Euclid pondering a problem in geometry in a painting at the Ducal Palace in Urbino, Italy (Courtesy Alinari-Scala/Editorial Photocolor Archives, Inc.).

Solving physics problems—how do we do it?

Inquiry into students' misconceptions about nature and into the interior processes of learning can help physicists teach problem-solving to others.

Robert G. Fuller

Scribblings on the back of an envelope are as much a part of the professional mystique of the physicist as they are of the story of the Gettysburg Address.

Solving problems is the heart of the work of a physicist. Yet which of us physicists has not suddenly discovered, in a group of our nonphysicist friends, that this propensity for problem solving sets us apart from many others? And which of us, during a time of reflection, has not puzzled on how we regularly solve problems that seem difficult to others?

In this paper I want to share with you some of the tentative answers that have been found to the puzzle of how people solve physics problems.

While solving problems is a long tradition in physics, research into how people actually solve physics problems is relatively recent. In fact, before the early 1970s it was thought that physicists knew how to solve physics problems and they knew how to teach other people to solve physics problems.

Just what triggered this interest is unclear to me. It may have been a combination of the decreased ability students show in doing physics problems and the arrival in strength of the information-processing branch of psychology. Anyway, since 1971 several preliminary steps toward understanding problem solving have been taken.

The first article that I remember seeing that raised new questions about whether we knew all we ought to know about teaching physics appeared in 1971, written by Joe McKinnon and John Renner.[1] Since that time many studies of reasoning and problem solving have been done, not only by physicists, such as Robert Karplus[2] and Arnold Arons,[3] but also by such others as Herbert Simon,[4] a Nobel laureate in economics, and Robert Glaser,[5] a psychologist, who chose physics as the context for their research in problem solving.

The diverse currents of this recent research have still to be unified, yet there are three aspects of it that I think are of particular interest to physicists: micconceiving natural laws, processing information and constructing solutions.

Misconceiving natural laws

Every physicist knows the importance of having the correct concept in mind before beginning to solve a problem. The beginning of the new physics of this century was marked by the

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attempts of physicists to create new concepts appropriate to the contexts of the new experiments.

Quantum effects seem to play a small role in the consciousness of most people. It would be comforting to know that most people can apply principles of Newton's physics for their understanding of physical events. It will not surprise you to learn that no such comfort is available.

Most of the basic research into the conceptions people have about the behavior of physical systems has come from the study of college students done using the oral semi-clinical interview techniques developed by Jean Piaget[6] in his work with children.

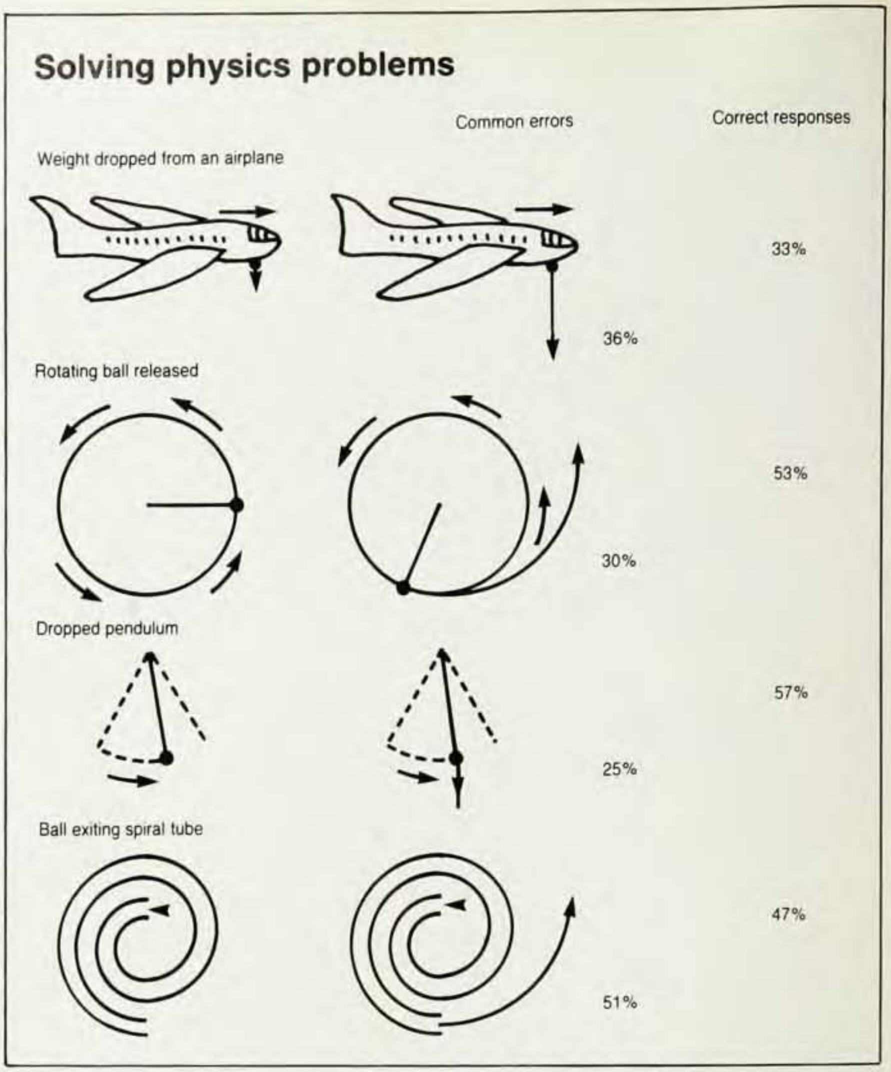
A number of groups—Lillian McDermott's at the University of Washington–Seattle,[7] Jack Lockhead and John Clement at the University of Massachusetts–Amherst,[8] and John Gilbert and his coworkers in England[9]—have developed systematic methods for interviewing students about physics and have detailed the students' misconceptions about how physics works. One such study received widespread distribution through an article in *Science* magazine.[10] In that study students at Johns Hopkins University gave written responses to four questions about moving objects (see illustration this page):

