Unitary field theories

Much of Einstein's life was devoted to searching for a theory that incorporates gravity and other fields into a generalized geometrical structure derived from the general theory of relativity.

Peter G. Bergmann

General relativity interprets gravitational fields as arising from a curvature in space–time, given by a tensor R_{ik}, as Einstein is discussing in this 1931 lecture. (Photo from Brown Brothers.)

The spirit that motivated Galileo, Newton, Einstein and Bohr to strive to understand nature by tracing unifying principles that underlie the diversity of physical phenomena is the motivating force behind unitary field theories. In its earlier stages, in the absence of physical facts that could have given it direction, the quest for unitary field theories was stimulated primarily by geometric notions. In recent years quantum physics and elementary-particle theory have suggested new directions, some of which may be successful. Before we discuss these recent developments, let us review some of the background.

Preliminaries

In the years after Hermann Minkowski's geometric interpretation of the special theory of relativity,[1] the relationship between physical force fields and space-time appeared straightforward. Particles would trace out trajectories—curves—in the four-dimensional space–time continuum, and fields were to be understood as collections of functions of the four space–time coordinates, say x, y, z and t. Space–time itself was endowed with an immutable structure, the Minkowski metric, which assigned to each pair of points in space–time ("events," localized both in space and in time) an "interval," independent of the choice of Lorentz coordinates.

A physical theory is Lorentz-invariant (or Poincaré-invariant, in the terminology used now) if the dynamical laws, presumably sets of differential equations, take the same form in all possible Lorentz coordinate systems. The Lorentz transformations, or Poincaré transformations, are defined as transitions between any two four-dimensional coordinate systems in which the mathematical form of the interval takes the standard form

$$\tau^2 = c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) \quad (1)$$

With the advent of the general theory of relativity in 1916,[2] the relationship between the structure of space–time and the physical fields changed profoundly. In particular, the clear distinction that one could previously make between the geometry of space–time in which the particles move and the fields that affect the motion of particles became much less clear. The physical basis for the change was the fact that in the presence of gravitational fields all ponderable bodies that are electrically and magnetically neutral follow trajectories that depend only on initial location and state of motion and not on masses of the bodies (the "principle of equivalence"). Einstein recognized that this principle would affect the physical definition or interpretation of the space–time geometry. Locally, at least, the variety of possible trajectories in a gravitational field resembled the variety of trajectories postulated by Newton's Law of Inertia in the absence of forces;

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that principle asserts that in a force-free situation trajectories are straight lines, traveled at a uniform speed. But whereas in Newtonian physics, and in Einstein–Minkowski physics as well, the four-dimensional coordinate systems (the *frames of reference*) in which the law of inertia is valid are defined throughout the physical universe, coordinate systems in which the trajectories of free-falling bodies are straight lines could now be constructed only in the vicinity of a point. Such local frames might be termed *free-falling frames of reference*. Any attempt to connect them into a single inertial frame of reference is bound to fail; it was precisely that failure which was to distinguish a gravitational field from a space–time from which gravity was absent.

The idea that global inertial frames of reference must be replaced by free-falling frames only locally defined lies at the root of the general theory of relativity. In that theory, too, there is an interval, but it is defined only for infinitesimally distant pairs of points,

$$d\tau^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\, dx^\mu dx^\nu \qquad (2)$$

The coefficients $g_{\mu\nu}$ are not constants, but are themselves fields, functions of the space–time coordinates. The coordinates, too, are much more flexible than are those in the special theory of relativity. There is no analog to the Poincaré transformations. Rather, all that is required of a coordinate system, and of its relationship to another coordinate system, is a unique, continuous and differentiable relation to the points that make up the space–time continuum.

The metric coefficients $g_{\mu\nu}$ that appear in equation 2 are fields. Their dependence on the four space–time coordinates describes the intrinsic geometric properties of the space–time continuum. By a mathematical analysis one can determine, for instance, that in a certain field identified physically with that corresponding to a single mass (the Schwarzschild field) the most nearly straight curves existing (the geodesics) are closed in space (helices in space–time), and are approximately elliptical. Thus, the coefficients $g_{\mu\nu}$ are also potential functions of a physical field, the gravitational field. This double role of the $g_{\mu\nu}$, to describe geometry and to be physical potentials, is the crucial formal characteristic of the general theory of relativity.

