a crucial role in such a division. We will come back to the details of this later on, but because we will need to use some of the notations here, let us now look at the salient features of chiral symmetry.

If we ignore quark masses in the modern theory of strong interactions, quantum chromodynamics—a particularly good approximation for the up (u) and down (d) quarks with which we are mainly concerned in nuclear physics—then there exists an invariance that stems from the fact that the equation of motion of the quark conserves the quark's helicity. This is the so-called chiral $SU(2)\times SU(2)$ invariance. If the vacuum respected this symmetry, then no interactions could ruin the invariance and we would have the usual multiplet structure frequently observed in nature, in this case massless u and d quarks. This is usually referred to as the Wigner–Weyl mode, or the Wigner mode for short.

Suppose now that the vacuum is unstable against the condensation of quark–antiquark pairs. A condensed pair must, like the vacuum, have no angular or linear momentum, and must necessarily fail to conserve helicity. When condensation occurs, the vacuum picks up a preferential direction and the quarks become massive. The $SU(2)\times SU(2)$ symmetry breaks down to an isospin group SU(2), with each broken group generator producing a massless particle known as a Goldstone boson. When we ignore their small masses, the familiar pions—π^+, π^-, π°—are such bosons. This mode of symmetry breaking, which is believed to be realized in nature, is referred to as the Nambu–Goldstone mode, or the Goldstone mode for short. The symmetry is said to be spontaneously broken.

It is natural to postulate, at least at a semiclassical level, that there is an interface between regions of the nucleon in which chiral symmetry is realized in different modes: the interior or Wigner mode where the confined

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quarks are nearly massless, and the exterior Goldstone mode where the mesons exist. Although we talk in terms of the Goldstone mode, this is not to say that the mesons are not composed of quarks and antiquarks. It is, however, convenient to use meson names to discuss the quark–antiquark ($q\bar{q}$) clusters, just as we've talked about α-particle clusters in the surface region of nuclei for many years. There are, however, two fundamental differences from the nuclear picture.

- ▶ Pions can only be present when chiral symmetry is realized in a mode different from that in which it is realized in the center of the nucleon
- ▶ Pions cannot be split into quarks and antiquarks, whereas α particles can be split into nucleons.

Because we cannot get very far toward delineating the two regions of the nucleon by long-wavelength electromagnetic probes, let's look at the nucleon's excitation spectrum. As we indicated earlier, the excited states of the nucleon with three quarks make up a much simplier picture than do the excited states of a nuclear three-body system such as H^3 or He^3. In the latter case, the continuum complicates the picture considerably; as soon as one of the nucleons has enough energy to form an excited state of the nucleon, it lies in the continuum and its wave function stretches to infinity. In the three-quark case, the quarks are confined, and a single quark cannot move very far from the other two. Thus, the excitation spectrum of the nucleon is exceedingly simple, as one can see from figure 2, which shows the ground state, the first excited even-parity state (known as the "Roper resonance"), and the odd-parity states. Other excitations begin at above 1700 MeV.

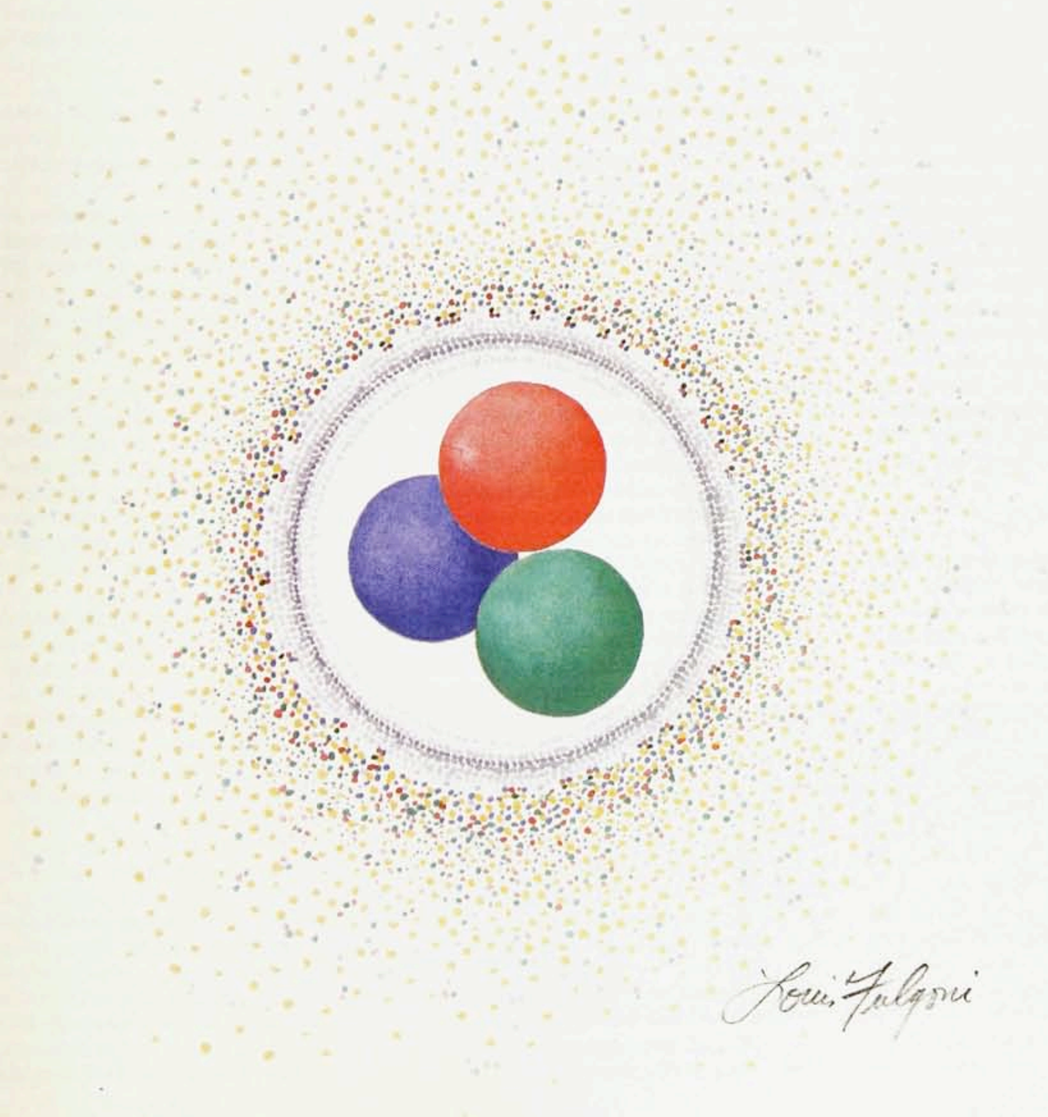
Harmonic oscillator potential. The degrees of freedom in the odd-parity excitations can be understood in a very simple way. Think, as many people have,[1] of the quarks being in an oscillator potential, as shown in figure 3. There will be five odd-parity excited states corresponding to those shown in figure 3b: The spins of the three quarks can be coupled to give a total spin S_{tot} of $\frac{1}{2}$ or $\frac{3}{2}$, and these in turn can be coupled to the orbital angular momentum L of 1 to yield a total angular momentum J as follows.

