Nickel says the 21-term brute-force calculation would probably have taken several years.

Once he had the series, he "had to play extrapolation games" to estimate the critical exponents. He made a series of estimates, keeping more and more terms, and found that a plot of estimates *vs.* number of terms is very flat up until 12 or 15 terms. If you had stopped calculating there, as previous theorists had done, you would assume that $\gamma = 1.250$ was close to the right answer.

This change can plausibly be interpreted, Nickel told us, as the result of competition between the leading divergence, $(T - T_c)^{-\gamma}$, and a weaker but still divergent term also near T_c, which is expected to be there. The effect of this weaker divergence on the estimates of γ will not be negligible unless you have calculated enough terms.

But are 21 terms enough? At present, Nickel finds $\gamma = 1.238 \pm 0.003$ and $\nu = 0.631 \pm 0.003$, in good agreement with the renormalization-group calculations. One recently quoted 12-term series estimate for γ was 1.250 ± 0.003, a 1% difference in result. Thus Nickel's work suggests that previous error estimates in series calculations were a factor of three or four too low.

The new error estimates made by Nickel may also not be realistic, he told us. So now he is collaborating with John Rehr (University of Washington) in analyzing the 21-term series by methods that explicitly account for the weaker divergent terms.

Although Nickel has only done such a long series expansion for the bcc lattice, according to the universality principle, the critical exponents are identical for whole classes of systems. So Nickel believes his bcc results can be applied to other lattice geometries because they are all in the same universality class. He says, "The good news is that at present the evidence from high-temperature series is consistent with the renormalization-group picture of the critical point. The bad news is that we have learned how easy it is to be misled by the apparent convergence of estimates from short series and that to exclude, with reasonable confidence, the possibility of a violation of hyperscaling or universality or the complicated critical region behavior as envisaged by Baker and John M. Kincaid [earlier this year] will require much more effort in deriving longer series."

Baker, although enthusiastic about Nickel's new series terms, says he doesn't believe Nickel has shown his extrapolation procedures to be better than either traditional methods or variational procedures. Concerning his old speculation that hyperscaling does not hold, Fisher says, "I'm sitting on the fence."

Kadanoff is convinced that Nickel has shown that the left- and right-hand sides of equation (2) are consistent. "We have other extremely good reasons for believing that the scaling phenomenology and renormalization-group theory are right. Nickel has weakened the strength of the apparent discrepancy. Now we can return to our theoretical prejudices." —GBL

Great undersea waves may be solitons

Peculiar striations more than a hundred kilometers long, visible on satellite pictures of the surface of the Andaman and Sulu Seas in the Far East, appear to be of interest in fields as far removed from oceanogaphy as quantum field theory. A recent report[1] of underwater current and temperature variations associated with such surface phenomena in the Andaman Sea, by Alfred Osborne, a physicist at Exxon Production Research (Houston), and Terrence Burch, an oceanographer with EG&G Environmental Consultants (Waltham, Mass.), suggests that these striations mark the propagation of "solitons," exotic solutions of nonlinear wave equations that have captured the interest of mathematical physicists studying a broad range of phenomena spanning 22 orders of magnitude in size.

The surface striations seen in the satellite pictures are interpreted as secondary phenomena that accompany the passage of "internal solitons," solitary wavelike distortions of the boundary between the warm upper layer of sea water and the cold lower depths. These internal solitons are traveling ridges of warm water extending downward hundreds of meters below this thermal boundary. Carrying enormous energies, these presumed solitons appear to be the cause of the usually strong underwater currents periodically experienced by Exxon's deep-sea drilling rigs between Sumatra and the Malay Peninsula in the Andaman Sea. They have even been implicated in the mysterious disappearances of several submarines. If one wants to continue deep-sea drilling for oil in areas where solitons occur, one will have to build drilling and production facilities that can withstand the large horizontal forces they generate.