▶ The first question concerned the path of an object dropped from an airplane traveling with constant velocity. A third of the students gave the correct, parabolic, path for the projectile, but more than a third of the students showed the object falling in a straight line to the ground, not moving forward with a velocity equal to the velocity of the airplane.

▶ The second question was about a ball being swung in a horizontal circle on the end of a string. Subjects were asked, if the string were cut, what direction would the ball go? While half said it would go forward in a straight line, 30% of them showed the ball moving in a spiral path.

▶ A pendulum problem asked students what would happen to the bob swinging at the end of the pendulum if the string were cut during its swing. Although more than half the students gave the correct answer, one-quarter showed the bob falling in a straight vertical line to the ground.

▶ A fourth question asked what happened when an object that was injected into a horizontal spiral tube left the end of it to roll on a horizontal table. Almost half the students predicted that it would travel in a straight line; slightly more than half said it would continue to spiral around on the table.



Incorrect predictions by a large percentage of students for the paths of moving objects released from constraints—weight dropped from airplane, ball on string, pendulum, ball in tube.

Let us examine the implications of these studies for two aspects of our work as physicists: first, our teaching of physics concepts to others and second, our work to carry out basic research in physics with the public support of our friends and neighbors.

For physics teachers the most obvious implication of this research is that our traditional view about teaching is inaccurate. The student is not an empty vessel into which professors pour the knowledge of physics equations, of functional relationships, and of problem-solving strategies. The mind of today's student is not empty. It is a jungle of Aristotelian and even pre-Aristotelian ideas about nature.

How do such false ideas arise in the minds of students? Perhaps it is because Nature appears to operate along non-Newtonian lines. Students have experiences of force producing a change of position according to Aristotle's laws. If we reduce the velocity of an object, such as a rolling piano, nearly to zero, then the next position of the piano seems to be in the direction of our push. We do not experience the persistence of its original motion. The ubiquitous force of friction interferes with objects performing as Newton's laws would appear to predict for the forces we apply. We push on the piano with great effort. It begins to roll, but it doesn't appear to accelerate across the floor. Similarly, students have had experience pushing with a constant force objects that do not go in straight lines with ever-increasing velocity. Therefore the explanation of the way objects move provided by the physics professor seem to be placed in a special category of unlikely and useless ideas to be mastered only for a particular course.

A recent study on unlearning Aristotelian physics shows these early ideas persist in college students even after a complete course in general physics.[11] The problem of replacing wrong ideas about nature is more difficult than

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trying to teach students with no ideas about physics whatsoever.

An instructor who wishes to teach students good problem-solving strategies has to consider the students' present understanding of nature and of the way the laws of nature work. A teacher also needs to understand the peculiar strategies for problem solving that students already use. How can we develop strategies in our classrooms to carve out paths through the jungles in the minds of the students? Are there ways we can use the misimpressions students have to build strong new roads?