$$J = \tfrac{1}{2},\ \tfrac{3}{2}\ (S_{\text{tot}} = \tfrac{1}{2})$$
$$J = \tfrac{1}{2},\ \tfrac{3}{2},\ \tfrac{5}{2}\ (S_{\text{tot}} = \tfrac{3}{2})$$

These would be degenerate in the harmonic oscillator, but, as we shall see, gluon exchange produces a spin–spin interaction

$$\delta H = \sum_{i \neq j} f(r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \qquad (1)$$

where f is a positive function of r_{ij}, so it is repulsive in states with parallel spins. The states with $S_{\text{tot}} = \frac{3}{2}$ are thus moved up in energy relative to those with $S_{\text{tot}} = \frac{1}{2}$ and this is the pattern one sees in figure 2; the triplet of states corresponding to $S_{\text{tot}} = \frac{3}{2}$ lies above the $S_{\text{tot}} = \frac{1}{2}$ doublet. The reason the states in the triplet and those in the doublet are not degenerate among themselves is that there are interactions in addition to the spin–spin interaction. We will discuss these later. But at this point we see that the five



The nucleon. This artist's conception shows 3 quarks in a spherical "bag" surrounded by a cloud of mesons. The quarks themselves should be thought of as pointlike particles; their finite extent in the picture may be taken as an indication of their probability distributions. (Painting by Louis Fulgoni.) Figure 1 lowest odd-parity states can be labeled by the five values of J listed above.

The centroid, or weighted average, of odd-parity levels is at about 1600 MeV, with the ground state of the nucleon at 940 MeV. The energy of a nucleon with $S_{\rm tot} = \frac{1}{2}$ will be pushed down by the same spin–spin interaction (equation 1) that occurs in the excited states, so that the unperturbed energy is closer to 1000 MeV. We identify $\hbar\omega$ of the oscillator with the difference in energy between the ground and excited states: 1600 MeV − 1000 MeV = 600 MeV. (see figure 2) Now, in the harmonic oscillator, $\hbar\omega$ is related to the radius by

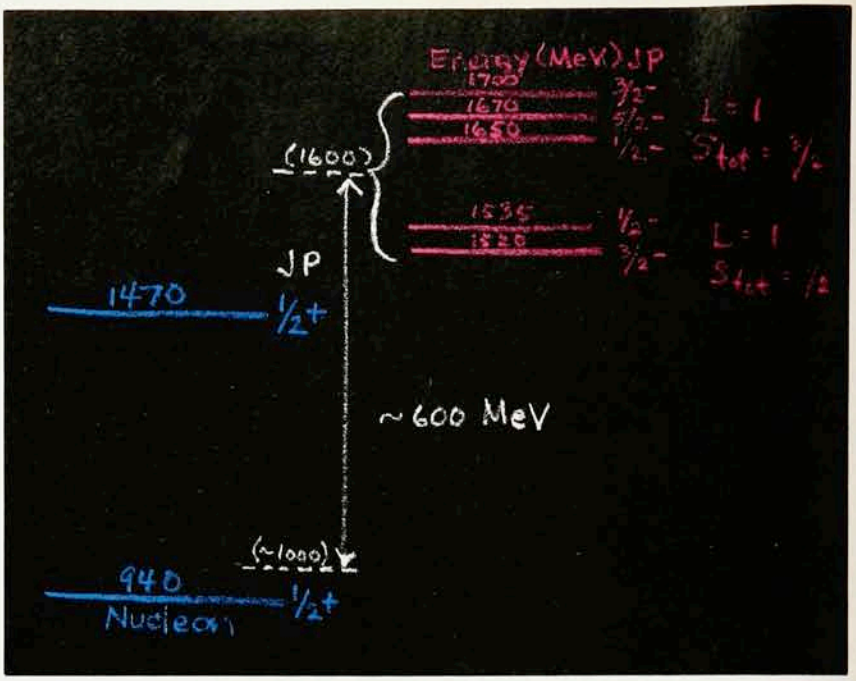
$$\hbar^2/m_{\rm Q}a^2 = \hbar\omega$$

where $m_{\rm Q}$ is the quark mass and a is the radius parameter. The question is, "What is the quark mass $m_{\rm Q}$?" The answer to this question isn't known, and possibly must await solution of the problem of quark confinement, although we shall make a suggestion later. We can say, however, from measurement of various properties such as magnetic moments, that the quarks in the nucleon act for some purposes as if they have masses about $\frac{1}{3}$ that of the nucleon, $m_{\rm n}$. This has a deceptively simple appearance; the most natural assumption, if three constituents make up a nucleon, is that their masses, aside from binding energies, should add up to the mass of the nucleon. Such a model with massive quarks is called the "constituent quark model" and may be viewed as something akin to the quasiparticle picture in many-body systems. The $m_{\rm Q}$ cannot, however, be masses in the usual sense; we shall give an example later of how such masses might come about.

With a quark mass $\frac{1}{3}$ that of the nucleon, the above equation yields a radius parameter a of 0.45 fm. The mean-square radius of the ground state is given by

$$\langle r^2\rangle_{\rm nucleon} = \tfrac{3}{2}\, a^2$$

But this contains a contribution from spurious center-of-mass motion. Removing this spurious motion reduces the number of degrees of freedom from 3 to 2. Thus we multiply the above result by $\frac{2}{3}$ and obtain an rms ground-state radius of a, or 0.45 fm. In a similar way, we arrive at 0.50 fm for the rms radius of odd-parity excited states, which must be somewhat larger in extent because they lie higher in the potential well.