John Apel and James Holbrook of the Pacific Marine Environmental Laboratory (Seattle) will report the results of their recent studies in the Sulu Sea, between Borneo and the Philippines, at the December meeting of the American Geophysical Union in San Francisco. Noting that these great internal waves usually appear in groups of up to ten, they prefer to describe them as damped "cnoidal" wave trains, a nonlinear hydrodynamic phenomenon closely related to solitons, and sharing most of their bizarre properties.

The history of solitons has its colorful beginning with a fortuitous observation in 1834 by the Naval architect John Scott Russell, while riding on horseback alongside a Scottish canal. "I was observing the motion of a boat... which suddenly stopped—not so the mass of the water in the channel which it had put in motion... Suddenly leaving it behind, (it) rolled forward with great velocity, assuming the form of a great solitary... well defined heap of water, which continued along the channel apparently without change of form or diminution of speed. I followed it on horseback... still rolling on at... eight or nine miles an hour, preserving its original figure. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel."

Standard linear dispersive wave theory does not permit such solitary waves of constant shape, even in the limit of a frictionless fluid. A "well defined heap of water" would rapidly lose its shape by frequency or amplitude dispersion. Not until D. J. Kortewegs and Hendrick de Vries wrote down the appropriate nonlinear wave equation in 1895 was it seen that localized nondispersive solitary waves could exist. In their (K–de V)equation, as in other nonlinear wave equations that admit of soliton solutions, the shape-preserving solitary waves result from a cancellation of the dispersive term by the nonlinear term. Unlike ordinary dispersive linear waves, the solitons have only crests, unaccompanied by troughs (with respect to the equilibrium surface).

It was another 70 years before Martin Kruskal (Princeton) and Norman Zabusky (now at the University of Pittsburgh) discovered the peculiar property of these solitary-wave solutions that led them to the name "soliton"—a coinage suggestive of a particle. From computer-generated numerical solutions of the K–de V equation, they discovered in 1965 that the solitary waves preserve their shape and velocity even when they pass through one another. We have then highly localized entities that preserve their identities as they propagate—and even when they "collide." Small wonder that quantum field theorists speculate that solitons may describe as-yet-undiscovered elementary particles—in particular, magnetic monopoles.

Although people had written down special soliton solutions of the K–de V and other nonlinear wave equations of physical interest such as the sine–Gordon and nonlinear Schrödinger equations, there was before 1967 no analytic procedure for finding the exact solutions of such equations with arbitrary initial conditions. In that year Kruskal and his Princeton colleagues Clifford Gardner, John Greene and Robert Miura discovered such a technique for the K–de V equation, allowing one to predict precisely the solitons that would emerge among (and run away from) the conventional linear waves in any given situation.

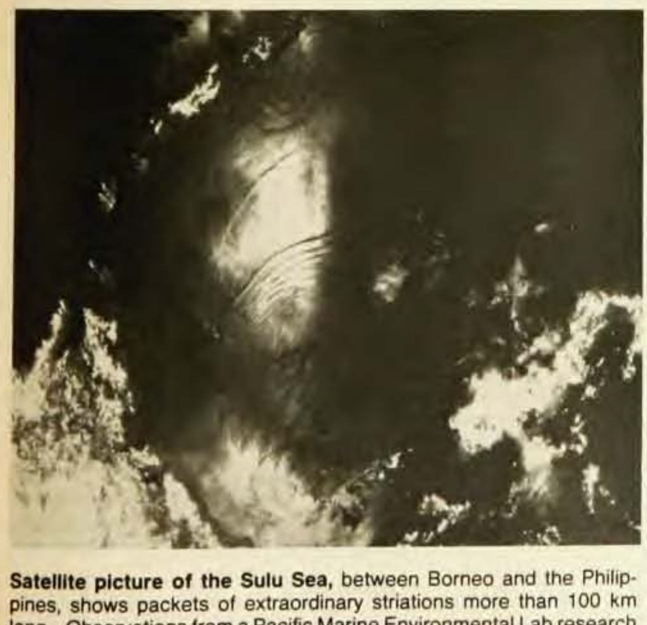
This "inverse scattering method" developed by Kruskal and his coworkers was quickly generalized to a large class of nonlinear wave equations that had previously been intractable, producing something of a revolution in mathematical physics. Among the newly soluble equations, those that have soliton solutions have found application in fields as diverse as magnetohydrodynamics, Josephson junctions, organic conductors, crystal-lattice theory and even the Great Red Spot in Jupiter. These successes, however, have for the most part been restricted to wave equations that are one-dimensional in space. Solitons have shown a tendency to be unstable in higher dimensionalities. Harvey Segur of Aeronautical Research Associates (Princeton) believes that the primary importance of the Andaman and Sulu Sea findings is the demonstration that stable solitons apparently do show up in the real multidimensional world—outside of special confined geometries cooked up in the laboratory.

