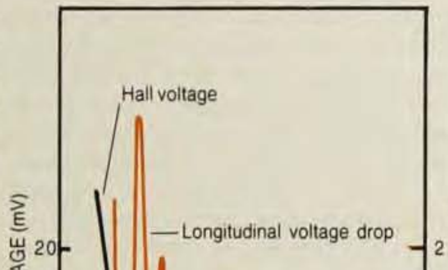
Quantized Hall effect yields e^2/h to a part per million

Several recent experiments investigating the Hall effect in its quantum-mechanical limit have made it clear that Nature is providing us with a fundamental unit of resistance. The extraordinary accuracy with which these experiments appear to be obeying the naive theoretical expression for the quantized Hall resistance is at once promising and puzzling.

If this agreement of experiment and simplistic theory—already close to one part per million—persists for yet another order of magnitude, it will provide us an independent determination of the fine-structure constant as good as any we now have. This new measurement of α would serve as an important test of quantum electrodynamics. Furthermore, the quantized Hall resistance would provide an absolute standard of resistance far more convenient than the arduous calculable-capacitor technique hitherto employed by the National Bureau of Standards.

The puzzle is: Why are the experimental results in such spectacular agreement with an expression whose derivation by Tsuneya Ando and his colleagues, six years ago at the University of Tokyo, passes lightly over the complexities of real solids and their imperfections? If the quantized Hall effect obeys Ando's expression to better than a part per million, a more fundamental theoretical explanation is required. Such an extraordinarily precise and simple regularity, independent of experimental parameters, is likely to be telling us something deep about Nature.

A year ago, Klaus von Klitzing and his coworkers demonstrated[1] at the University of Würzburg (Germany) that the Hall resistance of a two-dimensional electron gas in the inversion layer of a silicon MOSFET transistor, subjected to high magnetic field and low temperature, exhibited quantized steps as the transistor's gate voltage varied its Fermi level. At each step, the Hall resistance is given (to about one part in 10^6 in their latest experiments) simply, in terms of fundamental constants, by $R_H = h/e^2n$, where the integer n is the number of "Landau levels" lying below the Fermi energy of



Hall resistance, as measured by Hall voltage across von Klitzing's MOSFET, falls in quantized steps as gate voltage increases, raising the Fermi energy through successive Landau levels. Resistance plateaus occur at deep minima in the longitudinal voltage drop, when current is lossless because Fermi energy is between Landau levels. Shallower minima are due to spin and valley degeneracies.

the gas, without further reference to experimental parameters. In MKS units, h/e^2 is about 25 813 ohms. In the same unit system, the fine-structure constant is given by $\alpha = (e^2/h)(\mu_0 c/2)$, where μ_0 is the permeability of the vacuum.

Last fall Daniel Tsui and Arthur Gossard at Bell Labs observed the same effect[2] under less stringent magnetic-field and temperature conditions at a GaAs–$Al_xGa_{1-x}As$ heterojunction. Barry Taylor's group at the National Bureau of Standards is currently investigating both techniques in hopes of ultimately producing a measurement of α and an absolute resistance standard good to a part in 10^8.

The quantized Hall effect was observed by von Klitzing and his colleagues in a two-dimensional, degenerate electron gas, formed at the inversion layer in the silicon just below the oxide surface of a metal-oxide-semiconductor field-effect transistor. At a temperature of 1.5 K, the electrons are confined in the ground state of the potential well generated by the transistor's positive gate voltage. The electric field under the gate, normal to the oxide–silicon surface, thus effectively prevents the trapped electrons from moving in the z direction, normal to the surface.

If one now imposes a modest magnetic field in the z direction and starts the electrons moving along the surface with an electric field in the x direction, one observes the conventional Hall effect: The Lorentz force on the electrons starting out in the x direction will produce some deflection in the y direction. But if one increases the magnetic field to very high values—150 kilogauss in von Klitzing's experiment—the phenomenon becomes much more intriguing. The degenerate electron ground state breaks up into well-separated Landau levels. An electron in the nth Landau level will have an energy of $(n + 1/2)\hbar\omega_c$, where ω_c, which is proportional to the magnetic field strength B, is the cyclotron frequency. These Landau energy levels represent electrons orbiting around the field lines with classical radii of about 70 Å (for the lowest level in silicon at 150 kilogauss).