States of the nucleon. The excitation spectrum of the nucleon, a system of 3 quarks, is much simpler than that of a 3-body system of nucleons. As quarks cannot separate to infinite distances, the nucleon has no continuum spectrum. The odd-parity states, shown on the right, result from promoting a quark from the 1s to the 1p level. Figure 2

Effects of gluon exchange

We wish now to discuss effects on the nucleon of gluon exchange between quarks. The work[2] of Alvaro De Rujula, Howard Georgi and Sheldon Glashow of Harvard University brings out the similarity between the gluon exchange interaction and photon exchange in atomic physics, and is a good starting point for our discussion. We consider only the lowest-order coupling of quarks and gluons, because at short distances asymptotic freedom pertains, and the coupling is weak. This, of course, sidesteps the fundamental problem of confinement, which is related to the behavior of interactions at larger distances. We simply assume that the quarks are confined, and we discuss their short-range interactions perturbatively. To lowest order, the coupling between quarks and gluons is given by a Lagrangian analogous to that for the coupling of photons to electrons, except that matrices for color degrees of freedom are included in the quark–gluon coupling. The quark–gluon coupling constant g replaces e, the electric charge.

Figure 4 illustrates the interaction resulting from the exchange of a virtual gluon. The interaction from gluon exchange has precisely the same form as the electromagnetic interaction between two relativistic particles, but with the square of the charge replaced by $-\frac{2}{3}g^2$; the sign of the interaction is that between a positive and a negative charge. Thus, the signs of the various effects will be those of the analogous electromagnetic interactions in the hydrogen atom, although the strength of interaction will be quite different because g^2, although small, is much larger than the electromagnetic e^2. The color coupling constant $\alpha_{\rm s}$, which is defined as $g^2/4\pi\hbar c$, is actually a function of momentum. When we talk about its magnitude, we mean its size for the momenta encountered inside the nucleon.

As in the hydrogen atom, one has in the baryon spin–spin, spin–orbit and tensor interactions. The spin–spin interaction in the baryon,

$$V_\sigma = (8\pi/3)(\tfrac{2}{3}\alpha_{\rm s})(\hbar/m_1c)(\hbar/m_2c) \times \mathbf{s}_1\cdot\mathbf{s}_2\,\delta(\mathbf{r}_{12}) \tag{2}$$

is known from the hydrogen atom as the Fermi–Breit interaction. (With m_1 and m_2 identified with the electron and proton masses, respectively, this gives the hyperfine splitting in the hydrogen atom.) For the moment, we will take the quark masses m_1 and m_2 to be those discussed earlier, namely $m_i \approx \frac{1}{3}m_{\rm n}$ for the nonstrange quarks.

The nucleon is made of three s-state quarks coupled to give $S_{\rm tot} = \frac{1}{2}$; the Δ(1230 MeV)-isobar is made of three s-state quarks coupled to give $S_{\rm tot} = \frac{3}{2}$. To lowest order, the spatial dependence of all quark wave functions is the same, so that the radial dependence of V_σ can be integrated out, and, for the nucleon or isobar, one has

$$V_\sigma = C\sum_{i\neq j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j \tag{3}$$

where C summarizes the constants and radial integrals. For the three-body systems, the sum is easily evaluated; it is -6 for the nucleon and $+6$ for the isobar. The $\Delta(1230)$ has an energy 300 MeV greater than that of the nucleon, so that C is 25 MeV if one attributes all of the splitting to the one-gluon-exchange spin–spin interaction. The tensor interaction will not contribute here, because it averages out in the spherically symmetric spatial states (although there have been suggestions that, as in the deuteron, there are substantial D-state admixtures in the nucleon and isobar), and the spin–orbit interaction does not appear because $L=0$ in these states.

Researchers have applied[1] these simple ideas with considerable success to describe the spectrum of odd-parity excited states of the nucleon. As figure 2 illustrates, $J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ triplet of states corresponding to an S_{tot} of $\frac{3}{2}$ lies higher than the doublet $J=\frac{1}{2}, \frac{3}{2}$ for which S_{tot} is $\frac{1}{2}$. With harmonic-oscillator wave functions, the interaction described by equation 2 would give the splitting of centroids of these states to be just half the Δ(1230)–nucleon splitting; that is, the triplet would be about 150 MeV above the doublet. This corresponds well to the actual situation.

Nathan Isgur of the University of Toronto and Gabriel Karl of the University of Guelph, Ontario, find[1] the mixing of states produced by the tensor interaction resulting from gluon exchange to be helpful in explaining certain observed phenomena. For example, without this small mixing, the upper triplet of states would be purely of total spin $\frac{3}{2}$, and none of these states could decay to the nucleon by electromagnetic dipole transitions, because $S_{tot}=\frac{1}{2}$ for the nucleon, and the dipole operator contains no spin dependence.

The situation is different with the spin–orbit interaction, which we know and love from atomic physics. Literal application of this interaction would absolutely ruin any agreement between experimental and theoretical spectra; matrix elements of this interaction in the states of figure 2 are several hundred MeV. Isgur and Karl[1] neglect the spin–orbit terms and hope that this interaction will go away. There is some support for this hope in charmonium calculations, where a spin–orbit interaction of opposite sign arises from the confinement potential, and this cancels to a good degree the two-body spin–orbit interaction from gluon exchange. Indeed, there are also definite indications[3] of such a cancellation in systems with light quarks, although calculations are much more difficult and complex here. In any case, nature tells us that we do not require much of a spin–orbit force to explain the spectrum of the nucleon, and it is one of our tasks to understand why. (The spin–orbit force cannot, however, be entirely absent because it is the only one of the above mechanisms that can split the $\Delta^*_{1/2^-}$ and $\Delta^*_{3/2^-}$ states around 1700 MeV, which is necessary because these two states are at somewhat different energies experimentally.)

We can understand the general features of the spectroscopy of mesons and strange baryons along similar lines. Gluon exchange, treated perturbatively, gives a simple and unifying first-order picture.

Magnetic moments

Initially, the anomalous magnetic moment of the nucleon was thought to come from the meson cloud. With the advent of the quark model, all of the magnetic moment was attributed to quarks. Now, as we will see, there are indications that the situation is about halfway between these two pictures.

The quark model received a great impetus with the realization that it gives a value of $-\frac{3}{2}$, compared with the empirical -1.46, for the ratio of the magnetic moment of the proton to that of the neutron. This development appeared in a paper[4] on SU(6), but only four types of quarks (up and down, each with two spins) are involved in the nucleons, so the magnetic-moment calculation actually concerned SU(4).

