The physics of white dwarfs

Increasingly sensitive and detailed astronomical observations coupled with calculations of the properties of matter under extreme conditions have given us new insights into their structure and evolution.

Hugh M. Van Horn

White dwarf stars, so called because of the color of the first few to be discovered, occupy a key position in astrophysical theory. Together with neutron stars and black holes, they are the terminal points of stellar evolution. Their properties thus provide clues to the physical processes that take place during the rapid and often spectacular evolutionary stages near the ends of stellar lifetimes. In addition, white dwarfs provide astrophysical "laboratories" for "measuring" the physical properties of matter under extreme conditions. These extend from conditions like those in laser-produced plasmas to those typical of the solid crusts of neutron stars. White-dwarf stars also occur as components of cataclysmic binary systems—novae, dwarf novae and related objects—and knowledge of the properties of white dwarfs is essential to the development of satisfactory theoretical models for these systems.

In this article, I shall describe our current understanding of the nature of the white dwarfs. The past ten years have seen a number of remarkable new discoveries about these stars. We now know, for example, that some white dwarfs possess magnetic fields of several hundred megagauss, much larger than are attainable in terrestrial laboratories, some others display periodic light variations, and still others may have solid cores that have crystallized at temperatures of several million kelvins. These discoveries and theoretical interpretations have resulted from a close interplay between astronomical observations and developments in physical theory.[1] This close relationship has characterized white-dwarf research from its beginnings, and important contributions to our understanding of these stars have come from almost every subdiscipline of physics, including statistical mechanics, nuclear and particle physics, solid state, and fluid dynamics.

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Stars as small planets

White dwarfs were first recognized as a distinct class of stars, having stellar masses but planetary dimensions, in the early years of the present century. By about 1920, the gravitational redshift of light predicted by Einstein's general theory of relativity had been measured in the white dwarf Sirius B (figure 1), confirming the small size and high density of this star (see the box on page 25). The existence of such compact stars constituted one of the major puzzles of astrophysics until the quantum-statistical theory of the electron gas was worked out by Enrico Fermi and P. A. M. Dirac in the mid-1920's. In accordance with the Pauli exclusion principle, the Fermi–Dirac theory showed that even the coldest electrons cannot all accumulate in the quantum state of lowest energy, and thus the total energy and pressure of an electron gas remain non-zero even at zero temperature. R. H. Fowler and others realized immediately that this was the explanation of the puzzle of the white dwarfs, and Fowler showed that the pressure of a degenerate electron gas, in which the low-lying quantum states are successively filled, is sufficient to support an object of stellar mass against its own self-gravitation at precisely the radii of the white dwarfs (see the box on page 28). For this reason white dwarfs are sometimes also called "degenerate dwarfs," and I shall use these two terms interchangeably.

By the early 1930's, Subrahmanyan Chandrasekhar, then a young graduate student at Cambridge, had extended the theory of the degenerate electron gas to include the effects of special relativity. As a result of this work, Chandrasekhar found that there is a critical stellar mass above which stable degenerate dwarfs cannot exist (see the box on page 28). This was a major discovery, for the existence of this critical mass—now called the "Chandrasekhar limit," $M_{\rm Ch}$, about $1.44 M_\odot$—has profound consequences for the final stages of stellar evolution. Stars that end their evolution with masses in excess of this limit have gravitational self-attraction too great to be counterbalanced by the pressure of the degenerate electrons and are doomed to ultimate collapse. Supernovae are external manifestations of such catastrophes. In some cases, even the violence of a supernova explosion does not wholly disrupt the star, however, and a stellar remnant of even greater density than a white dwarf may be left behind: a neutron star or black hole. Investigations of evolutionary paths by which stars become white dwarfs are therefore important not only of themselves, but also because they impose limitations on the possible pathways leading to supernovae and their exotic stellar remains.

Knowledge of the thermal structure of white dwarfs began to develop early in the 1940's. The first step was Robert E. Marshak's application of the quantum theory of electron conduction in metals, worked out earlier by Arnold Sommerfeld and Hans Bethe, to conditions appropriate to the electron gas in a degenerate dwarf. Marshak, then a student with Bethe at Cornell, found the thermal conductivity to be so high that the electron-degenerate core of a white dwarf is virtually isothermal, while, in contrast, heat transfer through the thin, non-degenerate

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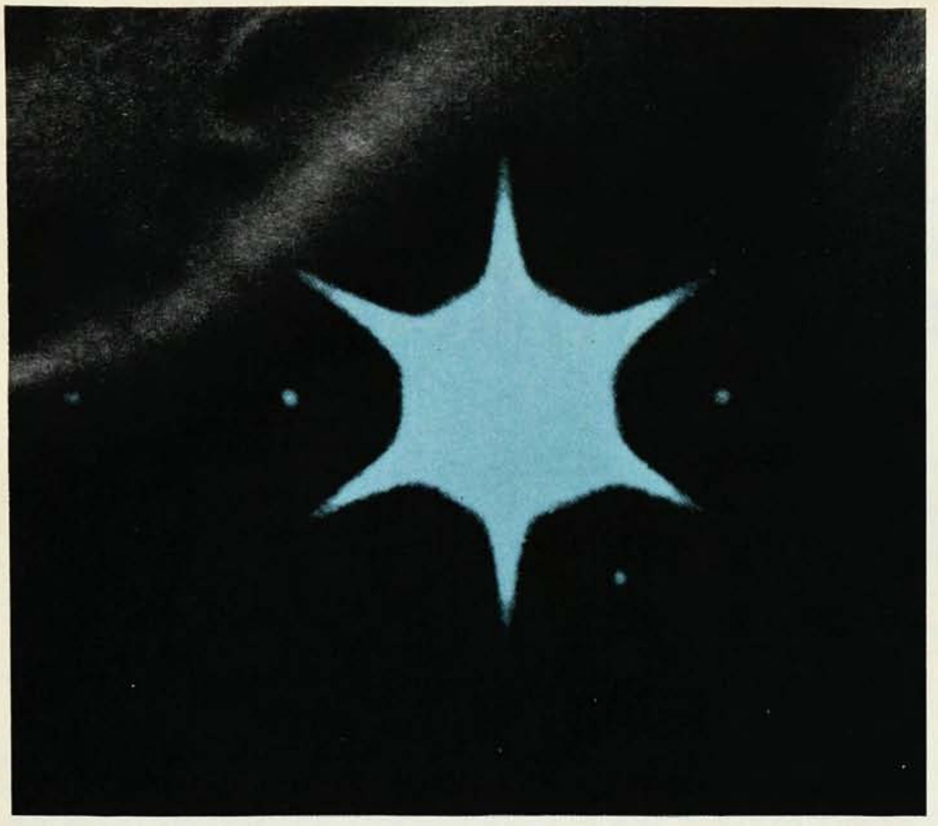


