boundary of a second stability region.

Although the complexity of the mhd calculation makes it difficult to predict precisely at what value of beta the second stability region will begin, it is unlikely that present steady-state tokamak experiments can get there. Two groups have therefore been availing themselves of pulsed toroidal devices, which can briefly attain very high beta, to look for the high-beta stability region. The belt-pinch experiments conducted by Helmut Zwicker, Rolf Wilhelm and their colleagues at Garching appear to have found the second stability region at a volume-average β of about 10%—albeit for times very short compared with the energy confinement time. In their small device, whose purpose is to study plasma conditions that are much more difficult to produce in steady-state machines, an elongated ("belt-shaped") toroidal plasma is brought abruptly to high beta by magnetic-pinch shock heating.

Robert Gross, Thomas Marshall, Gerald Navratil and their co-workers have achieved much the same results[4] in a similar pulsed machine at Columbia, Torus II. "We're not sure yet, but we think we've reached the second stability region—at least briefly," Gross told us. At a beta of about 10% the Torus II plasma has remained stable for pulse times much longer than the characteristic growth time (a few microseconds) of the ballooning modes.

The Columbia group is also investigating the unstable region between the two stable beta regimes. "We want to know whether this unstable middle ground (roughly between $\beta = 3\%$ and 10%) is an abyss, a swamp, or merely a wading pond," Gross explained. In a steady-state tokamak, where one has to make one's way through this region to get to the hoped-for high-beta stability, it is important to know how unstable the plasma would become during the crossing.

Advanced fuel cycles become interesting if one can in fact keep tokamak plasmas stable for beta well above 10%. One must compensate for the fact that D–D fusion, for example, yields only $1/30$ the power of the D–T reaction at a given beta, by operating a D–D reactor at a beta level more than five times as high. But the D–D reaction offers the great advantage of burning only deuterium, an abundant, nonradioactive fuel. There would be no need for a lithium blanket to breed tritium. One could also remove the He^3 produced in a D–D reactor for use as fuel in a D–He^3 reactor.

To ignite D–He^3 would require a temperature in excess of 40 keV. Coppi believes that after D–T ignition, a first-generation Ignitor might well reach sufficient temperature to start a D–He^3 burn. His idea is that one would shut off the tritium supply after the burning D–T plasma gets hot enough, and then start feeding the Ignitor helium-3. The D–He^3 reaction produces a 14.7-MeV proton in place of the neutron coming out of the D–T reaction. Because the charged protons can be deflected, they do much less damage to the reactor walls than do neutrons; furthermore, their energy can be directly converted to electricity, without requiring an intermediate thermal blanket.

At the high temperatures required for advanced-fuel ignition, the decreased Coulomb scattering cross section takes a longer time to thermalize the energetic fusion products. Robert Conn and Geoffrey Shuy (UCLA) have recently calculated that this longer time scale allows various nuclear scattering processes to become important in kicking deuterons up to higher energies—permitting D–D and D–He^3 ignition at values of the confinement parameter significantly lower than previous estimates. At these high temperatures and magnetic fields, energy loss by electron synchrotron radiation becomes a concern. Steven Tamor of Science Applications has recently produced a detailed computer code that appears to show that the plasma energy losses from synchrotron radiation would not be as severe as had been feared; such losses would not be a serious impediment to advanced fuel ignition.

Prospects. George Miley (University of Illinois) has suggested a national energy scheme based largely on D–He^3 and D–D fusion reactors. Small D–He^3 tokamaks of a compact, Ignitor-like geometry would produce electricity in the vicinity of metropolitan areas. Much larger "catalyzed" D–D reactors, requiring more cooling water and producing more radioactivity, would operate in remote areas. Much of the He^3 produced in the D–D reactors would be extracted for use by the smaller D–He^3 reactors. Most of the energy produced in the large D–D machines would be used to produce synthetic chemical fuels (methanol, hydrogen, and so on) at the reactor site, Miley suggests. Early in the next century, he argues, we will need twice as much synthetic fuel as electricity.