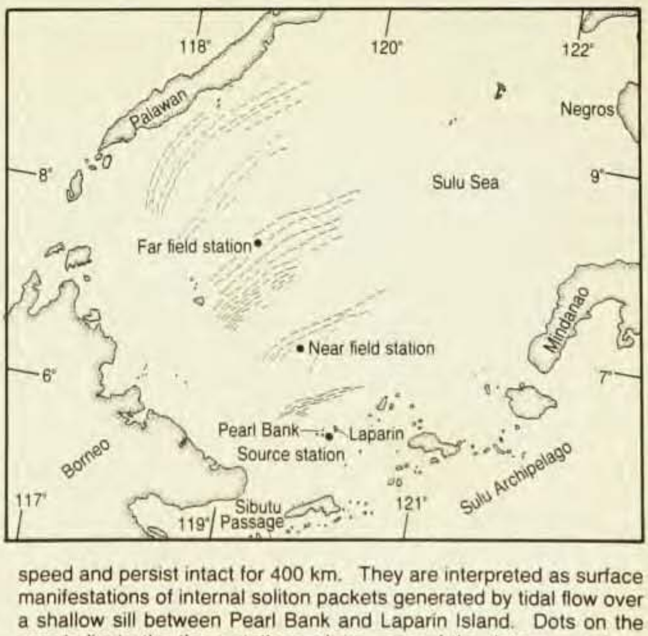
Osborne and Burch were asked by Exxon in 1976 to investigate the unusual underwater currents that were periodically arriving at an oil rig drilling in 3600 ft of water at the southern end of the Andaman Sea basin, near the entrance of the Malacca Strait, which separates Sumatra from the Malay Peninsula. Before setting out to do underwater measurements in the fall of 1976, they had seen Apel's survey of computer-enhanced LANDSAT pictures and other satellite photos showing 100-km-long striations on the Andaman Sea, separated by 6 to 15 km and grouped in packets of typically 4 to 8. The average period between arrival of successive packets at their research vessel, 12 hours 26 minutes, made it obvious that this was a tidal phenomenon of some sort. From the spacing between these semidiurnal packets it followed that their group velocities were as high as 2½ meters per second, an unusually high speed that indicated this was a strongly nonlinear wave phenomenon.

Osborne and Burch took data for four days with the research vessel sitting five miles from Exxon's drill ship *Discoverer 534*, which was at the time setting a world record for deep-water drilling. They measured underwater currents and temperatures from a specially instrumented mooring anchored to the sea floor 3600 ft below and buoyed up by a subsurface float. Additional temperature measurements were made with an "expendable bathythermograph," a weighted thermistor cast into the sea, that sends back temperature measurements over a fine magnet wire as it falls to the bottom.