What about physics research? It seems to be an ark afloat in the flood of a pre-Newtonian culture that has a few islands of Newtonian and post-Newtonian decision makers. We have clearly underestimated our task to help our fellow citizens understand the laws of physics and appreciate physicists view of nature. Our future as a profession may depend upon our ability to lead our nation to appreciate physics, not just for the new gadgets that can be derived from our research, but for the ability of our vision of the interaction between humans and Nature to guide us into a secure future.

There are two different schools of researchers who have been studying reasoning and problem solving: information processors and constructivists. Information processing describes the way people solve finite and particular problems, while constructivism applies more to the creative and intuitive work of an Einstein or Heisenberg. The relation between the two areas appears to be fertile ground for future research.

Processing information

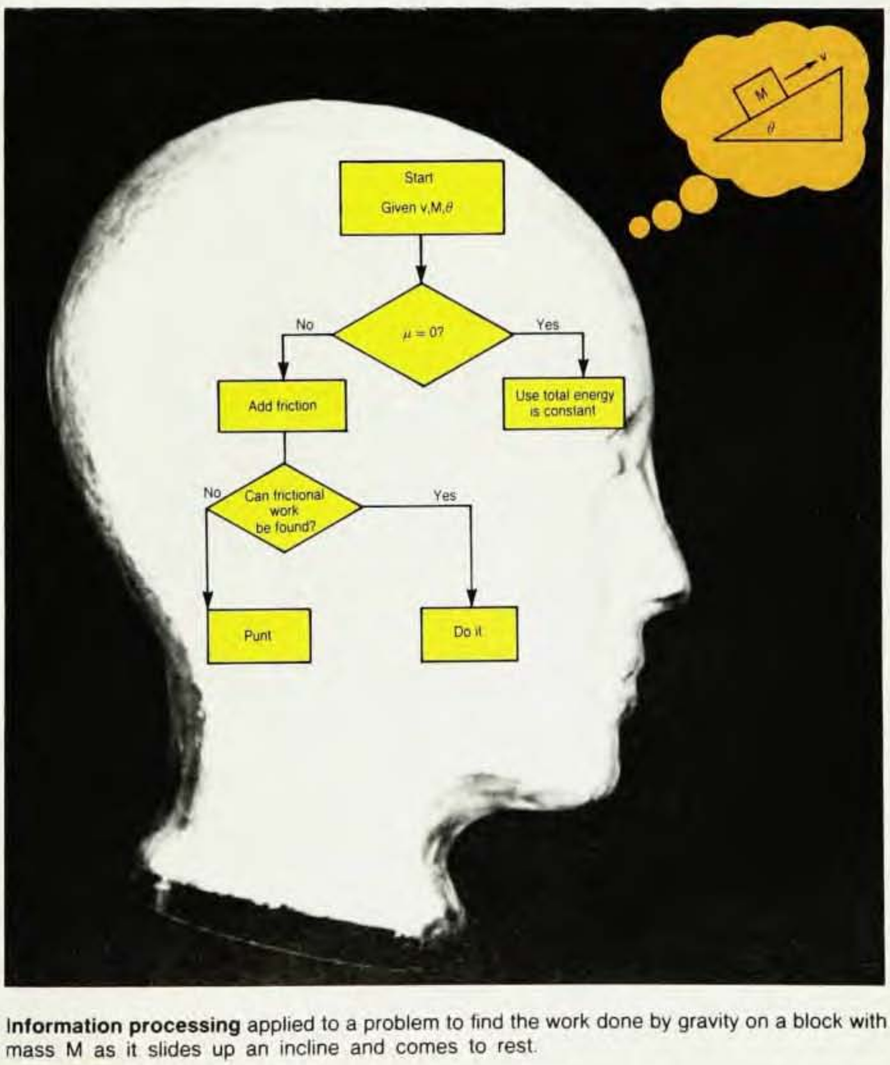
Information processing has made two significant contributions that can be very helpful in the understanding of problem solving. First, these researchers have been extremely skilled at analyzing tasks.[12] Second, they have fruitfully compared expert and novice problem solvers. Many of them have performed very clever task analyses and have devised systems of questions about a physics problem that allows them to determine the processes that are going on in the mind of the subject.