If Coppi's optimism is vindicated, the small D–He^3 reactors could be operating well before the D–D plants could begin to supply He^3. He^3 is not abundant in Nature; but there appears to be enough surplus He^3 available as a by-product of thermonuclear weapons production for the short run. Miley also suggests that the breeding of fission fuel in depleted-uranium blankets would be an important function of the D–D reactors in their early years.—BMS

References

1. B. Coppi, A. Taroni, in *Theory of Magnetically Confined Plasmas*, B. Coppi et al. (eds.), Pergamon, N. Y. (1979), page 287.
2. B. Coppi, A. Ferreira, J. J. Ramos, Phys. Rev. Lett. **44**, 990 (1980).
3. C. E. Wagner, Phys. Rev. Lett. **46**, 654 (1981).
4. R. A. Gross, T. C. Marshall, Comments Plasma Phys. Cont. Fusion **5**, 233 (1980).

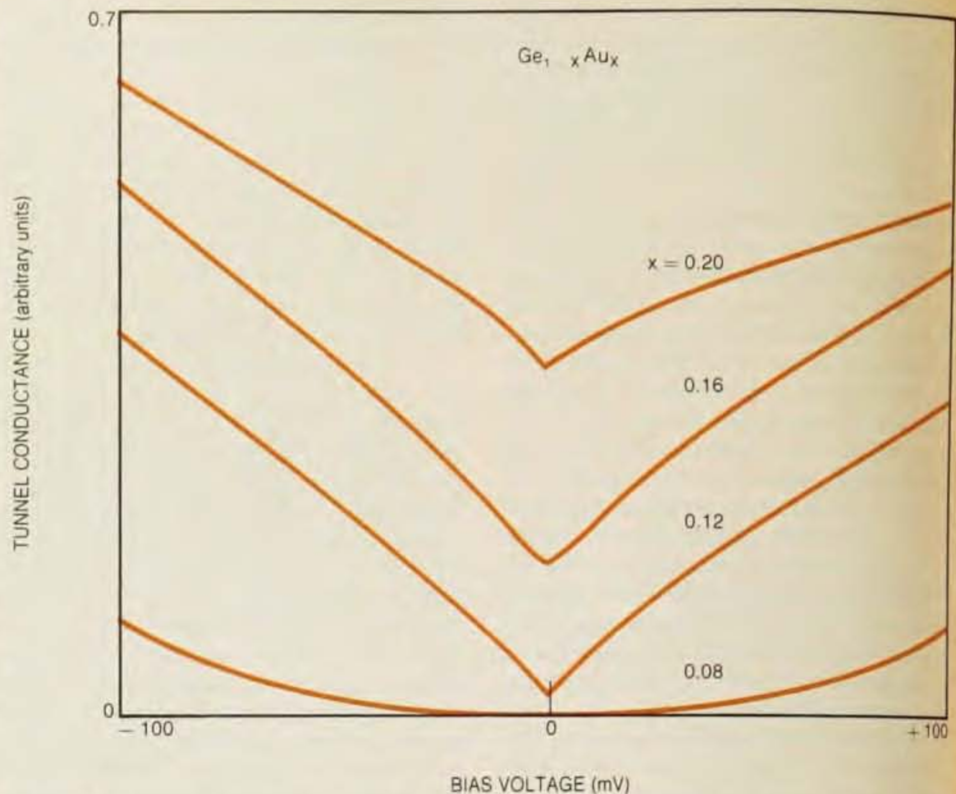
Metal–insulator transitions

In one, two and now three dimensions, the behavior of amorphous materials is challenging the traditional concepts of metals and insulators. Over a year ago experiments showed that one- and two-dimensional disordered materials appear to move continuously rather than abruptly from insulating to metal-like phases as their randomness decreases. Even in the metallic phase, their conductivities do not exhibit the temperature dependence typical of metals with truly periodic lattices. These results confirmed a scaling theory of localization which predicted that at least in two dimensions the electrons remain localized for arbitrarily small amounts of disorder. This theory did not include any effects the electrons might have on one another, although other treatments have indicated that such effects might be important. Recent experiments have now measured properties in addition to conductance, and some have discovered effects indicative of interacting electrons. Clearly, any theory must now incorporate features both of localization and of electronic interactions, although no one knows in exactly which circumstances each effect might dominate. The one theory so far to incorporate both features has received considerable attention, and will no doubt stimulate greater theoretical and experimental activity, aimed at creating order out of these disordered systems.