The white dwarf Sirius B is the companion of the ten-thousand-times brighter star Sirius. Sirius B is below and to the right of the bright image of Sirius in this photo by Irving Lindenblad of the US Naval Observatory (reference 2). The hexagonal shape of the image of Sirius as well as the supernumerary images to the left and right are caused by a hexagonal diffraction grating. Figure 1

surface layers is very slow and inefficient. Numerical calculations showed that for typical white-dwarf luminosities of roughly $10^{-2}L_{\odot}$, the temperature of the isothermal core is about 10^7 K. Even at these high temperatures—as hot as the center of the Sun—the electron gas is degenerate because of the high density of matter in the white-dwarf core. Thus, a white dwarf came to be pictured as a massive, hot, conducting sphere sheathed in a thin, but very effective, blanket of insulation.

By this time, astronomical observations had long established that hydrogen is the most abundant element in the universe and that it makes up about 70 percent of the mass of ordinary stars. In 1938, Bethe calculated the rates of thermonuclear reactions involving hydrogen under conditions appropriate to stellar interiors, and he showed that these reactions provide the energy source from which the luminosities of most ordinary stars are derived. For this work, Bethe was awarded the Nobel Prize for Physics in 1967. Application of Bethe's results to white dwarfs, however, led to a startling contradiction: the predicted energy generation rates led to stellar luminosities many orders of magnitude greater than those observed! The inescapable conclusions were:

- Thermonuclear reactions are *not* the source of white-dwarf luminosities, and
- Hydrogen must be entirely absent from the white-dwarf interiors.

These results showed clearly that the white dwarfs are stars that have exhausted their thermonuclear energy sources and are indeed among the terminal stages of stellar evolution—the burned-out embers of dying stars. What, then, is the source of the energy they radiate? This question went unanswered until 1952, when Leon Mestel[3] pointed out that the thermal energy content of the hot white-dwarf interior, leaking slowly away through the insulating surface layers, is entirely sufficient to explain the observed luminosities. Mestel's theory also predicts a statistical distribution of the luminosities of the white dwarfs that agrees satisfactorily with existing observations. Except for details, this theory of white-dwarf evolution has remained unchanged over the past quarter century, and it is the accepted theory today.

Spectra and surface layers

Observations of white dwarfs require the light-gathering power of a large telescope, because these stars are intrinsically very faint. Completion of the 5-meter (200-inch) Hale reflector on Mt Palomar in 1948 provided the first opportunity for detailed studies of these stars. In the following decade, Jesse Greenstein and his collaborators at the Hale Observatories began extensive spectroscopic and photometric investigations using this instrument, and their work has provided much of our present observational knowledge of the white dwarfs. In the past two decades, other large telescopes have been constructed in various parts of the world, and several other groups have also begun white-dwarf observations. As a result, detailed spectroscopic information has now been acquired for more than 400 of these stars. More recently, satellite and rocket-borne observations have begun to open up the far ultraviolet and x-ray regions of the spectrum that are inaccessible from ground-based observations, and important new results have already been obtained from this research.

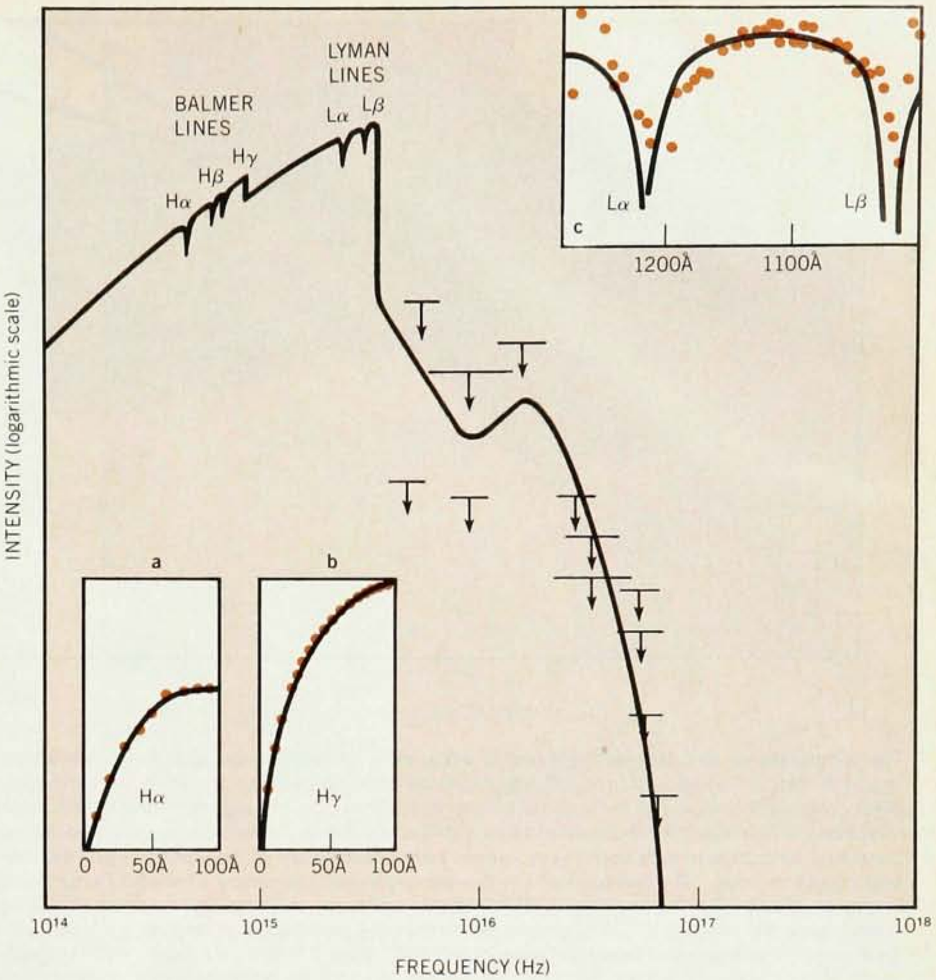
It became apparent quite early that white-dwarf spectra are very different from those of ordinary stars. One difference is that the spectral lines are very broad, regularly being 20 to 50 Å wide at half the central line depth. This is a direct consequence of the high surface gravities of degenerate dwarfs: $g \sim 10^8$ cm/sec^2, some 10^4 times larger than that of the Sun. The high gravities result in high pressures in the atmospheres of these stars that in turn cause large Stark broadening of the spectral lines.