Let us consider the "equal-arm-balance" task: to predict and explain the behavior of a balance when integer amounts of mass are placed at integer distances from its fulcrum. The experimenter holds the arms of balance after placing the masses on it according to the subject's instructions, and the subject is asked for a prediction and an explanation before seeing how the balance behaves. The task is interesting because its solution is related mathematically to the concept of proportionality; its rule for generating correct solutions, once known, is easy to execute; and its rule is not trivial to deduce in the first place. Many people, including Piaget, have studied how students approach this task; in addition, it is appropriate for a wide range of ages. Small children have had experiences with teeter-totters, yet some college students, even after a year of physics, do not readily solve this task.

In analyzing this task, Robert Siegler[12] developed a dichotomous tree model with four different rules, as shown on the next page. Siegler was able to deduce the rule being used by each subject by presenting seven different arrangements of the masses on the balance arm. For example, a subject using rule I would wrongly predict a balance for 3 mass units, m, at a distance of 3 length units, l, on the left versus $3m$ at $2l$ on the right while all the other rules give a correct prediction. Subjects using rule III will give chance responses to the situation of $2m$ at $3l$ plus $2m$ at $2l$ versus $2m$ at $4l$ on the other side of the fulcrum, but rules I, II and IV give correct answers for this situation. Clearly only subjects using rule IV will be able to solve this task for experiments that conflict with the other three rules, for example, $3m$ at $2l$ versus $6m$ at $1l$.

Many of us have been solving physics problems for so long that we have not recently thought systematically about the problem-solving demands of the questions that we tackle or those that we ask others. The research on information processing helps us understand how to go about the process of analyzing physics problems. A most notable area of this research is the comparison[5,14] between expert and novice problem solvers—between physicists who have solved physics problems for 20 years and beginners who have been solving physics problems for 20 days.

There appear to be two important characteristics of expert physics problem solvers. First, the experts approach solutions to physics problems through a process of successive refinements. They start with a gross description of the problem in words and drawings. Only later do they examine the details of problems and introduce mathematics. Second, physicists have organized their knowledge into large, coherent "chunks" of information, more accessible than individual princi-



Information processing applied to a problem to find the work done by gravity on a block with mass M as it slides up an incline and comes to rest.

ples and equations. By having knowledge organized in useful and interconnected unities, they can call upon the one or two strategies that are likely to be the most successful. Students seem to lack this connectedness of knowledge that experts have. Students begin by searching through all of the trees in the forest for some possible way of making a path to the solution.

How can physicists go about helping novices develop the "chunks" of knowledge in ways that can help them in problem solving? How can they help people organize their knowledge in more global ways so they can see how to apply various pieces of it to different kinds of problems? One answer to these questions is that a general problem-solving strategy needs to be taught explicitly. Students need to learn how to organize their own thinking for solving problems.

Frederick Reif, Jill H. Larkin and George C. Brackett described[15] in 1976 a "primitive" scheme of the four steps that experienced problem solvers use. The first step, "D," is to describe the problem. People should be encouraged to explore the problem verbally and pictorially. They must be sure to understand exactly what is given, what the assumptions are and what can be neglected. This is one of the most difficult things to get beginning problem solvers to do. They do not like to pause to think. They want to begin immediately toward a solution, and the first thing we must teach beginning problem solvers is to describe problems in their own words so they understand the conditions of the problem.

The second step, "P," is to plan a solution. How can one proceed from what is given to the solution? What kinds of knowledge will be useful in solving this problem? How can this knowledge be systematically used to solve this problem? Frequently in physics this step calls forth some algebraic relationships and equations that give the relationship among the various quantities in the problem. Planning a problem-solving strategy makes use of empirical and algebraic relationships.

The third part, "I," of this strategy is implementation. To implement the plan of solution often means putting numerical values for quantities in algebraic equations and computing a numerical result. Proper implementation of a planned solution saves nearly all the numerical calculation to the end. Beginners start by putting numbers into the equations, and they lose sight of the relationships between the variables. They cannot simplify their results nor see how the quantities are related to each other.

After a solution has been obtained, the final part, "C," of problem solving is to check the result. Does the result make sense? How does the answer fit with one's own experience of nature and one's own sense of how the problem might have worked out if one had guessed at the beginning? What are some variations on the problem? What happens if the mass is doubled or the force is doubled or a quantity goes to zero? Do the results obtained for the problem still hold true?