When one adjusts the gate voltage to set the Fermi level between two such Landau levels, so that all the Landau levels below the Fermi energy are fully occupied, and all above are empty, a curious thing happens. In response to any electric field in the x–y plane, say E_x, the electrons in the two-dimensional gas of occupied Landau levels will drift only in the y direction, perpendicular to E_x, in response to the magnetic field in the z direction; but there can be no electron current in the direction of the electric field. This comes about because (in the absence of imperfections) the electrons cannot suffer dissipative scattering. With their own Landau levels filled, the only place to scatter into would be an unoccupied Landau level. But that involves a jump across an energy gap of at least $\hbar\omega_c$—energetically impossible at 150 kilogauss and 1.5 K. In the absence of a dissipative mechanism, the electric field can do no work on the current, and hence will always be orthogonal to it.

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This is seen in a well-known student lab demonstration, where crossed electric and magnetic fields in a vacuum tube results only in a drift of the electron cyclotron orbits in the direction orthogonal to both.

In the language of the Hall effect, one describes the condition when the current flows only normal to the electric field by saying that the diagonal elements of the conductivity matrix, σ_{xx} and σ_{yy}, vanish. In this case, in the absence of scattering, the off-diagonal "Hall conductivity," σ_{xy}, which usually has a complex dependence on material parameters, takes a very simple form. The Hall conductivity becomes simply Ne/B, where N is the two-dimensional density of electron carriers.

But N also has a very simple form that cancels the B dependence when the Fermi level lies between Landau levels. With n Landau levels filled, the carrier density is simply nBe/h, giving $\sigma_{xy} = ne^2/h$, or a two-dimensional Hall resistance $R_H = h/e^2n$, independent of experimental details.

The experiment of von Klitzing and his colleagues was designed to show this stepwise decrease of the Hall resistance as the MOSFET gate voltage pushes the Fermi level up through consecutive Landau levels. A constant current was made to flow from the source to the drain of the transistor along a channel 400 microns long and 50 microns across. MOSFET's of different sizes were also used, to show that the effect was quite independent of experimental dimensions. Probes measured the voltage drop along the current direction and the Hall voltage generated across the width of the channel by the concentration of electrons near one edge in response to the Lorentz force.

As the gate voltage was continuously raised, the voltage drop along the current exhibited periodic deep minima, indicating the almost complete absence of ohmic scattering loss—presumably when the Fermi level was passing between Landau levels. At these times, the only electric field in the conducting channel is the transverse field due to the Hall pile-up of electrons at one edge. During these deep minima, when the current was flowing essentially without loss along the transistor, the Hall voltage across the current flow exhibited a series of plateau steps. The Hall resistance at each step, measured by the ratio of Hall voltage to the current, turned out to be precisely h/e^2n, within the accuracy of the experiment, with n determined simply by counting the minima through which the longitudinal voltage drop had passed.

The Bell Labs group found essentially the same result, with epitaxially grown GaAs-Al_xGa_{1-x}As heterojunctions of very high electron mobility replacing the MOSFET's of von Klitzing's group. In this case the potential well is provided not by a gate voltage, but rather by the ionized donors in the modulation-doped Al_xGa_{1-x}As layer, and the bending of the conduction band edge as it drops several hundred millivolts to the GaAs layer (PHYSICS TODAY, April 1979, page 20). In the absence of a gate voltage, Tsui and his colleagues moved the Fermi level through successive Landau levels by varying the magnetic field.

The cyclotron frequency, and hence the separation of Landau levels, is inversely proportional to the effective electron mass. Because this effective mass in GaAs is only about a third of its value in silicon, the Bell Labs group was able to make do with a magnetic field of only 80 kilogauss, at a temperature of 4 K. This makes the heterojunction technique particularly attractive as a possible standard of absolute resistance for widespread laboratory use. There is a significant price break in superconducting magnets at about 90 kilogauss. Below this intensity, relatively inexpensive niobium–titanium superconducting coils suffice; above it one requires far more expensive niobium–tin magnets.

Theoretical problems. If the reader finds our justification of the expression h/e^2n for the quantized Hall resistance too cavalier to be good to better than 10 or 20%, he is in good company. Ando and his colleagues had originally anticipated neither the high precision of their expression, nor the seemingly contradictory fact that zeros of σ_{xx} and the steplike plateaus of Hall resistance persist over quite wide intervals of the gate voltage (or magnetic field). If the Landau levels were perfectly narrow and the density of states between them vanished, as one would expect in the absence of localized imperfection states, the Fermi level would simply jump from one Landau level to the next with a very small change of gate voltage, producing almost no observable Hall resistance plateaus. But if the surprisingly broad plateau widths attest to an abundance of localized states between Landau levels, due to impurities and surface imperfections, why is the precise h/e^2n result not badly spoiled?