A second difference from ordinary stars is the striking abnormality of the element abundances in white-dwarf atmospheres, for reasons that are not yet fully understood. In ordinary stars, differences among the spectra are primarily due to differences in effective surface temperatures $T_{\rm eff}$ (and, to a lesser extent, in surface gravities), rather than to large intrinsic differences in abundances. Near $T_{\rm eff} \approx 10\,000$ K, normal stellar spectra are dominated by the strong Balmer absorption lines of hydrogen. At higher temperatures most of the hydrogen becomes ionized, causing the Balmer lines to weaken; stars with $T_{\rm eff} \gtrsim 15\,000$ K are characterized by strong absorption lines of helium, the second most abundant element, which is less easily ionized than hydrogen. Conversely, in stars much cooler than 10 000 K (like the Sun, with $T_{\rm eff} = 5800$ K), the spectra are dominated by the lines of various metals and, in extreme cases, by molecular bands. These species dissociate or ionize much more readily than hydrogen with increasing $T_{\rm eff}$, and their lines accordingly disappear in the hotter stars.

In marked contrast to this well-understood behavior of ordinary stellar spectra, the white dwarfs display few explicable spectral regularities. By far the largest numbers of these stars, comprising about two-thirds of all white dwarfs, exhibit only the pressure-broadened Balmer lines of hydrogen. These spectra are all the more remarkable because we know that hydrogen must be completely absent from the deep interiors of these stars! This group of hydrogen-line white dwarfs is termed spectral class DA. The spectrum of Sirius B, which is a DA white dwarf, is shown in figure 2. The remaining third of the degenerate dwarfs, which display other than pure hydrogen spectra, are distributed among a number of different spectral classes. About 8% of all white dwarfs have essentially pure helium line spectra and are classified as spectral type DB. Another 14% of the total display only featureless continua; these are termed spectral type DC. Still other groups of white dwarfs, containing only a dozen or so stars each, exhibit weak metallic lines or molecular bands, generally of carbon-bearing molecules.

In the DA white dwarfs, the absence of lines due to elements other than hydrogen cannot be simply the result of peculiarities of ionization equilibria in dense stellar atmospheres. The enormous range of T_{eff} covered by the DA spectral class extends from less than 5000 K to more than 70 000 K (for HZ43 and Feige 24, the hottest white dwarfs known). The associated variations in ionization balance would inevitably lead to spectral changes analogous to those in ordinary stars if other elements were present in significant amounts. Instead, the elements other than hydrogen appear to be truly deficient in the DA white dwarfs, relative to normal cosmic abundances. The probable explanation of the remarkable purity of these hydrogen atmospheres, which was proposed some time ago by the French astrophysicist Evry Schatzman, is that gravitational settling has taken place in the strong gravitational fields of the degenerate dwarfs, with heavier elements sinking below the atmospheric layers and the hydrogen floating up to the stellar surface. The estimated timescale for this process appears short enough to give rapid element separation in the high gravity fields of the white dwarfs, thus satisfactorily accounting for the spectral appearance of the DA stars.

The existence of white dwarfs with non-DA (hydrogen-deficient) spectra, however, has not yet been satisfactorily explained. The complete absence of hydrogen from these stars appears to require ejection of the residual, hydrogen-bearing surface layers sometime after the completion of thermonuclear hydrogen-burning and before these stars become white dwarfs. Stars known to be among the ancestors of white dwarfs and that are observed to have ejected as much as several tenths of a solar mass of material, do exist. They are the stellar nuclei of the so-called "planetary nebulae." It is not clear whether the nuclei of planetary nebulae are completely devoid of hydrogen, however, and it also appears possible that their rate of formation may significantly exceed the estimated birthrate of the non-DA white dwarfs. The evolutionary connections between the planetary nuclei and specific white dwarf spectral classes thus remain still to be established. In addition, the situation is complicated further by the existence of several other types of hydrogen-deficient



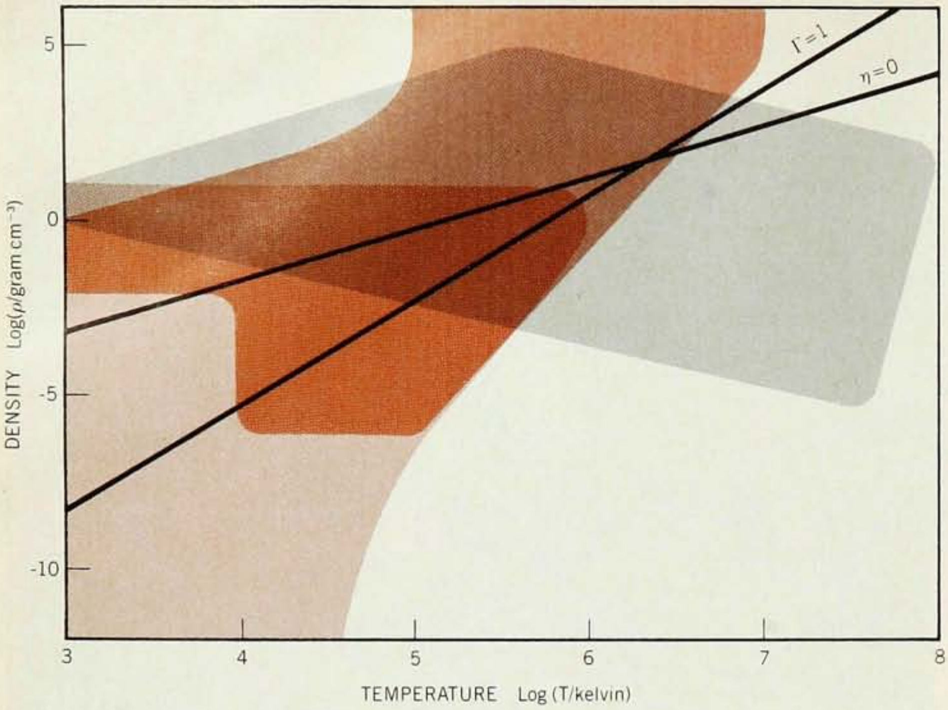
The spectrum of Sirius B. The curve in the main figure gives the emergent flux distribution computed by Harry Shipman for a pure hydrogen stellar atmosphere with T_{eff} = 32 000 K and log g = 8.65. The bars with arrows give upper limits of extreme-ultraviolet emission from rocket-borne measurements (references given in reference 4); they indicate a temperature somewhat lower than 30 000 K for the star. Insets **a** and **b** show line profiles of the Balmer lines $H\alpha$ and $H\gamma$ with intensity plotted as a function of $|\Delta\lambda|$, the magnitude of the distance to the line center (data from reference 5; theoretical curves for T = 33 000 K, log g = 8.8). Inset **c** shows the Lyman lines $L\alpha$ and $L\beta$ with data from Copernicus satellite measurements (reference 6) and a theoretical curve calculated by François Wesemael for a T = 27 000 K log g = 8.65 pure hydrogen atmosphere. Figure 2