The proton has two up quarks and one down quark; the neutron has one up and two down quarks. Thus each nucleon has two quarks of the same flavor. Because the color wavefunction is totally antisymmetric, the flavor wave function must be symmetric. Thus, the two like quarks must be in a spin-one state. One can then use vector addition to add the magnetic moments of these two like quarks and that of the third quark to produce the nucleonic magnetic moment:

$$\mu_{tot} = \tfrac{4}{3}\mu_a - \tfrac{1}{3}\mu_b \qquad (4)$$

where μ_a is the magnetic moment of either of the two like quarks, and μ_b is that of the third quark.

In the model discussed so far, the up and down quark moments are

$$\mu_{u,d} = \tfrac{1}{2}(\tau_3 + \tfrac{1}{3})(m_n/m_Q)$$

in units of the nuclear Bohr magneton. Here the isospin component τ_3 is $+1$ for up quarks, -1 for down quarks. One easily finds from the above two equations

$$\mu_{proton} = m_n/m_Q$$
$$\mu_{neutron} = -\tfrac{2}{3}m_n/m_Q$$

where we have taken $m_u = m_d = m_Q$. For $m_Q = \frac{1}{3}m_n$, $\mu_{proton} = 3$ and $\mu_{neutron} = -2$, close to their physical values. By taking $m_Q c^2 = 336$ MeV one can come even closer to experiment.

Theorists extended[2,5] this simple picture to the Λ particle by assuming the mass of the strange quark to be

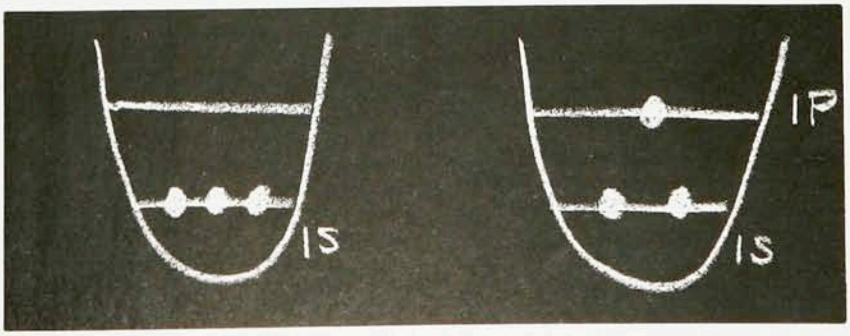
$$m_s = m_u + (m_\Lambda - m_p) \qquad (5)$$

where m_Λ and m_p are the masses of lambda particle and proton. (This picture is referred to as "broken SU(6)." The broken-SU(6) model contains 6 objects: the up and down quarks, each with two spin projections, and the strange quark with spin up and down. The symmetry is broken because the strange quark has a different mass.) The lambda particle has isospin zero, so the up and down quarks must be coupled to give a net spin of zero if they are to be symmetric in spin and isospin. Therefore, the magnetic moment of the Λ should be that of the strange quark,

$$\mu_\Lambda = -\tfrac{1}{3}\, m_n/m_s = -0.61$$

Measurement[6] subsequent to this calculation produced the predicted result precisely. Such great success gave the quark model further momentum.

In spite of this great success, the above results for the nucleon are difficult to understand. Previous to these developments, the vector-dominance model[7] seemed successful in producing the anomalous moments of proton and neutron by coupling the photon to admixtures of ρ and ω vector mesons in the nucleon ground state. The chief assumption in the vector dominance model is that the γ-ray coupling in the electroproduction of ρ and ω mesons is the same as it is coupling to the magnetic moment, that is, the coupling is the same for timelike and small spacelike momentum transfers. Calculation in the vector-dominance model under this assumption produces proton and neutron magnetic moments within



Oscillator potential with quarks. The diagram shows the ground-state configuration (left) and the excited-state configuration (right). Figure 3 the errors given in the latest experimental data. No wonder the vector-dominance model was considered successful! Because the vector mesons are coupled to the nucleon via pionic interactions, they can be incorporated into the meson cloud.

But now we have two pictures that give the nucleon magnetic moments, one in terms of quarks, and one in terms of coupling via vector mesons. In such a situation, one often finds that at a deeper level of understanding the two pictures give different aspects of the same underlying theory. This does not seem to be the case here. The SU(6) picture attributes all of the magnetic moment to the three-quark piece of the nucleon wave function, whereas the vector dominance model attributes it to an admixture of vector mesons. The latter are quark–antiquark pairs. Thus, in a Fock-state decomposition of the nucleon,

$$|\text{nucleon}\rangle = \sqrt{Z_2}|3\text{q}\rangle + b|4\text{q}1\bar{\text{q}}\rangle + \dots$$

the SU(6) model seems to say that the magnetic moment comes from the first term (with Z_2 taken to be about 1); the vector dominance model seems to attribute the moment to the second term.

Two-component moments? Some doubts that the physical situation can be as simple as the SU(6) model indicates come from measurement[8] of the Ξ° and Ξ^- moments. Consideration of the Ξ^- moment, found experimentally to be -0.69 ± 0.04, is instructive. The Ξ^- is made up of two s quarks and one d quark; thus, from equation 4, its magnetic moment is $\mu_s - \frac{1}{3}(\mu_d - \mu_s)$. Now μ_s is set from the Λ moment, so that the magnetic moment of the Ξ is given by $-0.69 \pm 0.04 = -0.61 - \frac{1}{3}(\mu_d - \mu_s)$. Remembering that μ_d and μ_s are negative, we see that satisfaction of this equality would require $|\mu_d| < |\mu_s|$, which runs completely contrary to the naïve quark model. But our own more detailed consideration indicates[10] that the nucleonic moments come about equally from the constituent quarks and the meson cloud. Let's look at this argument.

Harry Lipkin of the Weizmann Institute has combined[9] baryon moments into two quite instructive "analysis functions":

$$R(\text{p},\Sigma^+,\Xi) = \frac{\mu_\text{p} - \mu_{\Sigma^+}}{-\frac{1}{3}[\mu_{\Xi^0} - \mu_{\Xi^-}]} = 2.7 \pm 0.8$$

$$R(\Xi,\Lambda) = \frac{\mu_{\Xi^0} + \mu_{\Xi^-}}{3\mu_\Lambda} = 1.05 \pm 0.04$$

The quantity $R(\text{p},\Sigma^+,\Xi)$ would vanish in the limit that $m_s = m_u$. In fact, equation 4 shows that $\frac{8}{9}$ of the proton moment comes from up quarks, and that somewhat more than $\frac{8}{9}$ of the Σ^+ moment comes from up quarks. The up-quark contributions enter into the proton and Σ^+ moments, in the same way, so it is surprising that $R(\text{p},\Sigma^+,\Xi)$ is so large. (The demoninator serves chiefly as a normalization.) Indeed, broken SU(6), the constitutent-quark model of equations 4 and 5, gives $R_\text{theo}(\text{p},\Sigma^+,\Xi) = 0.34 \pm 0.05$.