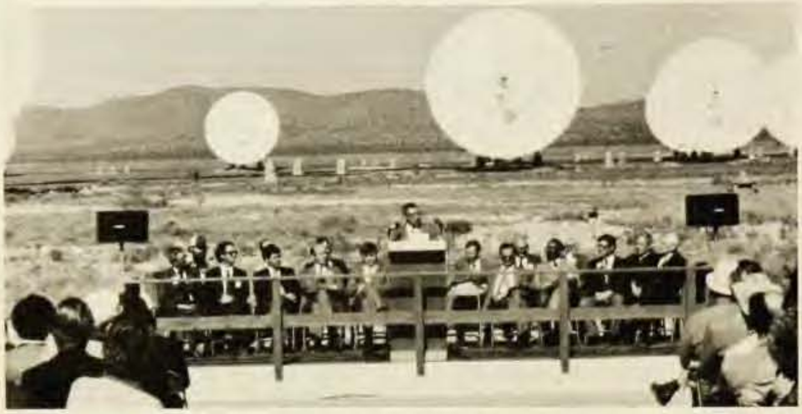
As observed on the surface, the striations seen by the LANDSAT, Apollo–Soyuz and ERTS-I satellites turned out to be kilometer-wide bands of extremely choppy water stretching from horizon to horizon, followed by about two kilometers of water "as smooth as a millpond." These striking bands of agitated water are called "tide rips," because seamen who have observed them in the past erroneously took them to indicate tidal movement over uncharted shoals. The tide rips reached the research vessel at intervals of about an hour (corresponding to spacings of 5 to 10 km), until a packet of 4 to 8 such bands had passed. This spectacle repeated itself with the regularity of the semidiurnal tidal period.

The submerged instruments showed that each tide rip corresponded to the leading edge of the two- to three-kilometer-wide traveling ridge of warm water protruding down from the warm "mixed layer," the upper 50 meters of the sea. Isotherms dipped as much as 60 meters below their quiescent depths at the maximum of the traveling internal wave. The amplitude of each succeeding soliton in the packet—measured by the downward distortion of the isotherms—was less than the one before. This is precisely what one expects for solitons. The velocity of a soliton increases with amplitude—a characterististically nonlinear phenomenon already noted by Scott Russell. Thus if a number of solitons are generated together, the larger ones pass through and outrun the smaller, presenting finally a ranked order of solitary waves of decreasing amplitude from front to rear. One must not of course confuse these internal solitons,





Satellite picture of the Sulu Sea, between Borneo and the Philippines, shows packets of extraordinary striations more than 100 km long. Observations from a Pacific Marine Environmental Lab research vessel show that these bands of choppy water travel at unusually high speed and persist intact for 400 km. They are interpreted as surface manifestations of internal soliton packets generated by tidal flow over a shallow sill between Pearl Bank and Laparin Island. Dots on the map indicate the three stations where most of the data were taken.



Dedication of the Very Large Array on the Plains of San Augustine, New Mexico on 10 October. The $78-million facility, whose construction began in 1972, consists of 27 dishes arranged in the shape of a Y; two arms are each 21 km long and the third is 19 km long. The design capability is one or two orders of magnitude higher in sensitivity and angular resolution than any existing array. Each of the 27 parabolic dishes is 25 meters in diameter and fully steerable.

Persons identifiable on the platform (from left) are: Manuel Lujan (Representative from New Mexico), Carl Heiles (Berkeley), Harrison Schmitt (Senator from New Mexico), Donald Langenberg (Deputy Director, NSF), Bruce King (Governor of New Mexico), Morton S. Roberts (Director of the National Radio Astronomy Observatory) at the rostrum,David S. Heeschen (former NRAO Director), Frank Press (President's Science Adviser), Frank Johnson (NSF assistant director), John Slaughter (Director Designate of NSF), Pete Domenici (Senator from New Mexico), Jack Lancaster (VLA project manager), and Gerald Tape (President, Associated Universities Inc).

propagating along a boundary between fluid layers, with Scott Russell's surface solitons.

From the spacing between successive waves in a packet and the rate of separation calculated from the K–de V equation, Osborne and Burch were able to estimate the distance the packet had travelled from its origin, and thus to identify possible source regions. They concluded that the solitons are generated by tidal currents washing over shallow "sills" off northern Sumatra or between islands of the Nicobar chain that extends beyond it.