Two-dimensional systems. The theory of localization that was so remarkably successful in predicting the unusual behavior seen in one- and two-dimensional disordered metals is a scaling approach proposed in 1979 by Elihu Abrahams (Rutgers), Philip W. Anderson (Princeton and Bell Labs), Donald C. Licciardello (Princeton and Bell Labs) and T. V. Ramakrishnan (Bell Labs and the Indian Institute of Technology, Kampur). (See PHYSICS TODAY, October 1979, page 17.) The features it described were observed in thin films and wires, and in MOSFET devices. These features include (in two dimensions) a logarithmic increase in resistivity as temperature decreases and the absence of any minimum metallic conductivity. The latter concept had been proposed by Sir Nevill Mott (Cambridge) as a sharp dividing line between metallic and insulating behavior. According to Mott's picture, any material with a critical (or greater) degree of disorder—as measured macroscopically by its conductivity—should be an insulator, with conductivity falling abruptly to zero at a temperature of 0 K. By contrast, Abrahams and his colleagues concluded that in two dimensions the random system never becomes a true metal and has a crossover in behavior rather than a sudden phase transition.

Also in 1979, Patrick Lee (Bell Labs) published results of an approximate numerical scaling calculation based on localization but differing in some predictions from those of Abrahams and his colleagues. Lee and Daniel Fisher (Bell Labs) reported at the APS meeting in Phoenix this March that recent calculations that dispense with the earlier approximations now agree with the localization scaling theory.

Despite the early success of the localization theory, many have still wondered whether electronic interactions aren't a part of the picture in disordered solids. Because electrons in these materials spend more time in a particular region of space, they are more likely to be affected by the charge of other electrons in the area. Boris Altschuler and A. G. Aronov (Leningrad Nuclear Physics Institute) tackled[1] the problem of electronic interactions in the weak scattering limit in three dimensions. Lee has recently worked with them[2] to apply the same approach to disordered systems in two dimensions. Their work predicts that the conductivity should vary logarithmically with temperature—the same behavior as that predicted by the localization theory. Thus conductivity measurements do not distinguish between these localization and interaction effects. Instead one must measure another property that might reflect the underlying cause for the changes in conductance. If localization effects dominate, for example, decreased conductance results from lowered mobility of the electrons. Thus the Hall effect, which measures the number of charge carriers and not their mobility, could sense which effect is at work. Altschuler, Aronov and Lee predicted that if electronic correlations are important the Hall constant should contain a term that varies logarithmically with temperature, with a coefficient that is twice as large as the log T term in the conductance. If only localization terms are important, this term should be absent.



Strong dip in tunneling conductance at zero bias voltage increases as the concentration of gold in germanium–gold alloys decreases, corresponding to greater disorder. The dip implies a depletion in the electronic density of states near the metal–insulator transition, and gives evidence for electronic interactions in disordered systems. (Figure from reference 12.)

The Hall effect was measured in MOSFET devices both at Bell Labs[3] by David J. Bishop, Daniel C. Tsui and Robert C. Dynes and at Cambridge University[4] by Michael J. Uren, Richard A. Davies and Michael Pepper. The two experimental groups found the log T term with the predicted factor of 2 in the coefficient (at high field) indicative of electronic interactions. At low fields, the Cambridge group recently found that the log T term enters with only a factor of one, implying that both localization and electronic interactions are present under these conditions.