Einstein[2] formulated the differential equations that represent the dynamical law of gravitation, corresponding in effect to Newton's inverse-square law of gravitation. In general relativity the dynamical law must take the same form in any chosen four-dimensional coordinate system; it must be generally covariant. The field equations are highly non-linear partial differential equations of the second order, whose variables are precisely

Valentin Bargmann, Einstein, and Peter Bergmann walking on the Princeton campus. They collaborated on several papers on unitary field theories. (Photograph copyright by Lucien Aigner.)

Hermann Weyl made many contributions to mathematical physics, including a generalization of Riemannian geometry. (Sketch by Elizabeth P. Korn.)

the metric coefficients $g_{\mu\nu}$.

The sources of the gravitational field are masses, mass flux, and stress; the analogous sources for the electromagnetic field, are the electric charge and current. All physical fields other than the gravitational field itself contribute to the sources of the gravitational field, and thus they appear on the right-hand side of Einstein's field equations. In turn, the laws of all physical fields contain as coefficients the gravitational potentials $g_{\mu\nu}$. Thus gravitational and non-gravitational fields interact with each other—they are *coupled.* Nevertheless, they enter into the scheme of the theory in a non-symmetric fashion. The coupling between the gravitational and all other fields implies that, because the gravitational field equations are not linear, no linear law can be generally covariant.

The gravitational field is the only physical field that in the general theory of relativity directly relates to the geometric structure of space–time. All the other physical fields do so only indirectly, through their linkage to the gravitational field. The physical justification for this lack of symmetry is that particles obey laws of motion independent of their own characteristics (that is, travel on geodesics) only if they are not coupled, through electric charges or otherwise, to non-gravitational fields. It must be admitted that this distinction is to some extent circular.

In the argument just presented it is taken for granted that the geometry of space–time is described by equation 2. The pre-eminent role of the gravitational field among all physical fields is then deduced from the appearance of the gravitational potentials alone in equation 2. Presumably, a different definition of what is to be understood by the term "geometry of space–time" might lead to different results. There remains one point that distinguishes the gravitational field from all others. If a particle's mass is considered its coupling parameter to the gravitational field, just as its electric charge is its coupling parameter to the electromagnetic field, then of all the conceivable coupling parameters mass is the one that has no kinematic effect on a particle's motion. Particles of different masses travel on the same trajectories in a gravitational field, whereas particles with different electric charges travel on distinct trajectories in an electromagnetic field. Even this assertion must be qualified, in that particles with different charges will travel on identical trajectories if their respective charge-to-mass ratios (e/m) are equal.

Almost from the inception of the general theory of relativity there have been attempts to construct theories in which gravitational and non-gravitational fields would assume similar roles, where *all* physical fields would lend themselves to a geometric interpretation. Moreover, it was hoped that such theoretical constructions would lead to a "unification" of all physical fields. Hence such theoretical constructions were termed *unified* or *unitary field theories.* Einstein himself searched untiringly for a truly satisfactory field theory, from the early 1920's to the end of his life. In the course of his endeavors, and those of many others, the meanings of the terms "geometry" and "unification" have undergone many subtle variations.

Manifolds

One concept that underlies many formulations of what is "geometry" is that of a *manifold.* In topology, a manifold is an infinite set of elements, called points, that has many of the local properties of an ordinary n-dimensional space. The points of a manifold can be identified with n-tuplets of real numbers, their coordinate values, so that points whose coordinate values differ by small numerical values are in each others' neighborhoods. It may not be possible to use a single coordinate system for the whole manifold; but it should be possible to cover a manifold by overlapping "patches" of coordinate systems. For instance, the surface of a sphere is a two-dimensional manifold. There is no way to cover the whole sphere with a non-singular coordinate system, but one can construct two patches, one perhaps centered on the north pole, the other on the south pole, and both extending beyond the equator. There will be a region of overlap, in the topics; within the overlap region each point may be identified either by means of the Northern or the Southern coordinates. (Incidentally, the usual coordinates of geographic latitude and longitude break down at the poles, in that a pole corresponds to an infinite set of longitudinal coordinates.)

A manifold is far from being the most general model of a topological space, but it is sufficiently close to what one conceives of intuitively as a space so as to be both popular and useful. A manifold may possess all kinds of special structures that represent its "geometric" properties.