Among the theorists now attempting to explain the high precision with which the quantized Hall effect gives h/e^2, independent of the detailed properties of the current-carrying material, are Ando himself and Richard Prange of the University of Maryland. Both have recently shown, in essentially opposite approximation limits, that the presence of localized impurity states should not affect the Hall current. Ando and his colleague H. Aoki have carried out[3] a renormalized perturbation calculation in the weak-scattering limit. Prange's, on the other hand, is an exact calculation[4] of a simplified strong-scattering model, with impurity scatterers described by delta functions. Both calculations conclude that while the localized impurity states, broken off from the Landau levels, carry no current, the extended electron states in the Landau levels compensate precisely for this loss of carriers by carrying just enough extra current to preserve the quantized Hall result. Neither calculation, however, deals with the additional problem of electron–electron interactions.

Robert Laughlin, a theorist at Bell Labs, began with the prejudice that a phenomenon of such precision and simple generality must not depend on specific models and approximations. He therefore sought, and believes he has now found, a quite general proof[5] of the precise e^2/h quantization of the Hall conductivity. His proof, he told us, depends only on the gauge invariance of the electromagnetic interactions, and on the existence of a "mobility gap" everywhere in the current-carrying material.

The Landau levels in a real, imperfect material are broadened to an extent that narrows, or even eliminates, the energy gaps. But the Landau tails, Laughlin argues, are due to localized states that do not contribute to the Hall current. There remains therefore a mobility gap between the extended wave functions in the central regions of adjacent Landau level, even when their tails overlap—and this is sufficient for his argument.

Laughlin invokes gauge invariance by putting forward a *Gedankenexperiment* in which the two-dimensional Hall current channel is closed upon itself to form a loop. In addition to the magnetic field piercing the surface, he considers an adiabatically increasing flux threading the loop, one flux quantum at a time. The current I in the loop will be given by the rate of change of the total energy with this flux. Localized electron states, which can be arbitrarily gauge transformed in phase without change, will contribute nothing to the energy change, and hence nothing to the current. But the extended electron states that persist coherently around the loop (as do superconducting Cooper-pair wave functions) are "pinned" in phase; they can change only by multiples of 2π in a complete circuit. Laughlin argues from gauge invariance that these states can respond to the addition of magnetic flux only by transferring precisely an integral number of electrons per flux quantum (h/e) from the local Fermi level at one edge of the loop to the Fermi level at the other edge. The energy change corresponding to a flux change $\Delta\Phi$ is thus precisely $neV_H\Delta\Phi/(h/e)$, where V_H is the Hall voltage between the edges. Recalling that $I = \Delta E/\Delta\Phi$, one has immediately the desired result that the Hall resistance $R_H \equiv V_H/I = h/ne^2$.

How precise is this result expected to be? Laughlin points out that although the virtually lossless Hall current is in some sense a "supercurrent," it differs from the true superconducting case by permitting a minute electric field component in the direction of the current. This residual nonvanishing σ_{xx} results from the "hopping conductivity" of carriers that persists at any finite temperature, even when the Fermi level is in the mobility gap. This hopping conductivity, which depends on the mean free path in the material in question and vanishes at absolute zero, should affect the Hall resistivity to much less than one part in 10^8 at the temperatures of the quantized Hall experiments, he estimates.

Using von Klitzing's technique, but with a more sensitive voltage measuring system, Taylor, Marvin Cage and their colleagues at NBS, collaborating with the Bell Labs group and Robert Wagner and Charles Levine at the Naval Research Lab, hope to have measured the fine-structure constant to within a few parts in 10^7 by the end of this year. At present, α is known to one part in 10^7 from two sources: The determination of α from the anomalous magnetic moment of the electron (the "$g-2$" experiments) depends on the correctness of quantum electrodynamics. The measurement of the proton's gyromagnetic moment, on the other hand, yields a QED-independent determination of α. —BMS