The quantity $R(\Xi,\Lambda)$ is predicted to be 1.06. Thus the agreement between theory and experiment is excellent for this quantity, but terrible for $R(\text{p},\Sigma^+,\Xi)$. The large empirical value for $R(\text{p}, \Sigma^+,\Xi)$ tells us that the up quarks contribute quite differently in the proton and in Σ^+. The agreement between theory and experiment for $R(\Xi,\Lambda)$ tells us that the strange quark obeys the additive quark model.

We can understand both of these features if some of the magnetic moment comes from the meson cloud.[10] The pion couples only to up and down quarks. The K meson is much heavier than the pion, so the kaon cloud, which couples to strange quarks, has very little effect. Therefore, strange quarks should follow the additive quark model. The baryons should differ in their coupling to the pion cloud. The nucleon has three non-strange quarks, which couple much more strongly to the pion cloud than do the two non-strange quarks in the Σ^+. In each case there are one-body couplings, but two-body couplings add to these in the case of the nucleon; these two-body terms are absent in all strange baryons because of a selection rule. Thus, in going from the nucleon to the strange baryons one can talk about the damping of the pion cloud. Assuming[10] that 50% of the nucleon magnetic moment comes from the meson cloud, we obtain $R(\text{p},\Sigma^+,\Xi) = 2.72$, which is in good agreement with the empirical value. Our estimate of the precise division between the meson-cloud contribution and the quark contribution may change as more accurate experiments are carried out, but it is fair to say that comparable contributions come from each of the two agencies. It thus seems that the near-perfect agreement with experiment found in both the vector-dominance model and the quark model, when each is applied individually, was coincidental.

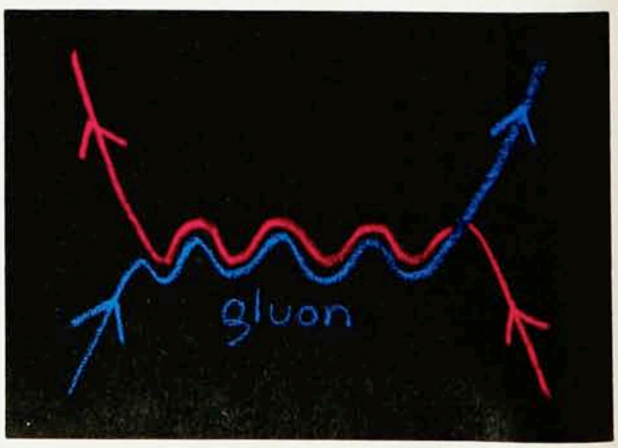
The MIT bag model

In the decisive demonstration that nucleons are made of quarks, the quarks behaved as if free and massless. In fact, that was just the remarkable feature of the high-energy deep inelastic electron scattering experiment (see figure 5). So what is the source of quark masses m_Q that we have been using so far?

Of course, our discussion has been in the low-energy domain, and couplings change according to momentum, with interactions becoming stronger at low momentum, so there may be mechanisms for quarks to acquire mass. The MIT bag model,[11] in which the up and down quarks are nearly massless is an attractive picture (we neglect the small up and down "current quark masses" of several MeV). Starting from essentially massless quarks it is easier to implement chiral symmetry, as we shall see.

In the MIT model, quarks are confined by fiat—by a boundary condition applied to the quark wavefunction at radius R, the edge of the "bag." Applying this boundary condition, one can verify that the normal component of the vector current is zero at $r = R$. Thus, no particles can escape from the bag. Ultimately, this confinement "by fiat" should be replaced by a real theory of confinement, but for the moment we don't have one. Given the MIT boundary condition, one can work out wave functions for the quarks, which, inside the bag, obey the Dirac equation for massless Fermions.

The physical vacuum is a horribly complicated colored "spaghetti" of gauge fields, with a sauce of quark-pair condensates. To allow colored quarks to exist locally, we must create a bubble, or bag, and this costs energy. The amount of energy is taken to be proportional to the volume: $\Delta E = \frac{4}{3}\pi R^3 B$, where B is the "bag constant."



Gluon exchange. Diagram depicts the exchange of a virtual gluon by quarks, whose colors are shown. The gluon, which must change one quark color into another, can be made of two colors. Figure 4

We can obtain the quark energies from the MIT boundary condition, which determines the eigenvalues; for the $1s_{1/2}$ ground-state quark, $E_Q = 2.04\hbar c/R$. Therefore, the energy of the bag with three quarks is

$$E_{\text{bag}} = \tfrac{4}{3}\pi R^3 B + 3(2.04)\hbar c/R \qquad (6)$$

People have added various additional energies to this functional, but the simple form shown here well represents the basic ideas. The parameter B is chosen so that when the energy is minimized with respect to R the energy of the bag is the experimental nucleon mass $m_n c^2$. One can see that the R^3 term keeps the bag from expanding too much, and the $1/R$ term, essentially the kinetic energy, keeps it from collapsing. These features persist through more detailed treatments.

Once B has been set in the manner described above, E_{bag} is a rather flat function of R about the minimum, and zero-point oscillations must enter in an important way; several groups are working to introduce them. Of course, the sharp bag surface (the sphere in the cover drawing) must be a crude approximation, but it simplifies greatly the coupling of mesons to the bag, as we shall see shortly.

Calculations. A great advantage of the MIT bag is that it gives a definite model in which one can carry out calculations—the bag boundary condition and minimization of the energy give a definite representation of quark wave functions. Gluon exchange is easily treated along the lines described earlier, with bag wave functions replacing those of the constitutent quarks. There is one important difference. Once B has been determined so as to minimize the energy at the correct value, the only parameter with dimensions of energy is $\hbar c/R_{\text{bag}}$, so that interactions like the one described by equation 2 must be proportional to g^2/R_{bag} rather than to a function of r_{12}. An interaction of the form of equation 3 nevertheless results, with C being proportional to the color coupling constant $g^2/\hbar c$ for either constituent quarks or the bag model. One sees, therefore, that only the combination g^2/R_{bag} enters into bag spectroscopy, so that one can obtain equivalent fits to the spectrum with large color coupling constants $\alpha_s(=g^2/4\pi\hbar c)$ and large radii R_{bag}, or with small α_s and small radii.