An example of such a structure is a metric, which assigns to curve segments arc lengths, such as suggested by equations 1 and 2. In all differentiable manifolds (manifolds in which the overlapping coordinate patches can be chosen so as to have differentiable transition equations) we may construct fields of vectors. The vectors at each point represent the tangents of curves passing through that

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Bergmann and Einstein working in Einstein's study at Princeton in early 1938, when they were actively engaged in attempts at constructing unitary field theories. (Photo by Lotte Jacobi.)

point. A metric endows every vector with a norm, which is a quadratic function of the vector components. General relativity, one might say, is based on a space–time manifold endowed with a metric. A manifold with a metric structure is known as a *Riemannian manifold.*

Given the existence of vectors at every point of a manifold, there is, *a priori,* no relation between the vectors at neighboring points, but such a relationship can be postulated. One might arbitrarily designate a particular vector at one point as the vector "corresponding" to a given vector at a neighboring point. There are certain reasonable conditions to be imposed on such a designation, the most important being that the designation of corresponding, or *parallel,* vectors should be compatible with the law of vector addition. Once the rule of correspondence has been introduced for vectors at nearby points, it can be applied for continuous progression along a curve, approximating the curve by point-to-point polygonal progression. If a manifold is to possess both a metric and a method for relating vectors at nearby points to each other (an *affine connection*), the two structures can be related to each other by requiring that the norms of two corresponding vectors have the same values.

What happens if a vector is displaced according to the adopted rule of correspondence along a closed curve, one that returns to the initial location? In general the resulting vector is not identical with the initial vector; one speaks of the *non-integrability* of the affine connection, measured by the (affine) curvature. Metric and affine connection are examples of the geometric structures that one can postulate on a manifold. There are many others that have been considered by geometers, such as the *symplectic structure* of phase spaces that play such an important role in Hamiltonian mechanics. Obviously, there may be promising structures that nobody has yet thought about. Most unitary field theories postulate structures that, for one reason or another, are believed to lend themselves to physical interpretation.

Fiber bundles

Many types of geometric structure can be described as *fiber bundles.* Fiber bundles have become popular among some physicists through their use in a discussion of isotopic spin by Chen Ning Yang and Robert L. Mills more than twenty years ago,[3] though they had been well known in mathematics long before. Given a manifold, such as space–time, called the *base* manifold, one attaches new manifolds to each point. These attached manifolds, all identical, are the *fibers.* They may have any dimensionality, not necessarily that of the base manifold. Each fiber can be subjected to mappings, or transformations on itself, which maintain the fiber's essential properties. As a case in point, in a metric manifold one may construct at each point a set of mutually perpendicular unit vectors; these might serve as the basis for all vectors at that point. This basis set can be rotated at each point, leading to mappings of each of these vector spaces on itself. Given a fiber and its permitted self-mappings, one may introduce a connection that establishes "corresponding" points on fibers at nearby points. Again, this connection will in general not be integrable.

A well-known fiber bundle in quantum mechanics is presented by the wave function defined on configuration space as the base manifold. The fiber consists of all complex values that the wave function may have at one point. As only the absolute square of the wave function has direct physical significance, one may multiply the wave function by an arbitrary complex factor of magnitude unity and choose this factor differently at different points. However, as phase relations play a role, for instance in the construction of canonical momentum operators, it is desirable to establish a rule by which "the same phase" is recognized at neighboring points. This rule must be of the form

$$\begin{aligned} i\hbar\nabla\psi + \frac{e}{c}\,\mathbf{A}\,\psi &= 0 \\ i\hbar\frac{\partial\psi}{\partial t} - e\,\Phi\,\psi &= 0 \end{aligned} \tag{3}$$

for the wave function of an electrically charged particle, where $\mathbf{A}$ and Φ are the vector and the scalar potentials, respectively. If the wave function is to be multiplied by a factor $\exp(iQ/\hbar)$, then $\mathbf{A}$ and Φ must be changed by the appropriate amounts,

$$\begin{aligned} \mathbf{A}' &= \mathbf{A} + \frac{c}{e}\,\nabla Q \\ \Phi' &= \Phi - \frac{1}{e}\frac{\partial Q}{\partial t} \end{aligned} \tag{4}$$

for equation 3 to remain valid. In this case the electromagnetic potentials represent the appropriate connection, and the *gauge transformation* (equation 4) shows how the connection must be changed if each fiber is mapped on itself by one of the permissible changes.