Sirius B

Sirius B, the faint companion of the "dog star" Sirius, was one of the first white dwarfs to be discovered. Some of its properties are given in the accompanying table. For comparison, the corresponding values for the Sun, the standard astronomical "yardstick" for measurements of stellar quantities, are also given. It is interesting to note that the radius of the Earth is 6.371×10^8 cm = $0.00915 R_\odot$, somewhat larger than Sirius B!

Quantity	Sirius B	Sun
Mass	$1.05 M_\odot$	$M_\odot = 1.989 \times 10^{33}$ gm
Radius	$0.008 R_\odot$	$R_\odot = 6.96 \times 10^{10}$ cm
Luminosity	$0.03 L_\odot$	$L_\odot = 3.90 \times 10^{33}$ erg/sec
Effective (blackbody) temperature	27 000 K	5800 K
Gravitational redshift	89 ± 16 km/sec	0.6 km/sec
Mean density	2.8×10^6 gm/cm³	1.41 gm/cm³
Central density*	3.3×10^7 gm/cm³	1.6×10^2 gm/cm³
Central temperature*	2.2×10^7 K	1.6×10^7 K

* Model-dependent quantities



The temperatures and densities relevant to white-dwarf atmospheres. The irregularly shaped region in color is the region of helium-dominated white-dwarf envelopes. The lower left-hand corner (light color) is the region of incomplete helium ionization. On the high-temperature boundary, ionization is dominated by thermal excitation of the electrons, and on the upper, high-density, boundary, ionization results from the increased Fermi momentum of the electrons and the overlapping of ion cores. The line marked $\eta = 0$ is the degeneracy boundary where the Fermi energy is approximately kT. The line marked $\Gamma = 1$ shows where the electron–ion Coulomb interaction energy is on the order of kT. The gray region shows the parameters of interest for laser-fusion plasmas. (Figure adapted from reference 7) Figure 3

stars that may also be precursors of some of the non-DA white dwarfs.

To make quantitative analyses of the various white-dwarf spectra observed, we need detailed numerical models of the atmospheres of these stars. Such calculations, first begun about a decade ago, have become increasingly sophisticated in more recent years. Calculations by Peter Strittmatter and D. T. Wickramasinghe at Steward Observatory, by Harry Shipman first at the Hale Observatories and now at the University of Delaware, by Karl-Heinz Böhm and his collaborators at the University of Washington (Seattle), and by Volker Weidemann and his group in Germany have now provided quantitative evaluations of the effective temperatures, surface gravities and atmospheric compositions for many of these stars.

An early result of these model atmosphere calculations that has had far-reaching consequences was Böhm's discovery in 1968 that convection occurs in the surface layers of these stars. Because convection produces efficient mixing in stars, Strittmatter and Wickramasinghe subsequently suggested that mixing of a thin superficial hydrogen layer into a deep subsurface helium convection zone might be capable of transforming DA (H-line) white dwarfs into DB (He-line) white dwarfs at a particular stage in the cooling of these stars. Although this hypothesis is now believed to be incorrect (it appears to be in conflict with the observed temperature distribution of the DA and DB stars, as pointed out by Shipman) the work of Strittmatter and Wickramasinghe was the first to consider the possibility of evolutionary transitions between different spectral classes of the white dwarfs.

More recent investigations have lately begun to establish a reliable framework for understanding the chemical evolution of white-dwarf surface layers. In particular, Shipman has pointed out that at some stage in the cooling of DB stars the helium lines must disappear. If the atmospheres of these stars contain nothing other than helium, their spectra at still cooler temperatures will thus display only featureless continua. Accordingly, Shipman suggests that the DC (pure continuum) white dwarfs are simply cooled-off DB's. Much additional work remains to be done before the many remaining spectral classes can be fitted into this scheme, however, and efforts are just beginning to explore the effects of convection, gravitational settling, accretion, and the thermal evolution of the star itself upon the spectral evolution of the white dwarfs.

A second important consequence of convection in white dwarfs is its effect upon heat transfer through the non-degenerate outer layers of these stars. Because the rate of heat transfer from the isothermal degenerate core of a white dwarf through the non-degenerate "envelope" to the stellar surface controls the rate of cooling of the star, and because convection is generally a much more efficient heat transfer process than radiative diffusion, the presence of a subsurface convection zone can greatly affect the rate of cooling of a white dwarf. Indeed, Böhm found that in the cooler white dwarfs the convection zones may actually penetrate into the degenerate cores, causing a great reduction in the insulating abilities of the stellar envelopes.