In the bag model, tensor and spin–orbit interactions are the result of gluon exchange. A great advantage of the bag model is that spin–orbit terms connected with confinement may be automatically included, and that these cancel[3] most of the unwanted spin–orbit interaction from gluon exchange.

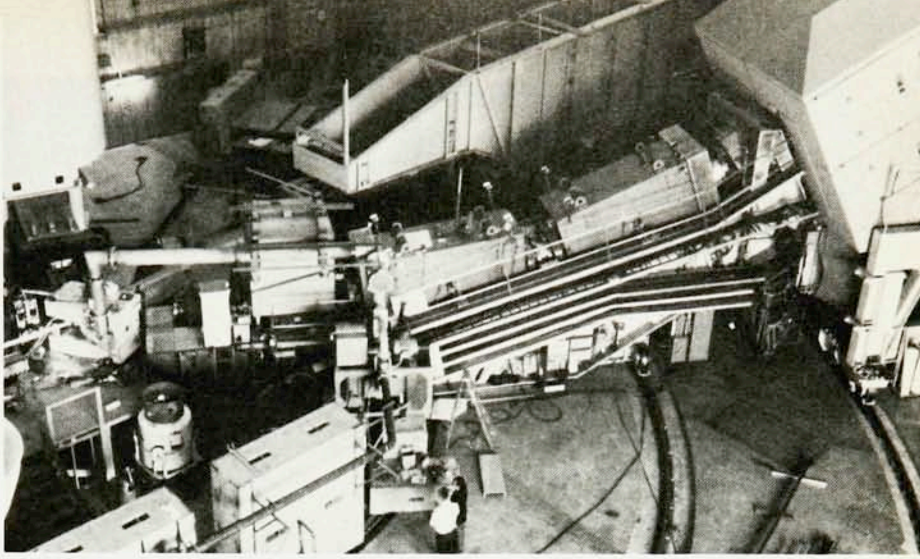
In calculations of baryon magnetic moments[11] the quark energies appear in the denominators of the magnetization density, just as the constituent quark rest masses m_Q appear in the nonrelativistic development.

So there is a sort of mapping of relativistic quark energies in the bag model onto constituent quark masses in the nonrelativistic theory.

Our discussion is somewhat imprecise, as is our present understanding, but it is clear that a broken-SU(6) picture, with some important differences, emerges from the bag model. At first sight this seems surprising because the broken SU(6) theory was first associated with nonrelativistic massive (constituent) quarks.

For the broken-SU(6) picture one needs six objects. The two up and down quarks, each with two spin projections, up and down, form an SU(4) foursome, which is not broken at the level we discuss because the quarks are treated symmetrically. The strange quark enters the bag model with an input mass, a "current quark" mass. The mass of the strange quark is not given by equation 5; it is chosen so that when the strange quark replaces one of the up or down quarks in the nucleon, and its mass is added to the Hamiltonian in the Dirac equation, the mass of the resulting Λ particle comes out correctly. From our discussion in the previous section, we see that once the mass of the Λ particle is given correctly, it is probable that the magnetic moment will be reproduced. In fact, one can easily evaluate Lipkin's analysis function $R(p,\Sigma^+,\Xi)$ in the MIT bag model, finding it to be 0.27, as compared with the broken-SU(6) constituent-quark model's value of 0.34, quoted earlier. Thus the ratios of magnetic moments in the MIT bag model behave very much as they do in broken SU(6).

Only with massless objects is it easy to enforce the chiral symmetry in the underlying theory, quantum chromodynamics. Thus the bag picture, in which one begins with massless up and down quarks, has great advantages, even if phenomenologically it does not do better than the naïve constituent-quark model. (We are not saying that it does or that it doesn't do better; at this stage that is a matter of opinion.)



Three spectrometers pointing at a common target at SLAC, where experiments in the deep inelastic scattering of electrons off of nucleons gave evidence for nucleon structure. Each spectrometer consists of bending and focusing magnets followed by a detector in concrete shielding. The large cylinder at the left is a concrete shielding hut on top of the 90° bending magnet of a spectrometer that can analyze momenta as large as 1.6 GeV/c. At top center is a 20-GeV/c spectrometer, which features two reverse 10° bends. The 8-GeV/c spectrometer at the right bends upward by 30°. The latter two spectrometers were part of the early experiments that contributed to the development of the parton model of the nucleon. (Photograph courtesy of Stanford Linear Accelerator Center.) Figure 5

Chiral invariance and pions

As we mentioned above, the QCD Lagrangian has a chiral invariance: when a quark field ψ is replaced by a field $\exp(i\gamma_5\theta)\psi$, where θ is an arbitrary constant, the Lagrangian remains unchanged. This would not be true if the quark masses (more precisely, the current quark masses) were non-zero. In fact, one knows that even up and down quark masses in the Lagrangian are not zero, because the $\pi^{\pm}$ and π^0 are not massless. However, the small current quark masses of several MeV are supposedly relics of interactions at an energy scale much higher than the QCD regime, and are irrelevant quantities from the point of view of strong-interaction physics. In fact, by pretending that the symmetry is exact, we can reproduce much of what we consider to be the success of soft-pion theorems established in the 1960s. In what follows, we will proceed as if the quark masses and the pion mass were strictly zero.

To the extent that chiral invariance

is a good symmetry, the axial-vector current $J_\mu^{(5)}$ is conserved:

$$\frac{\partial J_\mu^{(5)}}{\partial x_\mu} = 0 \qquad (7)$$

The axial-vector current is the current that enters into the weak interaction for β decay. One goes from the vector current to the axial-vector current by inserting a γ_5; the latter current is conventionally written in terms of Fermion wave functions $\psi(x)$ as

$$J_\mu^{(5)}(x) = i\bar{\psi}(\tau/2)\gamma_5\gamma_\mu\psi(x) \qquad (8)$$

As usual, one transcribes this into quark wavefunctions by replacing $\psi(x)$ by the quark wave function $\psi_q(x)$.

This brings us to a problem. With the MIT boundary condition, the normal component of the axial vector current is by no means zero. At the boundary of the bag this current has nowhere to go because there is nothing outside the bag to carry it. Obviously, the current cannot be conserved and equation 7 cannot hold.

Actually this difficulty is much more general than the above derivation within the bag model indicates. When a massless quark is reflected by a boundary to form a standing wave, its chirality changes sign. But for the chirality of the system to be conserved, something must carry the chirality out through the physical vacuum. And that "something" must be colorless if it is to propagate outside the confinement region. Curtis Callan, Roger Dashen and David Gross of Princeton University and the Institute for Advanced Study proposed[12] that the pions carry the axial current in the region external to the confinement region.