Another feature of two-dimensional systems that should be sensitive to the mechanism of electron behavior is the magnetoresistance. Altschuler, Dimitri Khmel'nitzkii and Anatoly I. Larkin (both of the Landau Institute, Moscow) and Lee predict[5] that localization effects will cause the material to have greater conductivity in the presence of a magnetic field, that is, to have a strong negative magnetoresistance. If electronic interaction dominates the sample's behavior, the magnetoresistance should be positive. Negative magnetoresistance has been measured in MOSFET experiments performed by Yoichi Kawaguchi (Gakushin Women's Junior College) and Shinji Kawaji (Gakushin University, Tokyo)[6], and both the Bell Labs and Cambridge teams. Other experiments have detected negative magnetoresistance in thin films. They include studies on copper[7] by a team consisting of L. Van den Dries, C. Van Haesendonck and Y. Bruyseraede, of the University of Leuven, Belgium, and G. Deutscher (Tel Aviv University, Israel) and those on platinum by Robert Markiewicz (Northeastern). At IBM, Praveen Chaudhari and Hans Habermeier (on leave from the Max Planck Institute, Stuttgart) observe positive magnetoresistive effects whose interpretation is unclear but which are in good part related to the quenching of superconductive fluctuations.

Lee commented to us that the ambiguous picture in two dimensions suggests the need for both localization and correlation effects in any models, and that the two may dominate in different regimes, such as in low versus high magnetic field. Pepper remarked that he and Mott now feel that localization is suppressed by high magnetic fields and that, when this suppression occurs, the material regains its minimum metallic conductivity at absolute zero.

Three-dimensional experiments have just recently begun to provide evidence for both localized and correlated behavior. In this dimension, the localization theories predict that a well defined metal–insulator transition does exist but that the metallic conductivity at zero K disappears gradually as the disorder increases, rather than dropping discontinuously to zero below some minimum value of conductivity, as Mott had predicted. The experimental case for or against a minimum metallic conductivity is so far inconclusive. Evidence for a remarkably sharp metal–insulator transition comes from studies of temperatures down to 1 mK in silicon doped with phosphorous. The work was done[8] by Thomas F. Rosenbaum, Klaus Andres, Gordon A. Thomas and Ravindra N. Bhatt at Bell Labs. As this group lowered the concentration of the dopant, the conductivity extrapolated to 0 K dropped precipitously over a very narrow range of concentration. The drop was so rapid, commented Thomas, that their uncertainty in measuring the dopant density leaves open the possibility of Mott's discontinuity, although the group found no abrupt change near the predicted value of minimum conductivity. They measured values of metallic conductivity that were 1000 times lower than this predicted minimum value.

Two other studies in three dimensions measure slow variations at the metal–insulator transition more similar to those seen in two dimensions. At the University of Illinois, Brian W. Dodson, William McMillan, Jack Mochel and Dynes formed[9] amorphous films of germanium–gold alloys and varied the conductivity by altering the concentration of gold. As the conductivity was lowered below that predicted to be the minimum metallic conductivity, the samples still displayed finite conductivity at absolute zero. On the insulating side of the transition, the conductivity of the samples decreased exponentially, as is typical of random systems with localized electrons. At Bell Labs, Dynes and John P. Garno observed[10] similar variations in the conductivity of granular aluminum films.

In a large number of highly resistive metals that are disordered but not to the point of localization, the resistivity rises as the temperature drops. Ramakrishnan remarked that this behavior could be a precursor to localization effects. In the recent experiments, which probed closer to the metal–insulator boundary, both Dodson and his colleagues, and Rosenbaum and his associates (including Lee)[11] studied the changes in conductivity as their samples were cooled. In each case, the temperature-dependent part of the conductivity varies as a power of $1/2$ or $1/3$. (By contrast the traditional Boltzmann transport theory predicts a strong temperature dependence with integer exponents.) In the germanium–gold samples, the conductivity decreases with lowering temperatures while in the doped silicon materials, it increases. The different direction of the conductivity changes in these two experiments may be related to the different ranges over which the electron charges are screened from one another, according to Lee. In the phosphorous-doped silicon samples the screening has a very short range while in most dirty metals the screening length becomes much longer. In either case the fractional-power dependence on temperature is consistent with predictions of Altschuler and Aronov.

