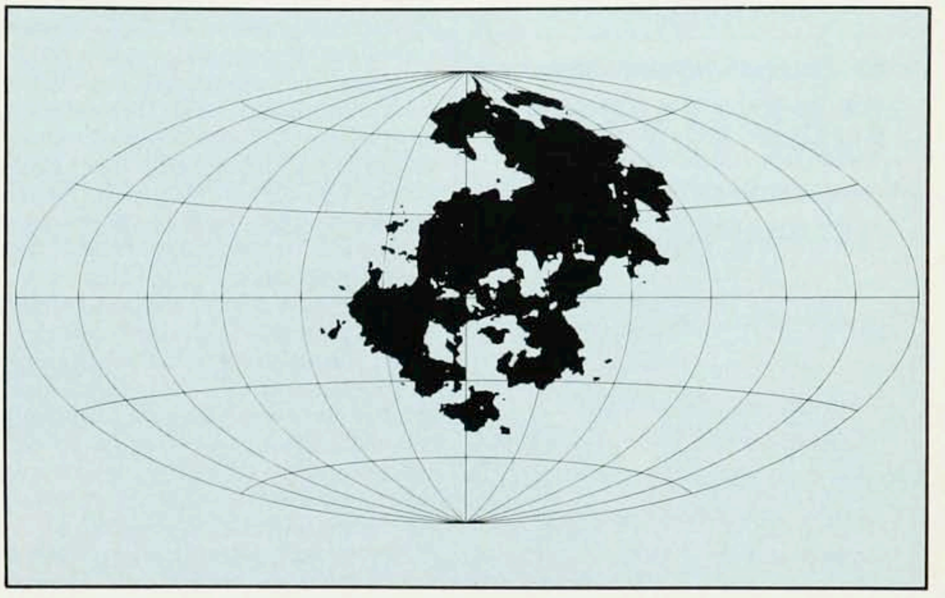
books

A look at coastlines, snowflakes and other natural geometries



Brownian Pangaea. The outline of a "primordial continent" randomly generated by computer from a fractional Brownian function. Benoit B. Mandelbrot discusses the techniques used to create this and other such figures in *Fractals: Form, Chance, and Dimension,* reviewed on this page.

Fractals: Form, Chance, and Dimension

B. B. Mandelbrot
365 pp. W. H. Freeman, San Francisco, 1977. $14.95

Reviewed by Michael Aizenman

Geometry provides the vocabulary for one of our fundamental modes of thought, and the geometrical constructs that we have at our disposition form an asset on which we often draw when constructing models or when laying the grounds for analytical investigations. *Fractals: Form, Chance, and Dimension* is a unique work that enriches our geometric resources by bringing to us some of the mathematical structure, physical significance and aesthetic appearance of shapes that may seem exotic to a mind accustomed to smooth forms.

Once one has a name by which to refer to them, "fractals" (a term coined by Benoit Mandelbrot) can be perceived in abundance. Examples from nature are the highly irregular shapes of coastlines, the very symmetric but rich-in-detail snowflakes, and our vascular system, which seems to combine regularity with randomness.

A set is *self similar* if it is a union of a number of translates of a scaled-down version of itself. Such sets—for example, the Cantor set—behave simply under well-chosen dilations. The self-similarity gives rise to the notion of *similarity dimension,* which is based on the law that relates the dimensions of intervals, squares, cubes and so on to their scaling properties. A new feature is that the dimension need no longer be an integer. A more complete terminology, which does not require the above regularity, is that of *Hausdorff–Besicovitch dimension.* This is but the simplest application of Hausdorff measures, which are among the basic tools provided by the rapidly growing "geometric measure theory." *Fractals* are sets whose H–B dimension differs from their topological dimension—in particular, the sets of non-integer similarity dimension.

In his book, Mandelbrot has taken full advantage of computer designs to visualize both completely regular and random fractals. A large part of the text is practically carried by the illustrations, the choice of which was guided to no small degree by aesthetic criteria. It seems that Mandelbrot has developed the ability to visually feel the dimension of a fractal and has made an effort to pass the skill and his fascination to the reader.

Mandelbrot touches upon various intriguing problems, which are better understood once one realizes that fractals play a role there and one takes into account different facets of the fractal geometry. An example is Mandelbrot's modeling of the distribution in time of transmission errors. These may seem peculiar in their tendency to occur in bursts. He points out that as a crude model one may consider the discretized random zero-sets of the Brownian function. This model exhibits both the independence of the successive error-free periods and the occurrance of bursts, the phenomenon being related to the fact that the dimension of the set of times in which an error occurred is smaller than one. In fact, one may construct a large family of related models for which the dimension is a useful parameter.

Mandelbrot strives to achieve such structural understanding, or rather modeling, for other phenomena in which one finds either irregularity, like in the shapes of continents, or lack of smoothness, as in the velocity distribution of a turbulent flow, or surprisingly long persistence, observed for example in the water discharges of rivers.