Models of the convective envelopes of the white dwarfs, however, involve theoretical calculations of the opacities, thermal conductivities, and thermodynamic properties of plasmas under rather complex physical conditions. Not only is ionization incomplete and pressure-ionization important, but also the electrons are partially degenerate and Coulomb interactions are strong enough that ion motions in the plasma begin to exhibit correlations. Rather interestingly, these conditions are very similar to those encountered in the laser-produced plasmas now being studied for thermonuclear fusion applications (figure 3). For the white-dwarf problem, the most extensive calculations of thermodynamic properties for this complex physical regime have been carried out by Gilles Fontaine[7] (now at the Université de Montréal), H. C. Graboske, Jr (of Lawrence Livermore Laboratories), and myself; by Francesca D'Antona, Italo Mazzitelli, and G. Magni; by M. A. Sweeney (now at Sandia Laboratories); and by Böhm and his collaborators. In some ranges of stellar temperatures, the white-dwarf envelope models constructed by the different groups agree rather well. However, at low temperatures serious differences arise due to the uncertainties in the calculations of the thermodynamic and transport properties for the deeper parts of white-dwarf envelopes. The resolution of these problems is important for both white-dwarf and laser-fusion calculations and provides a point of contact between white-dwarf research and a developing area of physical theory.

Diamonds in the sky

An important milestone in the theory of degenerate dwarfs was Edwin Salpeter's publication[8] in 1961 of his calculations of the thermodynamic properties of fully degenerate matter, including the corrections to Chandrasekhar's theory arising from electrostatic interactions among the ions and degenerate electrons. These corrections are primarily attractive and reduce the energy and pressure of the plasma by as much as several percent at low temperatures. The reduced pressure support in turn causes similar reductions both of the radius of a star of a given mass and of the limiting mass of a stable white dwarf, compared to the values given by Chandrasekhar's theory (see figure 4).

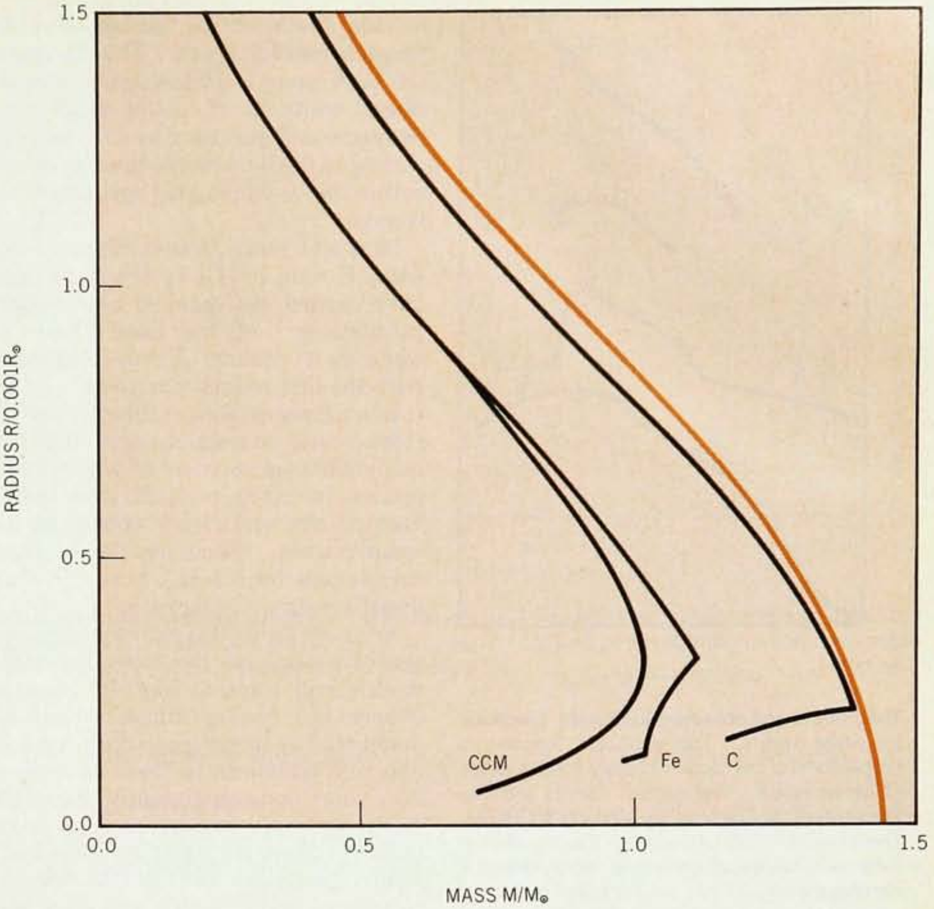
Because the electrostatic interaction energy depends upon the charge $+Ze$ of the positive ions, Salpeter's results depend upon the composition of the degenerate matter. Unfortunately, except for the certainty that there can be no appreciable hydrogen in a white dwarf, there is meager information about the internal compositions of these stars. Evolutionary calculations indicate that the progenitors of the white dwarfs have probably undergone both hydrogen- and helium-burning, while it appears unlikely that carbon ignition occurs in evolutionary sequences leading to the white dwarfs. Thus the most plausible composition for the core of a degenerate dwarf is a mixture of carbon and oxygen, the main products of helium-burning reactions.

One of the most interesting consequences of Salpeter's work was his recognition (also pointed out independently by A. Abrikosov and D. A. Kirzhnits in the Soviet Union) that at very low temperatures the Coulomb interactions in a high-density plasma cause the ions to freeze into a regular crystalline *lattice* within the nearly uniform "sea" of degenerate electrons. This crystallization occurs because an ion lattice has lower energy than a random distribution of point charges under these conditions. At high temperatures, of course, the thermal motions of the nuclei are sufficiently energetic to prevent lattice formation. Thus the interesting question arises: At what temperature does white-dwarf matter crystallize?

A preliminary answer to this question was obtained in 1966 in a now-classic work[9] by Stephen Brush, H. L. Sahlin and Edward Teller of the Lawrence Radiation Laboratory. In this paper, the statistical mechanics of a classical, one-component plasma, consisting of positive point charges in a uniform neutralizing background-charge distribution, was investigated numerically by Monte Carlo calculations. The properties of this system depend only upon the ratio of the Coulomb interaction energy to kT (effectively the ratio of potential to kinetic energy), and Brush and his colleagues accordingly found it convenient to express their results in terms of the dimensionless parameter

$$\Gamma \equiv (Ze)^2/akT \qquad (1)$$

where a is the radius of a sphere containing one single ion at the given ion density. High temperatures (or low densities) correspond to small Γ, and the ions behave as an ideal gas with weak electrostatic corrections. For $\Gamma \gtrsim 2.5$, however, the ion distribution begins to develop short-range order, as in a conventional liquid, and at $\Gamma \approx 125$, Brush, Sahlin and Teller found direct evidence in their Monte Carlo results of the expected first-order phase transition from this Coulomb liquid to a regular crystalline solid. More recent Monte Carlo calculations, carried out principally by Jean-Pierre Hansen and his coworkers in France, have greatly refined and extended this work, and have made the one-component plasma now one of the best understood nonelementary systems in statistical mechanics. As one result of these calculations, the Γ-value of the liquid-solid phase transition has been evaluated with considerably greater accuracy, and it is now believed that crystallization occurs near $\Gamma = 150$.