The underwater current meters observed a rapid circulation of water associated with the solitary waves, in good agreement with what the K–de V equation predicts for internal solitons. The underwater current, with speeds up to 4 knots, goes downward at the leading edge of the wave, passing under it and re-emerging on the lee side. It is the interaction of this internal circulation with ordinary surface waves that produces the tide rip—and also the unusual band of calm water, as it "sweeps the sea clean" of little waves behind the agitated leading edge. The horizontal forces experienced by Exxon's drilling rigs appear to result from the horizontal component of the internal circulation accompanying each soliton.

The solitary waves predicted by the K–de V equation have widths inversely proportional to the square root of their amplitude, and corrections to the linear wave velocity proportional to the amplitude. That is to say, the taller (in this case deeper) waves are faster and skinnier. For the case of a thin, warm surface layer above a deep colder layer, the K–de V equation predicts internal waves of depression, the warmer water protruding down. Osborne and Burch conclude that, despite the irregular geometry of the Andaman Sea, their observations are in good overall agreement with the characteristics of internal solitons predicted by the K–de V equation.

Apel and Holbrook set out last April to study in detail the production mechanism for these great internal waves in the Sulu Sea. From numerous satellite pictures they had concluded that the waves were radiating northward from a very localized source near the south end of the Sea—a shallow sill between Pearl Bank and Laparin Island in the Sulu archipelago. Having been warned by Philippine authorities that Pearl Bank was a favorite rendezvous point for the Moro pirates who infest the region, they added 50-caliber machine guns and Phillippine marines to their inventory of thermistors, current meters and echo sounders.

To study the evolution of the waves from source to extinction, they put out three sets of instrumented moorings: at the presumed source, in the "near field" (100 km downstream), and in the "far field" (200 km downstream). Their ship also profiled the waves with towed instruments and echo sounders. The echo sounders were used to map surfaces of constant density by bouncing acoustical signals off the underwater plankton that bob down and up again with the passage of the internal waves.

The ship followed one wave packet more than 400 km north to its extinction on the shoals of Palawan Island. At $2\frac{1}{2}$ meters/sec, the wave packet thus travels more than 2 days intact as it spreads across the sea—an extraordinarily coherent phenomenon in what Apel describes as "a very noisy medium."

He and Holbrook conclude that the wave packets are produced by "lee wave formation"—a mechanism similar to the undulation of the jet stream as it comes out of a mountain range. As the semidiurnal tidal current flows south out of the Sulu Sea, an internal lee wave forms on the southern (outer) edge of the shallow sill between Pearl Bank and Laparin Island. As the tidal flow goes to zero six hours later, the lee wave, trying to maintain its group velocity relative to the current, escapes north over the sill barrier back into the Sulu Sea, emerging as a packet of internal waves. This phenomenon had previously been studied on a smaller scale in fjords, by David Farmer and James Smith of the Institute of Ocean Studies in British Columbia, and in the laboratory by Tony Maxworthy at the University of Southern California.

Apel argues that it is a contradiction in terms to speak of a packet of solitons. He points out that the K–de V equation has a more general solution that he believes to be a better descriptor of their data than is the limiting soliton case. When the nonlinear parameter, n, is at unity, the K–de V equation generates solitons. In the other limit, $n = 0$, one gets ordinary sinusoidal waves. In the intermediate case, one gets cnoidal waves, a train of Jacobi elliptic functions. Apel and Holbrook find from the data that n is in fact less than 1, going as low as 0.43. They feel that the internal wave packets are well described as damped cnoidal wave trains with a wavelength of 6 to 10 km. These cnoidal waves would share most of the peculiar properties of solitons, such as high speed and preservation of shape. But the cnoidal description, Apel told us, treats the relations between successive waves in a less *ad hoc* manner than does the soliton solution. Burch points out, on the other hand, that the rigorous inverse-scattering method describes the general time-dependent solution of the K–de V equation entirely in terms of solitons and linear waves, without resort to cnoidal functions. —BMS☐

Reference

1. A. R. Osborn, T. Burch, Science **208**, 451 (1980).