We will briefly describe two of the historic unitary field theories, one proposed by Hermann Weyl,[4] the other by Theodor Kaluza,[5] and then present a third, "supergravity," which currently is very much in the limelight.

Weyl's conformal geometry

Weyl proposed to change the metric structure of space–time to one known as *conformal.* Instead of every vector possessing a norm, in Weyl's geometry only the ratio between the norms of two vectors (at the same point of the space–time manifold) is assumed to have an absolute value. If in Riemannian geometry the set of mutually perpendicular vectors at each point can be rotated at will, then in Weyl's geometry it can also be enlarged or reduced by an arbitrary factor. As a result, the affine connection of Riemannian geometry must be enriched by an extra rule that tells when two vectors at neighboring points have equal lengths (though the lengths themselves are not determinable). This extra affine structure was interpreted by Weyl as the electromagnetic potentials, long before these potentials were postulated to be the connection required by the indeterminate phase of the wave function of quantum theory.

As a generalization of Riemannian geometry Weyl's construction is at least mathematically attractive. Maxwell's equations of the electromagnetic field can be stated against the background of a Weyl geometry. On the other hand, those of particle fields with nonvanishing rest mass can not. That the affine geometry did not survive among physicists is because of the fact that stable particles have definite masses, and hence definite Compton wavelengths. The four-dimensional wave proportional to its relativistic linear momentum and its relativistic mass, respectively, thus has a norm proportional to the square of the mass, which can be used as a standard throughout the physical universe, a standard against which all vectors can be evaluated. Weyl's postulate of non-absolute norm thus appears to contradict what we encounter in nature.

It is perhaps remarkable that the electromagnetic potentials appear in Weyl's geometry as the connection for the transfer of the norm (or magnitude) of a vector, in the absence of any relation to the wave functions of quantum physics. The formal aspect that provides the analogy is that in both cases the degree of arbitrariness in the description of a geometric structure is a real scalar. In Weyl's geometry that scalar is the choice of norm of a vector, in wave mechanics it is the choice of the real variable Q in the transformation

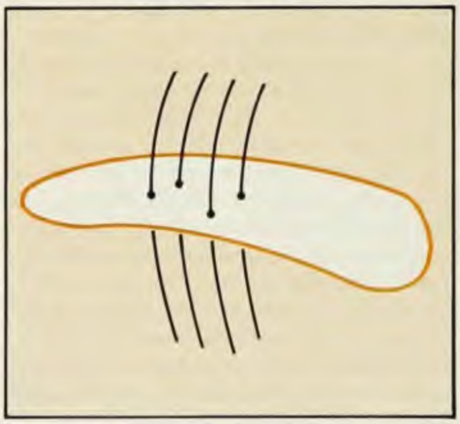
$$\psi'(x) = e^{iQ(x)/\hbar}\,\psi(x) \qquad (5)$$

of the wave function. Either way, the change in the connection (which must be a four-vector) is the addition of a (four-dimensional) gradient, just what is now called a gauge transformation of the electromagnetic potentials. Incidentally, the term "gauge transformation," now commonplace in electrodynamics, is due to Weyl, who spoke of the re-gauging of a vector norm.

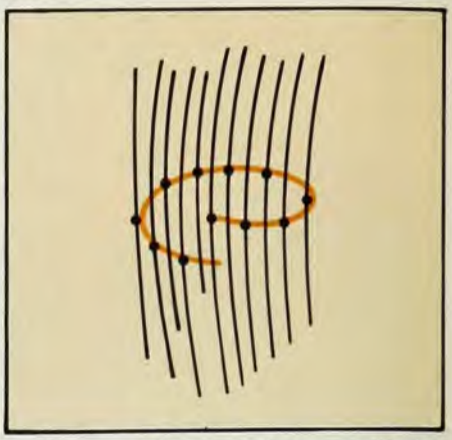
In Weyl's geometry the "fiber" erected over each point of space-time consists of all tetrads of mutually perpendicular vectors of equal lengths, and the permissible mappings of a fiber on itself are composed of Lorentz rotations and dilatations. This set of mappings forms a group, to be sure, but not a "simple" group; the term "simple" applied to a group implies that the group cannot be decomposed further into a (semi-direct) product of smaller groups that is preserved under the actions of the group itself. The group of Weyl permits such a decomposition: the Lorentz rotations of determinant unity, which in current notation are often designated by the symbol SO(3,1), and the isotopic dilatations. As a result of this group-theoretical decomposability of the group of fiber mappings, the connections also split, and the "unification" of gravitation and electromagnetism is partly illusory.