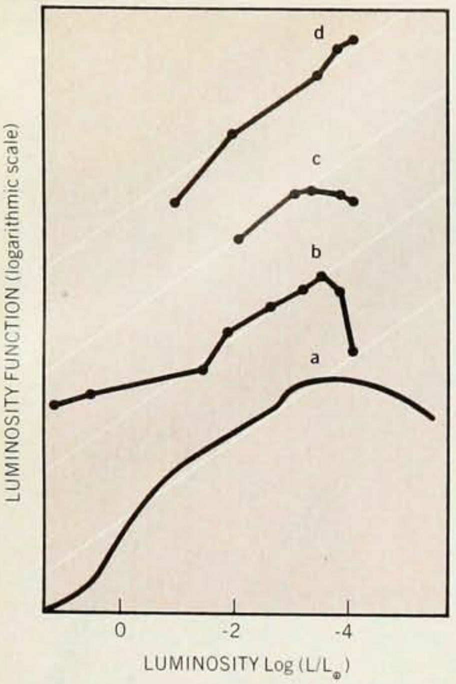
Mass–radius relation for white dwarfs. Chandrasekhar's pressure–density relation for a zero-temperature, non-interacting electron gas with $\mu_e = 2$ is shown by the colored line (see the box on page 28). The black curves show Salpeter's pressure–density relations for zero-temperature dwarfs for the composition indicated. CCM is cold-catalyzed matter, in which the iron nuclei have captured electrons from the Fermi sea to produce the lowest energy state of nuclei plus electrons. (Adapted from reference 8) Figure 4

To appreciate the relevance of this result for white-dwarf evolution, we can use equation 1 to express $\Gamma = 150$ as a temperature-density relation for the plasma crystallization curve. For a C^{12} plasma we obtain

$$\frac{T}{10^7\,\mathrm{K}} = 0.238\left(\frac{\rho}{10^6\mathrm{gm\ cm^{-3}}}\right)^{1/3} \qquad (2)$$

For densities and compositions that are typical of white-dwarf interiors, crystallization thus takes place at temperatures characteristic of the cores of the cooler degenerate dwarfs. The coolest white dwarfs are thus, almost literally, "diamonds in the sky," except with a body-centered cubic lattice (that is, the lattice cell is a cube with ions located at the eight corners and at the center), rather than an actual diamond lattice structure.

Crystallization of the plasma in a degenerate dwarf has a number of consequences for the evolution of these stars, as pointed out some time ago by Mestel, Malvin Ruderman, and myself.[10] First, the crystallization temperature given by equation 2 is quite sensitive to the core composition of the white dwarf, varying approximately as $Z^{5/3}$. Perhaps this sensitivity may eventually enable us to place more stringent limits upon the allowable range of core compositions in these stars. In addition, crystallization leads to the release of latent heat associated with the liquid-solid phase transition, which slightly reduces the rate of cooling of the star during this stage. Early calculations raised the hope that this would slow the evolution enough to be observable, thus marking the composition-dependent locations of the crystallizing white dwarfs. However, the effect has turned out to be relatively small, and observations have so far been unable to distinguish it. Still another consequence of crystallization is the rapid decline of the heat capacity when the core temperature of the white dwarf falls below the Debye temperature, which is determined



Theoretical and observed luminosity functions for white dwarfs. The luminosity function is proportional to the number of stars in each range of luminosities. The various curves are displaced vertically by arbitrary amounts for clarity. The straight diagonal lines are the theoretical luminosity function predicted from Mestel's cooling theory. Curve **a** is the result of a theoretical calculation (reference 11). The experimental curves are based on measurements by: (**b**) Luyten (1958), (**c**) Luyten (1963–66) and (**d**) Eggen and Greenstein (1965). Figure 5

by the characteristic lattice vibration frequencies of the ions. This decline of the heat capacity greatly accelerates the rate of white-dwarf cooling at low temperatures and may even lead to complete cooling of the more massive white dwarfs within the 10-billion-year age span of the Galaxy.

In recent years, Don Lamb and I and Attay Kovetz and Giora Shaviv in Israel have carried out detailed evolutionary calculations[11] of the later phases of white-dwarf cooling. These calculations were the first to incorporate full and accurate representations of the effects of the electrostatic interactions upon the thermodynamic properties of white dwarf matter, including crystallization, latent heat release, and Debye cooling at low temperatures. Some results of these calculations for a 1 $M_\odot$, pure C^{12} white dwarf are shown in figure 5.

From the figure it is clear that the most sensitive tests of the new theoretical models will come at high luminosities (where the cooling times, which are dominated by energy losses due to copious neutrino emissions in these phases are very short and consequently the stellar statistics small) and at very low luminosities (where the stars are intrinsically faint, sometimes difficult to identify clearly as white dwarfs, and thus very hard to study). Progress is being made at both extremes, however. Recent space-borne observations in the extreme ultraviolet have revealed a (small) number of white dwarfs with much higher surface temperatures than previously realized. At the other end of the range, a systematic search for very faint white dwarfs has been undertaken by James Liebert (at Steward Observatory), Conrad C. Dahn (at the United States Naval Observatory), and their collaborators. This work has already begun to yield results that suggest discrepancies between the observations and the theories. Although it is premature to conclude that there is a serious conflict, further investigations are clearly necessary. Undoubtedly the Faint Object Spectrograph currently being planned for the Space Telescope will provide important data bearing on this problem when it is orbited early in the next decade.