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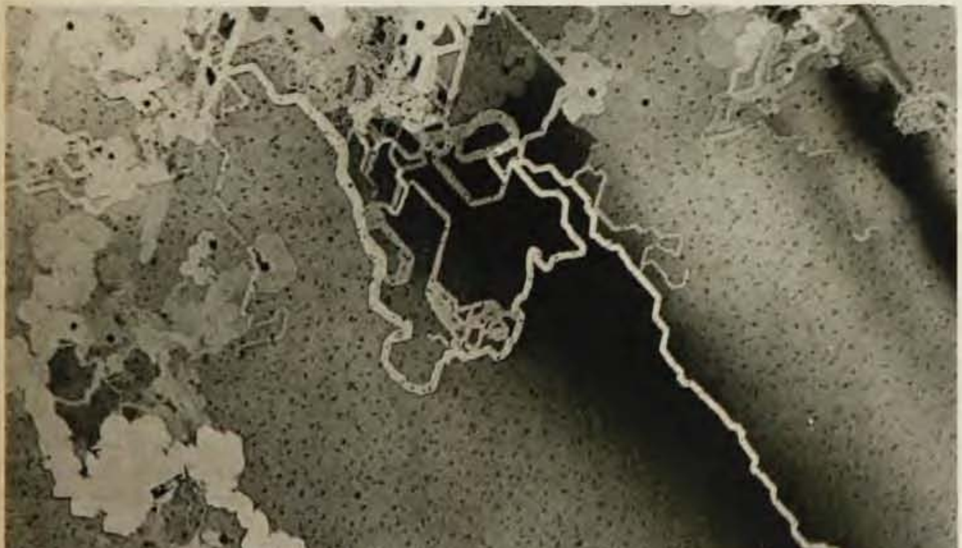
Catalysis of carbon gasification

The conversion of carbon to its oxide gases or methane, by interaction with oxygen, steam, hydrogen or carbon dioxide, are reactions of great practical importance. The oxidation of carbon to CO_2 and CO provides much of mankind's heat and power, and the formation of "syngas" (a mixture of carbon monoxide and hydrogen) by steam gasification of carbon is an important step in the conversion of coal to synthetic petroleum (PHYSICS TODAY, August 1978, page 32). These reactions, together with the hydrogenation of carbon to form methane, and its interaction with CO_2 to form CO, are the principal gasification reactions of carbon.



Catalytic channeling by microparticles of platinum catalyst, burrowing along the surface of a highly purified flake of crystalline graphite in a 5-mm atmosphere of oxygen at 750°C. This unique catalytic action in the oxidation of graphite and other carbon-gasification processes is observed in progress by the controlled-atmosphere transmission electron microscope developed by Terry Baker at Exxon. In an oxidizing atmosphere, the catalytic particles dig preferentially along the ⟨10$\bar{1}$0⟩ axis of the graphite basal plane.

In the absence of impurities, these reactions are all surprisingly sluggish. Although it has long been known that small amounts of naturally occurring or deliberately added impurities can alter the rates of these reactions dramatically, the precise catalytic mechanisms by which these additives act have for the most part remained a mystery.

A peculiar phenomenon that appears to play a central role in these catalytic gasification reactions is the cutting of channels and pits by catalytic microparticles as they spontaneously burrow along and into the crystalline surfaces of graphite. Over the past decade, Terry Baker, first of Harwell (England) and more recently at Exxon, has developed an electron-microscopy technique for observing this channeling action in progress. With a controlled-atmosphere transmission electron microscope (CAEM) he is now able to film catalytic particles as they cut channels along the various crystal axes of the graphite basal plane in the presence of the reactant gases at temperatures up to 1350° C, with a resolution of 25 Å. At the March APS Meeting in Phoenix, Baker reported recent results of his CAEM studies of catalytic graphite gasification.

Although these catalytic particles and the channels they cut are frequently smaller than 100 angstroms, far below the resolution of optical microscopes, they were first observed in optical studies by John Thomas, at the University of Bangor in Wales in the 1960s. In the early 1970s Douglas McKee at General Electric extended these techniques by the use of "hot-stage" optical microscopes, which could observe the catalytic gasification processes in progress. McKee and Philip Walker's group at Penn State have also performed extensive analytical and thermodynamic investigations to elucidate the chemical reactions involved.

Hot-stage optical microscopes can observe the gasification reactions at high temperatures and high reactant-gas pressures. But the details of the unique catalytic channeling in graphite eludes their limited resolution (about one micron). The particle beams of conventional electron microscopes, on the other hand, would be badly degraded while passing through a reactant gas atmosphere on the way to the surface under inspection.

The controlled-atmosphere electron microscope addresses this problem by holding the sample in a differentially pumped gas reaction cell. A