Kaluza's five-dimensional theory

Kaluza's attempt at unification has also considerable historical interest. Kaluza introduced a five-dimensional manifold that was to take the place of four-dimensional space-time, and endowed it with a five-dimensional metric. Four dimensions were to be space-like, one time-like. To account for the observed four-dimensionality of space-time, Kaluza introduced a congruence of space-like curves, so that through each point of his manifold one, and only one of his curves should pass. All points along one curve should be equivalent in every respect, so that the richness of his geometric structure could be sampled completely along any four-dimensional hyper-surface that would intersect each of the curves.



This implies, of course, that given two of Kaluza's curves, their distance from each other is constant up and down the curves. These distances are interpreted as the metric of the physical four-dimensional space-time. Any of the intersecting four-dimensional hypersurfaces may be accepted as a model of space-time. But though rigid sliding up or down would by assumption have no effect on the internal structure of such a hypersurface, the local angles of intersection with the curves would. If one could construct a hypersurface that is everywhere perpendicular to the curves, that would be a natural choice, but in general one cannot do that. Consider any closed curve and then slide the points of intersection with the curves of Kaluza up or down to make the closed curve everywhere perpendicular to Kaluza's curves. On returning to the original curve of Kaluza one will discover that it is now intersected at a different point.



The non-existence of a perpendicular hypersurface represents a twist of the congruence of the curves of Kaluza, which has the same formal properties as the electromagnetic field. Again, one can go over to the language of fiber bundles and interpret Kaluza's five-dimensional manifold as a bundle whose base manifold is space-time and whose fibers are one-dimensional. Each fiber can be mapped on itself by translation. The "connection" is the assignment of a point on a neighboring fiber that is reached by travel at right angles to the fibers, and this connection is non-integrable.

There is a scalar in Kaluza's geometric construction, which was originally a source of embarrassment. If a four-dimensional hyper-surface is slid "rigidly" along Kaluza's curves, the displacements along the various curves must be related to each other. In terms of the five-dimensional metric the magnitudes of the displacements need not be equal to each other, and their ratios constitute an intrinsic scalar field. As he could not see any physical interpretation for this scalar, Kaluza simply set it equal to unity, and thus in effect eliminated it. Later on, a number of different investigators, most recently Carl Brans and Robert Dicke,[6] capitalized on precisely this scalar field and attempted to give it a role in cosmology. They called this proposal the scalar-tensor theory. Though a very reasonable extension of Kaluza's original proposal, the scalar-tensor theory is not favored by present observational evidence. Nevertheless, it should, in my opinion, not yet be discarded, as the evidence is at the frontiers of present technology, and there may be alternative ways for incorporating a scalar into a unitary field theory. From a theoretical point of view it should be emphasized that, just as in Weyl's geometry, Kaluza's construction, and its subsequent modifications, does not accomplish a real unification.

Another generalization of Kaluza's five-dimensional approach was explored by Einstein, Valentin Bargmann, and myself.[7] We replaced the requirement of perfect equivalence of points along his

His office at Princeton in April, 1955 after Einstein left it for the last time. In pain, he went to the hospital and died a few days later in his sleep, of a burst aneurism. The equations on the blackboard refer to his continuing work on unitary field theories. (Photo by Alan Richards.)

special curves by the less restrictive assumption that the five-dimensional universe was closed in the fifth direction, with a circumference of subatomic magnitude. The five-dimensional manifold would thus have the appearance of a thin tube. Fields would not be constant around the tube, but would be free to vary, with a periodicity that would correspond to the tube's circumference. We hoped at the time that the periodicity might account for quantum phenomena; but it turned out that the resemblance was at best superficial, and we eventually discontinued these efforts.

Supersymmetry and supergravity

An entirely new approach to unitary field theories is afforded by the so-called *supersymmetry* theories. These theories take their physical motivation from the fact that all known particles in Nature belong to one of two classes. Particles having an integral spin are *bosons,* whose with odd half-integer spin are *fermions.* Fermions satisfy Pauli's exclusion principle, which is to say that no two fermions of the same kind can have all quantum numbers the same. The electrons contained, for instance, in an atom or a crystal cannot all crowd into the lowest energy state, but must distribute themselves in such a manner that each state is either empty or occupied by a single electron. Bosons are not constrained in this way. An arbitrarily large number of photons can, for example, be found in any one mode of, say, a beam of light. The idea of supersymmetry is to permit transformations in which bosons and fermions are converted into each other; its motivation is the existence of supermultiplets among elementary particles in which particles of different spin are grouped together.