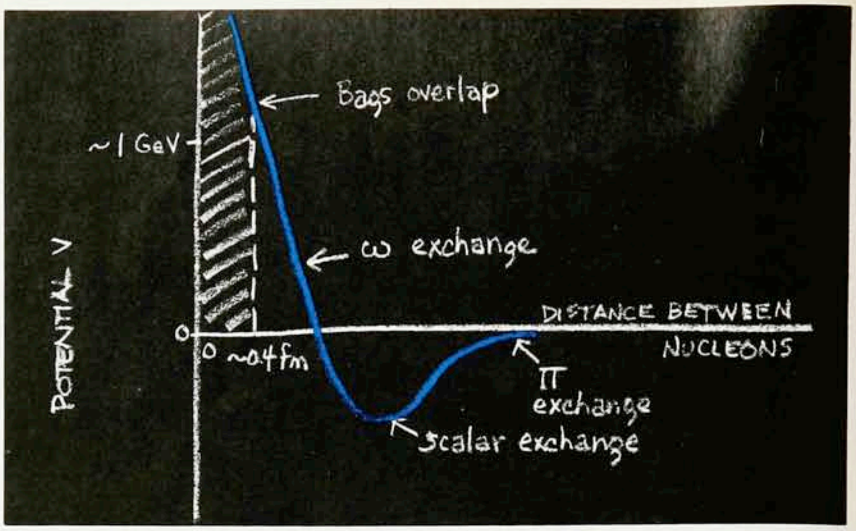
It was clear in earlier chiral theories involving nucleons and mesons that in a particular model one could write the axial current in terms of the pion field ϕ_π as

$$J_\mu^{(5)} = f_\pi D_\mu \phi_\pi \qquad (9)$$

Here D_μ is the nonlinear derivative $(1+\phi_\pi^2/f_\pi^2)^{-1}\partial/\partial x_\mu$, and f_π is the pion weak decay constant.

To make the axial-vector current continuous at the surface of the bag, the axial current inside the bag (equation 8) has to turn into the current outside the bag (equation 9). One can do this by analogy with electrodynamics, as we outline in the box on page 32. The pion coupling is chosen to make the normal component of $J_\mu^{(5)}$ continuous at $|\mathbf{x}| = R$. Outside the bag, the pion obeys the usual free-field equation. With these conditions one can solve for the pion field in terms of $\psi_q(x)$, the quark wave function in the bag.

Pion pressure. One can readily convert weak pion decay constant f_π into the strong pion–nucleon coupling constant f (whose experimental value $f^2/4\pi$ is about 0.08) by using the Goldberger–Treiman relation, $f_\pi = g_A m_\pi/2f$, where g_A is the axial-vector coupling constant. One can use this relation which is correct in any theory that has chiral invariance, to show[13] that the pion field external to the bag, when properly quantized, is the usual pion field that would follow from pseudoscalar pion–nucleon coupling in old-fasioned Yukawa theory. Because the pion exists only external to the bag, the bag acts as an extended source for the pion field. This model, with the pion coupled to restore chiral symmetry, is called the chiral bag model[13] or cloudy bag model.[14] Within the framework of this model, theorists have carried out many calculations, especially calculations involving pion–nucleon interactions.



Nucleon–nucleon interaction. The ω meson, scalar meson and pi meson couple to the nucleon with relative strengths of about 11, 5, and 0.08, respectively. Thus, the coupling in mesons becomes stronger with decreasing range. Figure 6

Because pions couple only to nonstrange quarks, the pion cloud couples more strongly to the nucleon than to the strange baryons. The pion cloud exerts a pressure—the "pion pressure"—on the internal confinement region, and because this pressure is greater for nucleons than for other baryons, it tends to make the nucleon smaller in extent than the other baryons. It is entirely possible that the nucleon is quite a small object, a fraction of a fermi in radius, whereas the cascade particles, Ξ^- and Ξ^0 could be as large as a fermi.

Of course, the pion, which we treat as a field, has a quark–antiquark substructure, just as the nucleon does. But for the purposes of constructing a pion cloud and calculating pion-exchange interactions between nucleons, it is convenient to treat the pion as a field. This is justified as long as one deals with wavelengths large compared with the dimensions of the pion's quark–antiquark substructure. Treating the pion as a field allows one to implement chiral symmetry in the simple way described above, and this implementation determines the coupling of the pion to the nucleon.

The pion cloud thus turns out to be a natural consequence of chiral symmetry, the pion being necessary to carry the chirality from the core. We are back to our original picture where we attribute the total physical extent of the nucleon to a quark core and a meson cloud. In calculations with the bag model, the total rms radius turns out to be insensitive to the precise radius at which one joins the cloud to the core. Without the meson cloud, everyone obtains too small a charge radius for the proton. Similarly, the total magnetic moment—core and cloud—is insensitive to the radius of joining. So one must look elsewhere to get a line on the size of the core.

As we noted earlier, the constituent-quark model gives 0.45 fm for the radius of the ground state of the nucleon, and 0.5 fm for that of the excited state. Similar calculations within the framework of the bag model[3] give a bag radius of 0.7 fm for the odd-parity excited states, or an rms radius of 0.4 fm. These figures are somewhat larger than those emerging from phenomenological QCD, which gives a nucleon radius of about 0.23 fm based on an analysis of J/ψ decay into $N\bar{N}$ and on high-energy data on nucleon form factors. Thus, our estimates may be somewhat too high, but there is considerable uncertainty in these numbers.

Forces between nucleons

The picture of the nucleon as a small quark core surrounded by a meson cloud removes certain conceptual difficulties that are encountered when the

MIT big-bag model is applied to interactions in nuclei, and it leads naturally to Yukawa's time-honored meson-exchange description. Once we couple the pion to the nucleon bag, it is straightforward to reconstruct the boson exchange model, because we can express the coupling of the other bosons in terms of the pion coupling. The intermediate range attraction comes from the exchange of two-pion systems in relative s-waves; the ρ meson consists of two pions in a relative p-wave. The ω meson is a three-pion system. Both ρ and ω mesons have an underlying $q\bar{q}$ structure at short distances.