Magnetic white dwarfs

In 1970, James Kemp and John Swedlund of the University of Oregon together with John Landstreet and Roger Angel, then of Columbia University, made the remarkable discovery[12] that the optical continuum radiation from the white dwarf Grw +70°8247 is circularly polarized. This was immediately interpreted as evidence for a strong magnetic field in this star. Since then, Angel (now at Steward Observatory), Landstreet (now at the University of Western Ontario), and others have discovered[13] about a dozen magnetic white dwarfs, half of them

The radii of white dwarfs and the Chandrasekhar limit

The structure of any star is determined by the condition of hydrostatic balance between internal pressure and self-gravitation. In an ordinary star like the Sun, the pressure is the ideal-gas pressure of the ions and electrons at the temperatures prevailing in the stellar interior. In a white dwarf, the pressure is dominated by the degenerate electron gas and is insensitive to the temperature. The mechanical equilibrium of a white dwarf can thus be calculated to a very good approximation by taking the pressure to be that of a fully degenerate electron gas at zero temperature. In this case, the radii of degenerate dwarfs and the value of the Chandrasekhar limit can be roughly estimated by the following argument, which emphasizes the physical principles underlying the structures of these stars.

Suppose the electron gas is compressed to a density n_e, and let r_0 be the radius of a sphere containing one single electron, that is, $r_0 = [3/(4\pi n_e)]^{1/3}$. From the Heisenberg uncertainty principle, the momentum of an electron confined to a region of dimension r_0 is of order $\hbar/r_0$. The quantum states available to the degenerate electrons thus range from states of nearly zero momentum (corresponding to electrons with macroscopic de Broglie wavelengths) up to states of momentum p_F (the Fermi momentum), $p_F \sim \hbar/r_0$. The Fermi energy ϵ_F, which is, for an electron gas, roughly the same as the mean energy per electron, is just the kinetic energy for electrons with momentum p_F. For extremely relativistic electrons ($p_F \gg mc$) the Fermi energy is proportional to $n_e^{1/3}$, while for non-relativistic electrons ($p_F \ll mc$) it is proportional to $n_e^{2/3}$. We can now use thermodynamics to compute the pressure of the gas, using the usual definition of pressure as the derivative of energy with respect to volume:

$$P \sim -\frac{\partial \epsilon_F}{\partial (n_e^{-1})}$$

For non-relativistic electrons

$$P \sim \hbar^2 n_e^{5/3}/m \qquad \text{for } n_e \ll (mc/\hbar)^3$$

and for relativistic electrons

$$P \sim \hbar c n_e^{4/3} \qquad \text{for } n_e \gg (mc/\hbar)^3$$

For the fully ionized matter in a white dwarf, the mass density and electron density are related by

$$\rho = n_e/\mu_e H$$

where $\mu_e = A/Z$ is the so-called mean molecular weight per electron, A and Z are the atomic mass and charge, respectively, and $H = 1.66 \times 10^{-24}$g is the unit of atomic mass. Together, these expressions for $P(n_e)$ and $\rho(n_e)$ comprise an approximate parametric equation of state for white-dwarf matter.

The condition of hydrostatic equilibrium is, for self-gravitating objects,

$$\frac{dP}{dr} = -\rho G M_r / r^2$$

where M_r is the mass interior to a sphere of radius r. A rough dimensional analysis of this equation replaces dP/dr by P/R, and r by R. In equilibrium the pressure and gravitational force must balance.

In the non-relativistic limit

$$P \sim GM^2/R^4 \sim \hbar^2 (M/\mu_e H)^{5/3}/mR^5$$

and we can always find a value of the stellar radius such that balance is achieved. For a star of one solar mass this argument gives the somewhat low value $R \sim 2 \times 10^8$ cm.

In the relativistic limit

$$P \sim GM^2/R^4 \sim \hbar c (M/\mu_e H)^{4/3}/R^4$$

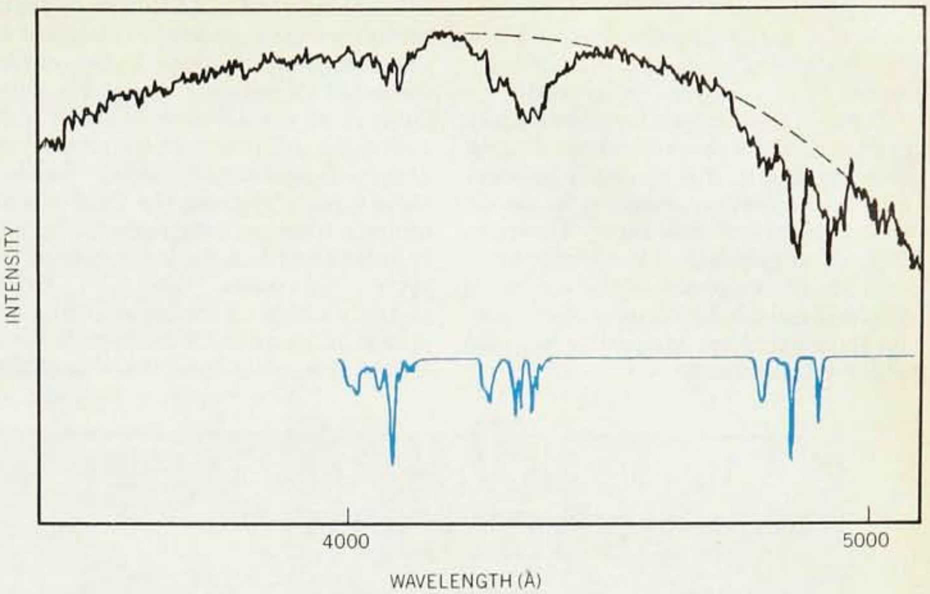
and equilibrium can exist only for a special value of the mass

$$M \sim (\hbar c/G)^{3/2}/(\mu_e H)^2$$

For $\mu_e = 2$ this mass is roughly $\frac{1}{2} M_\odot$ or 10^{33} gm. Exact calculation yields the value $M_{Ch} = 1.44\ M_\odot$ quoted in text. For masses $M > M_{Ch}$, the gravitational potential, GM^2/R^4, overwhelms the pressure and leads to collapse. For $M < M_{Ch}$, the pressure dominates and expands the star to lower (less relativistic) densities, where more exact calculations show that hydrostatic balance can again be achieved.

from polarization studies and half by observations of Zeeman splitting in spectral lines of H, He, or CH. The inferred magnetic field strengths range from about 5×10^6 to more than 10^8 gauss. In the higher fields, the Zeeman splitting is so extreme that field gradients over the stellar surfaces often broaden the lines into unrecognizability. In addition, some of the magnetic white dwarfs exhibit atmospheric composition anomalies, which are presumably caused in some way by the strong fields.