In systems with many identical particles it is convenient to introduce a formalism, often referred to as field quantization, in which the actual number of particles of a given species can be changed by operators that are known as *creation* and *annihilation* operators. As their names imply, these operators act on the state vector of a quantum system to convert it into one in which there is one particle more, or one particle less, than before. There must be one creation operator and one annihilation operator for every possible quantum state. In fermions it is impossible either to create or to annihilate two identical particles in the same quantum state; hence the square of each such operator must vanish. They are *nilpotent.* Moreover, it can be shown that any two distinct fermion annihilation operators must anticommute. This is, for any two such operators, say a_k and a_l,

$$a_k a_l = -a_l a_k$$

One can form a complete algebraic system of operators that obey such anticommutation laws, except that, because of their nilpotence, combinations of the a_k's formed by addition and multiplication in general possess no reciprocals, although they will if the polynomial formed from the anticommuting operators also contains a nonvanishing ordinary number term. For instance, the reciprocal of $(1 + a_k)$ is $(1 - a_k)$, as the reader can easily verify. (Note that $a_k{}^2 = 0$.) Algebras formed from ordinary numbers, anticommuting quantities, and their powers are called Grassmann algebras. It is easy to verify that odd products of anticommuting numbers also anticommute with each other, for example,

$$(a_1a_2a_3)(a_4a_5a_6a_7a_8) = -(a_4a_5a_6a_7a_8)(a_1a_2a_3)$$

whereas even products commute with each other, and with odd products. Nevertheless, even products of a_k's are not ordinary numbers, they are nilpotent.

In supersymmetries one introduces side by side integral-spin fields composed of commuting (thought not necessarily ordinary) numbers and half-odd-spin fields whose components are anticommuting. It is assumed that they obey field laws that are Poincaré-invariant. On top of these assumptions one introduces the notion of supersymmetry, invariance with respect to transformations generated themselves by anticommuting operators, which carry fermion fields into boson fields and *vice versa.* Julius Wess and Bruno Zumino[8] were among the first to construct such a theory.

Supersymmetry transformations are remarkable in that they also act on the coordinates of space–time (which in the original supersymmetry theories was assumed to be the Minkowski universe). In fact, the commutator of two supergauge transformations seems to behave as a translation, which, however, differs from an ordinary translation in that the change in the coordinates of a point of space–time is nilpotent. The generator of these "translations," which should behave as the total linear momentum of the physical system, cannot be an ordinary quantity, either.

From supersymmetry to supergravity[9] is a step that is closely patterned on the experience gained with fiber-bundle formalisms: Instead of having a single supersymmetry transformation throughout space–time, one introduces local transformations, called *supergauge* transformations; they map a rather complicated fiber on itself. There is need for a connection, which has both fermion and boson components. Any "superfield" that is introduced as the carrier of physics will consist of both boson and fermion components, which under supergauge transformations change into each other.

Supergravity theories are truly unified in the sense that the fields that occur in them cannot be decomposed invariantly. In any chosen supergauge frame they will decompose, but that decomposition is not left intact by supergauge transformations. The groups that have been considered in supersymmetry supergravity theories are simple groups, the so-called graded Lie groups. In some theories the fundamental symmetry group can be chosen so as to contain as subgroups those that are important in elementary-particle physics.

The Grassmann numbers that occur in both supersymmetry and supergravity theories are difficult to interpret. They assume at least some of the roles of ordinary numbers, serving, for example, as coordinates of points. In quantum physics we have learned how to obtain from operators ordinary numbers that are the expectation values of the physical quantities represented by these operators. A corresponding algorithm for the Grassmann quantities of supersymmetry and supergravity does not as yet exist. It is conceivable that such an algorithm will be forthcoming once the theories have been fully quantized. For the time being, supersymmetry and supergravity theories have so many attractive aspects, and are of such recent origin, that one ought not to abandon them just because right now their physical interpretation leaves much to be desired. Neither should one disregard the fact that in many respects these theories have remained fragmentary. The next few years will probably teach us much that should lead to a definitive evaluation.

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