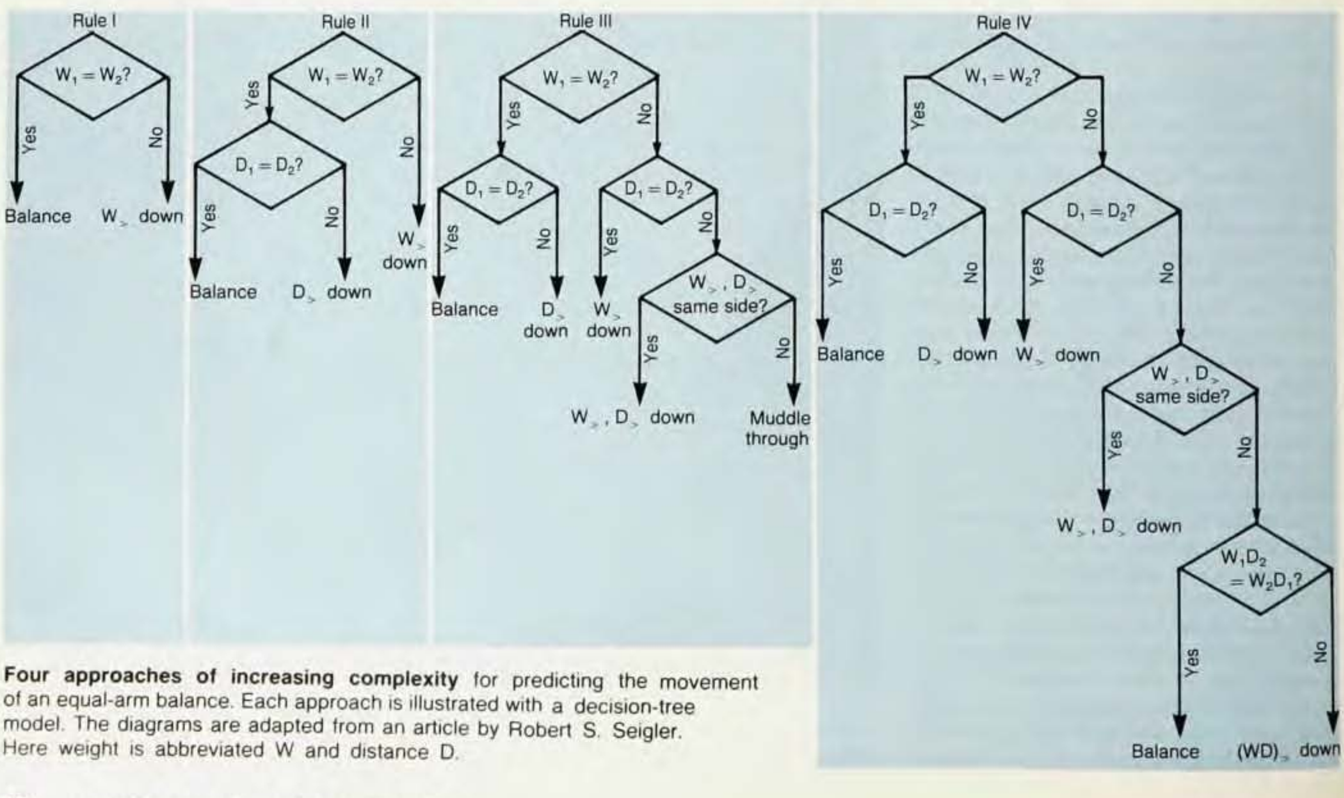
This "D, P, I, C" strategy should be applicable to a wide range of problems. Perhaps, just as physics problems serve as good examples for research into problem solving, physicists can take a more active role in modeling good problem-solving strategies for others.

Constructing solutions

A second group of researchers are called constructivists. Their research has grown out of the work of Jean Piaget, the Swiss psychologist. In contrast to the information-processing people, who have tended to focus more on the conscious steps of problem solving, the constructivists have talked more about the internal mental processes by which strategies of problem solving are constructed and how these strategies (and the fundamental concepts they imply) are changed as one grows and learns. (This is what I have called the "mental modeling clay" model, because it emphasizes the plasticity of cognitive and mental structures.) This school emphasizes philosophical understanding of what knowledge is and how new knowledge develops. Modern physicists and Piagetians are what might be called radical constructivists.

The philosophical school of empiricists believes that the laws of Nature are external to the minds of humans and that anyone who looks at Nature can discover exactly the same laws. For example, they believe Newton's laws need not be named for Newton; these laws are the laws of nature. The empiricist finds Nature objective and impartial and claims physical observables as the essence of Nature.