Mandelbrot suggests looking for the fractals that are the geometrical carriers of the various *scaling laws* found in abundance in different branches of science, such as in statistical mechanics. He expresses a hope that some of the scaling exponents are simply related to naturally defined fractals. Unfortunately, in many cases this hope is still no more than a vision (but not less than a vision either).

A topic worth mentioning to PHYSICS TODAY readers is the discussion of the mass distribution in the universe. Mandelbrot discusses the fractal aspects of Fournier's universe and of Fred Hoyle's model for the genesis of galaxies. In particular, the fractal models mentioned there permit one to avoid Olbers's paradox of the blazing sky without necessitating the expanding universe.

Another area of flourishing activity in which fractals have recently been encountered is that of dynamical systems, specifically in the study of phenomena related to turbulence.

Mandlebrot, not ignoring the aesthetic pleasure, wrote *Fractals: Form, Chance, and Dimension* to stimulate interest and open new avenues of thought, rather than to provide a solid and complete grasp of the subject or teach hard analytical tools. The explanations that he seeks at this point are *structural* rather than *dynamical.*

As is often the case with consciousness raisers, the book is somewhat overwritten and suffers from a degree of repetitiveness, especially in its second chapter. However, as a remedy, which is also one of the causes of the problem, Mandelbrot organized the material in a way that facilitates skipping. I urge the reader who loses interest to follow Mandelbrot's advice and to proceed to the next or another section. It is hard to repress the thought that Mandelbrot, following his developed perception, has replaced the line by a fractal in the design of his book.

The unique commitment of Mandelbrot to the subject, or rather to its discipline, is reflected throughout the book and in particular in the very personal chapter devoted to biographical and historical sketches. This chapter is fascinating by itself. Writing about his masters, he gives us his perspective on science and on the scientist's life and toil.

In summary, this work aims effectively at enriching our collection of the most basic tools of analysis; thus exposure to this book may produce unforseeable consequences. It is highly recommended to every natural scientist.

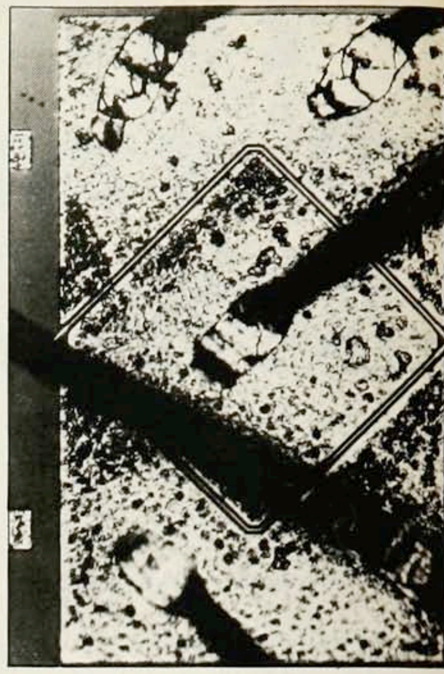
* * *

Michael Aizenman is an assistant professor of physics at Princeton University. His research interest is in mathematical physics with a focus on statistical mechanics and dynamical systems.

Metal–Semiconductor Contacts

E. H. Rhoderick
201 pp. Clarendon (Oxford U.P.), New York, 1978. $18.50

The field of metal–semiconductor contacts (called Schottky barriers or Schottky diodes) is of importance in applications to practical devices as well as a test of our fundamental understanding of semiconductor physics. Device work related to metal–semiconductor contacts include metal-oxide–semiconductor diodes, metal–insulator–semiconductor diodes, switches, transistors and solar cells. Fundamental studies have treated the role of interface states and attempted to replace the traditional descriptions involving work function and electron affinity with a more basic approach in terms of electronegativity, pseudopotentials and the band structure of the composite system.



Microwave field-effect transistor developed in 1968 by the IBM Research Laboratory, Zurich and now widely used. The theory behind such applications of metal–semiconductor contacts are described in E. H. Rhoderick's book.

Emlyn Huw Rhoderick, as professor of solid-state electronics at the University of Manchester Institute of Science and Technology, has made significant contributions to the metal–semiconductor field through his experimental work. His *Metal-Semiconductor Contacts,* which he frankly states in the preface is meant to update H. K. Henisch's *Rectifying Semiconductor Contacts* (Oxford U.P., 1957), contains many important experimental details not readily available elsewhere. It does not contain any significant accounts of practical devices nor recent theoretical work of a basic nature. Treatment of such work can be found in such texts as S. M. Sze, *Physics of Semiconductor Devices* (Wiley, New York, 1969) and A. L. Milnes and D. L. Feucht, *Heterojunctions and Metal-Semiconductor Junctions* (Academic Press, 1972). Both of these contain more comprehensive treatments and more extensive bibliographies than Rhoderick's book.

Rhoderick's treatment is a very readable account of the standard theory of metal–semiconductor contacts. He has treated the various cases in detail, separating out those features that control barrier height, such as surface state, image forces, oxide layers and tunnelling mechanisms, from the basic current-limiting mechanisms of diffusion and thermionic-emission. At each stage of the presentation he uses experimental results and details of the sample preparation to support the theory or reveal its shortcomings.