Figure 6 shows the nucleon–nucleon interactions, an important feature of which is the strong repulsion resulting from ω exchanges. This repulsion is seen directly in the nucleon–nucleon interaction and manifests itself in many phenomena, not least of all the spin–orbit interaction, which appears in the formalism when vector-meson exchange potentials are used in relativistic equation. The potential from ω exchange is

$$V_\omega(r_{12}) = g_{\omega\mathrm{NN}}{}^2/4\pi \times m_\omega c^2 \left(\frac{\exp(-m_\omega r_{12})}{m_\omega r_{12}}\right)$$

where $g_{\omega\mathrm{NN}}{}^2/4\pi$ is 10 to 12, and $m_\omega c^2 = 783$ MeV. Although this interaction drops off sharply with r_{12}, it has a value of about one GeV when $r_{12} = 0.4$ fm, and is still appreciable when $r_{12} = 1$ fm. Of course, in calculations of the bag-model type, the ω-exchange potential will begin to cut off when the bags begin to merge, but this is a gradual process and some of the repulsion will remain down to small distances. The coupling constant $g_{\omega\mathrm{NN}}{}^2/4\pi$ is so large that even a small remaining part of the ω-exchange potential will play a more important role than perturbative gluon exchange in the interior of the bag. If gluon exchange is to be perturbative, then the coupling constant $g^2/4\pi\hbar c$ must be small compared with unity.

It is the great strength of the short-range nuclear interactions that makes them essential in nuclear physics, even if they are cut off considerably, as compared with interactions from gluon exchange.

Bags in nuclei, then, do not move freely in an uncorrelated way, because the highly repulsive interaction arising from vector-meson exchange works to keep them apart. One cannot, therefore, say that if the bag radius is R_{bag} and the average distance between nucleons in nuclei is r_0, that the probability of bags overlapping goes as $(R_{\mathrm{bag}}/r_0)^3$. Because of the anticorrelation produced by the strongly repulsive potentials between bags, the actual probability will be less. This repulsion between bags, more then any other agency, may be the reason why the quark substructure of the nucleon seems to have so little direct influence in nuclear physics and why no low-energy nuclear phenomenon to date exhibits clearly this quark substructure. Of course, in proceeding to higher energies or higher densities the core regions of the nucleon will push over the repulsive barriers. The transition must, however, occur smoothly and involve no drastic change in physical character. Indeed, in our particular model,[16] the interaction becomes progressively more repulsive once bags merge, due to the loss of attraction from pionic couplings. We feel that it is the vector-meson exchange, highly nonperturbative from a QCD angle, and, possibly, the nonperturbative coupling to pions, that distinguishes nuclear physics from high-energy physics in the sense described above.

The precise behavior of the repulsion that remains after two bags merge needs still to be determined. Does it come from perturbative quark–gluon exchange, as many authors suggest? Or does it come from nonperturbative pionic effects? This issue remains to be settled.

Effects from quark substructure may be hard to see or recognize in low-energy nuclear physics. Not only is the quark core of the nucleon small in radius compared with the average distance between nucleons in nuclei (r_{12} about 1.2 fm), but the repulsions from vector-meson potentials tend to keep these cores apart. One can thus understand that the interaction of two nucleons is well described by the boson exchange model, regularized at short distances to take account of the quark core.

It may seem that we have relegated the quark substructure of the nucleon to a minor role in this description, and, in terms of describing low-energy nuclear phenomena, we have. Models of the structure of the nucleon do, however, provide a natural mechanism for cutting off the boson exchange potential at short distances, and this is important in making the interactions finite. Once we understand the highly nonperturbative aspects of QCD, knowledge of the structure of the nucleon will provide constraints on how and what boson exchanges intervene.

* * *

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Analogy with electrodynamics

In quantum chromodynamics, the axial current carried by the pion plays the role of Maxwell's displacement current $\dot{\mathbf{D}}$ in electrodynamics. Starting from Ampere's law with Maxwell's displacement current, $\nabla\times\mathbf{H} = 4\pi\mathbf{j} + \dot{\mathbf{D}}$, taking the divergence and using Poisson's law we see that the vector current is conserved in electrodynamics: $4\pi(\nabla\cdot\mathbf{j} + \partial\rho/\partial t) = 0$.

To obtain the expression for the conservation of the axial current in quantum chromodynamics, we construct $J_\mu{}^{(5)}$ out of massless quarks in the core region and out of the pion field in the exterior region:

$$J_\mu{}^{(5)} = i\bar{\psi}_q \frac{\tau}{2}\gamma_5\gamma_\mu\psi_q(x) \qquad |\mathbf{x}| < R_{\rm bag}$$
$$= f_\pi D_\mu \phi_\pi(x) \qquad |\mathbf{x}| > R_{\rm bag}$$

Glossary

A few words of explanation about some technical terms appearing in the text.

Asymptotic freedom: a feature of non-Abelian gauge theory in which interactions become weaker at shorter distances or at higher momenta.

Chiral symmetry: symmetry associated with massless fermions (quarks).

Chiral SU(2)×SU(2): chiral symmetry associated with two flavors of massless quarks; up and down quarks show such symmetry when their masses are ignored.

Constituent quark mass: an effective mass that arises in non-relativistic quark models.

Current-quark mass: mass that appears as terms in current algebra relations and also in the QCD Lagrangian.

Down quark: a light quark (slightly heavier than the up) of isospin ½, isospin z component −½ and charge −⅓e. With the up quarks, this non-strange quark is the main component of the nucleon.

Goldstone mode: manifestation of a continuous symmetry that arises when the vacuum does not possess the same contineous symmetry. This realization is referred to as a "spontaneously broken" symmetry mode.

Helicity: the handedness (left-handed or right-handed) of a particle described by how it spins with respect to the direction of its momentum.

"Little" bag: a hadron bag predicted to have a confinement radius much smaller than the typical hadron charge radius measured by electron scattering.

Quantum chromodynamics (QCD): non-Abelian gauge theory of the strong interaction with colored gluons and quarks as fundamental constituents. Hadrons are bound states of the quarks, the binding force mediated by the gluons.

Strange quark: a massive quark (though still considered light in particle physics) with isospin zero, charge −⅓e and strangeness 1. This quark is a constituent of hyperons.

Up quark: the lightest quark, with isospin ½, isospin z component +½, charge +⅔e. With the down quark, this non-strange quark is the main constituent of the nucleon.

Vector dominance model: model of strong interactions in which the vector and axial-vector currents of the weak and electromagnetic interactions are assumed to be dominated by vector mesons of the appropriate quantum numbers. Such vector mesons include ρ, ω, and A_1.

Wigner mode: manifestation of a symmetry that remains unbroken in the equation of motion while the vacuum possesses the same symmetry. It manifests itself in a degenerate multiplet structure, familiar in atomic and nuclear physics.

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