At the opposite extreme are nativ-



Four approaches of increasing complexity for predicting the movement of an equal-arm balance. Each approach is illustrated with a decision-tree model. The diagrams are adapted from an article by Robert S. Seigler. Here weight is abbreviated W and distance D.

ists, who believed that Nature is a jumble of random processes and that the laws of Nature exist innately in the minds of human beings. The nativist view may be expressed as "We humans think and reason; therefore the laws of Nature can be found in rational thought. According to the nativists, logic and mathematics are innate to humans and are unique structures to explain the processes of Nature.

Modern physics has rejected both the empiricist and nativist views of Nature, and holds that the laws of physics and human minds are somehow combined together. It is in the experience–mind interaction that understanding is constructed. The laws of Nature are built at the interface between our sensory experiences of the external world and our reasoning about those experiences. Nature is an open system—always inviting us to understand its works in different ways as we transform our sensory data through ever-evolving mental constructs.[16]

Piaget suggested the dynamic interaction model of assimilation–accommodation–equilibration as the way knowledge and problem-solving strategies are constructed.[17] This model is the mental equivalent of the homeostatic process that takes place in living systems; it is the process of self-regulation. This model sees the development of knowledge as a self-regulating process in which one's experience of nature through sensory input is compared with one's interior understanding of nature through one's use of mental structures. When our experience does not match our understanding, disequilibration occurs. Piaget argued that human beings are made uneasy by disequilibration. Humans are naturally led to seek additional experiences or to reorganize their understanding. We mentally evolve to a state of equilibrium in which our concepts are adequate to understand the things that previously confused us. We are in temporary equilibrium until we are challenged again by new experiences that do not fit our understandings.

Let me illustrate this process with the following story. Once a person went away on a trip. The day she returned home on a commuter airline, her husband, who was to pick her up, had car trouble and couldn't get to the airport in time to meet her. Meanwhile she had arrived and was waiting for the agent to get her bags from the airplane. When the telephone rang, she performed her civic duty and answered for the agent,

"Commuter airlines."

"Would you please get a message to my wife arriving on your 7:00 pm flight?" the caller said.

(Of course, she immediately recognized the voice on the phone; her husband's mind would not let him recognize his wife.)

"Yes, of course. How do you spell the name?" the supposed agent responded.

As you can imagine, a very interesting conversation ensued while the caller described his wife and spelled her name to who the supposed "agent." When she giggled at the description, the jig was up.

I think this simple story illustrates the main features of the Piagetian model. We do not receive our sensory input from Nature without preconceptions. During our lives we build up a whole array of mental structures to make sense of our experiences. When we obtain an input that does not match our present mental schemes, we are disequilibrated and need additional input to modify our mental processes or our interpretations of our experiences.

In this model of the dynamic interaction between the minds of people and their external experiences, the time when we are most likely to develop new understandings and new strategies is when our present experiences do not fit our mental preconceptions. This period of disequilibration, of being slightly confused, is the time when we are most likely to make intellectual growth.

One implication of this Piagetian model is that physics teachers need to provide concrete experiences for students to analyze, experiences that are likely not to match the students' preconceived ideas of the way Nature behaves. To develop reasoning, people need to be puzzled by their own experiences, not by the verbal explanations given by a teacher. Laboratory activities and classroom exercises need to be designed to be slightly puzzling to the students, given their present mental constructs. The students need to confront these puzzles in an environment where understanding them makes a difference, not just to please the professor, but for their own self-esteem, self-confidence and mental equilibrium.

A detailed teaching strategy called the Learning Cycle has been suggested by Robert Karplus as a way to bring these Piagetian ideas into classroom practice.[2] At the University of Nebraska–Lincoln we use the ADAPT program[18] based on the work of Piaget and Karplus. In the ADAPT physics class we spend less time talking about the laws of physics and the teacher's understanding of them. Rather, the students experience first-hand the behavior of nature and are required to construct their own sense of meaning from its performance.

In conclusion, these three avenues of problem-solving research do not seem to admit any simple unity. How can the constructivist view help us understand people's misconceptions about the laws of nature? How do task analyses and expert–novice differences relate to the self-regulation model of cognitive development? I do not know. This young branch of cognitive science awaits its first Newton. However, there are two unifying aspects of this research.

First, the research is empirically based on the performance of problem solvers. This research changes the emphasis of problem solving from the *problem* to the *solver*. This change may offer us new insights into the teaching–learning environment. Second, *physics* problems serve as a key role in this research. Perhaps that is enough to invite us physicists to participate in the struggle to understand problem solving.

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