Aside from the property of conductivity, one feature of three-dimensional systems that might be sensitive to the presence of electron correlations is the electronic density of states. This density can be determined by the tunneling conductance, since the tunneling is a direct measure of the states available to the electrons on the sample side of the barrier. For many years, experiments on slightly disordered metals had manifested a zero-bias anomaly in the tunneling conductance. Several groups tried to interpret the experiments in terms of a gap in the electronic density of states but none produced a consistent explanation. In their model of electron interactions, Altschuler and Aronov proposed[1] an explanation that was valid only in the weak scattering limit.

McMillan speculated that this tunneling effect might grow very large for systems sufficiently disordered to approach the metal–insulator boundary. In McMillan's picture of the tunneling effects, the distance over which electrons would be screened from one another goes to infinity at the metal–insulator transition and the electrons then see one another's charge. The resulting repulsion would strongly decrease the density of states at the Fermi surface and in fact make it go to zero at the insulator boundary. McMillan and Mochel measured[12] the tunneling conductance on germanium–gold alloys and found surprisingly large dips at zero bias, including a total suppression of tunneling at the insulator region (see figure). Dynes and Garno also measured the tunneling conductance in their granular aluminum samples and studied the way in which the tunneling dip develops as the disorder of their system increases. The observed behavior is consistent with McMillan's predictions.

Tunneling behavior and the fractional-power-law dependence of conductivity are among the predictions of a new theory of McMillan that is the first to incorporate effects both of localization and of electron correlations. In this scaling theory, McMillan defines three characteristic energies. The first two relate to electron diffusion and electronic spacing, and their relative values determine whether the electron is localized or extended. The third energy scale is the Coulomb potential with a screening factor. From these three energy scales, McMillan forms two dimensionless parameters, one of which is that used in the scaling theory of localization by Abrahams and his colleagues. This theory makes some predictions that still must be verified. If the recent flood of papers in the literature is any indication, that theory may not remain untested for long. —BGL

References

1. B. L. Altschuler, A. G. Aronov, Solid State Comm. **30**, 115 (1979).
2. B. L. Altschuler, A. G. Aronov, P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).
3. D. J. Bishop, D. C. Tsui, R. C. Dynes, Phys. Rev. Lett. **46**, 360 (1981).
4. M. J. Uren, R. A. Davies, M. Pepper, J. Phys. Chem. **13**, L985 (1980).
5. B. L. Altschuler, D. Khmel'nitzkii, A. J. Larkin, P. A. Lee, Phys. Rev. **B22**, 5142 (1980).
6. Y. Kawaguchi, S. Kawaji, J. Phys. Soc. Japan **48**, 699 (1980).
7. L. Van den Dries, C. Van Haesendonck, Y. Bruynseraede, G. Deutscher, Phys. Rev. Lett. **46**, 565 (1981).
8. T. F. Rosenbaum, K. Andres, G. A. Thomas, R. N. Bhatt, Phys. Rev. Lett. **45**, 1723 (1980).
9. B. W. Dodson, W. L. McMillan, J. M. Mochel, R. C. Dynes, Phys. Rev. Lett. **46**, 45 (1981).
10. R. C. Dynes, J. P. Garno, Phys. Rev. Lett. **46**, 137 (1981).
11. T. F. Rosenbaum, K. Andres, G. A. Thomas, P. A. Lee, Phys. Rev. Lett. **46**, 568 (1981).
12. W. L. McMillan, J. M. Mochel, Phys. Rev. Lett. **46**, 556 (1981).

Exact solution of Kondo problem

The Kondo problem concerns a simple physical model of magnetic impurities in metals that has required the most sophisticated of mathematical techniques. To theorists, the model not only represents a first step towards understanding the far more complex problem of ferromagnetism but also appears tantalizingly close to exact solution by the techniques employed for renormalizable field theories. In 1975 Kenneth Wilson of Cornell University successfully used renormalization-group techniques to calculate numerically a key parameter in the Kondo problem. This past year Natan Andrei of New York University and independently P. B. Wiegmann of the Landau Institute in Moscow executed a second coup by exactly diagonalizing the Hamiltonian to allow a solution in closed form.

The renormalization scheme used in the new solutions departed so far from the customary approaches that the solutions at first met with some skepticism. However, Andrei and John Lowenstein (also at NYU) recently carried the calculations far enough to obtain a value for a universal para-