Quantitative analyses of the spectra of magnetic white dwarfs require calculations of the Zeeman shifts of atomic energy levels. Although the shifts are linear with magnetic field strength for weak fields, the term in the Hamiltonian that is quadratic in the field intensity dominates when the fields become as large as those in the magnetic white dwarfs. For such cases numerical computations of the level shifts are required. Calculations have been carried out for the levels giving rise to optical transitions in hydrogen and helium, the most detailed work having been done by S. B. Kemic and Roy Garstang at the Joint Institute for Laboratory Astrophysics. An example of the computed splittings of the Balmer lines H_β, H_γ, and H_δ (which have unperturbed wavelengths of 4861Å, 4340Å, and 4101Å, respectively) is shown in figure 6 for a relatively weak field of 5×10^6 gauss. The widths of the line subcomponents result from averaging over a uniform distribution of fields ranging from 4.5 to 5.5×10^6 gauss. Also shown is an observed spectrum of the magnetic white dwarf GD90, with the same wavelengths marked as in the theoretical spectrum. Evidently the magnetic shifts account very well for the locations of the observed features.



Observed and predicted spectra of the magnetic white dwarf G 90. The theoretical spectrum (lower curve) is based on calculations of the Zeeman splitting of the Balmer lines in a magnetic field of 5×10^6 gauss. (Adapted from reference 13) Figure 6

The origin of the magnetic white dwarfs poses an interesting problem. Calculations have shown that the time scale for ohmic dissipation of the magnetic field is of the order of several billion years. This is long enough for the white dwarfs to cool down to surface temperatures near $T_{\rm eff} \approx$ 5000 K, and it may be significant that no magnetic white dwarf has yet been discovered with temperature much lower than this. The long magnetic-field decay times are consistent with the view that the fields are primordial (frozen in when the star became a white dwarf), rather than being generated later by some process intrinsic to cool degenerate stars. How the fields were produced in the ancestors of the white dwarfs is still not known, however. Are they frozen-in relics dating back to the main sequence origins of the magnetic white dwarfs? Were they produced in a stellar dynamo perhaps driven by the final stage of thermonuclear burning prior to the white dwarf phase? The answers to such questions, which are still being sought, can teach us much about the relationship of the white dwarfs to the earlier stages of stellar evolution.

Oscillating white dwarfs

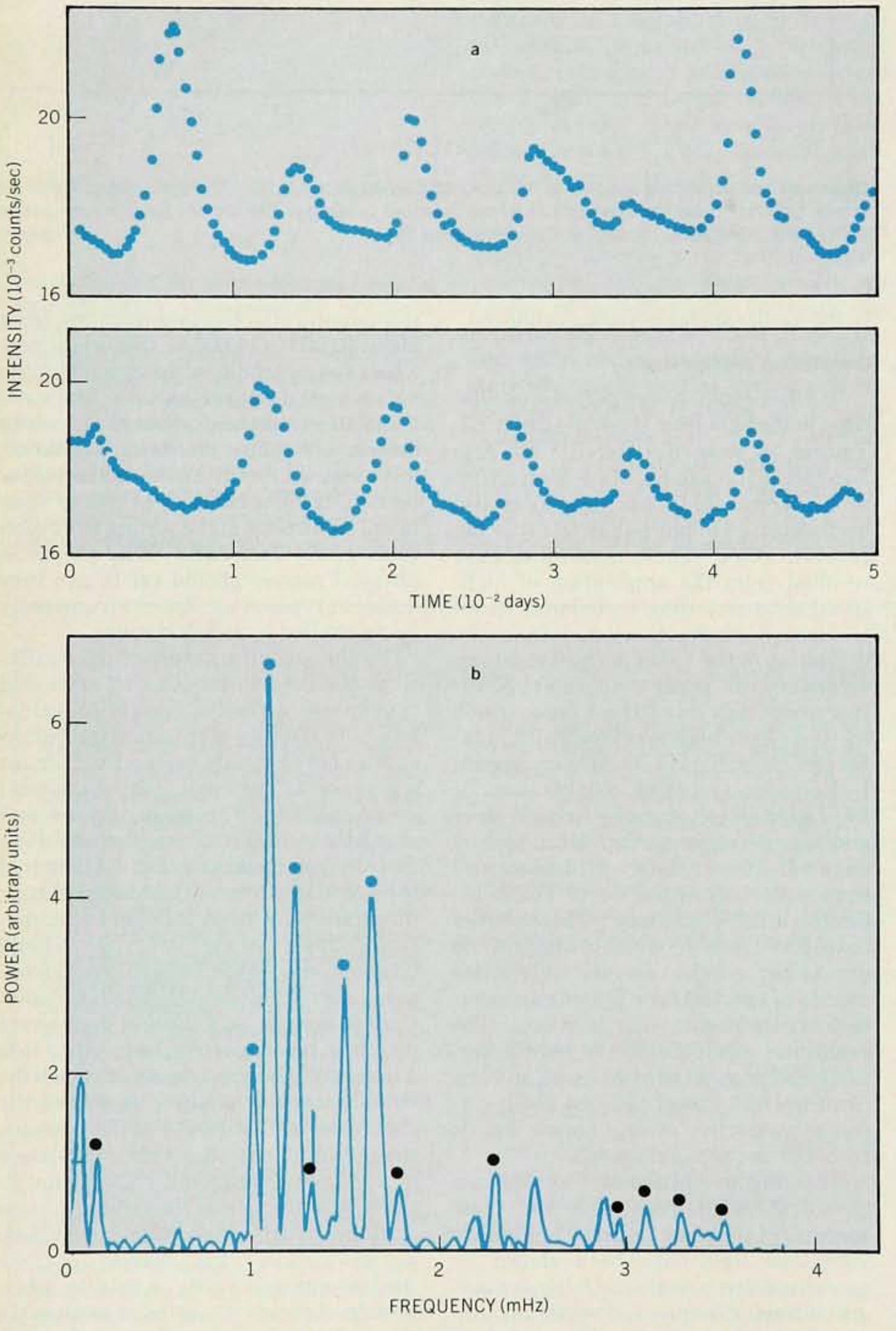
In 1968, rapid, quasi-periodic oscillations in the light from the white dwarf HL Taurus 76 were discovered[14] by Arlo Landolt of Louisiana State University. Since then, rapid variations have also been detected in more than 30 other degenerate stars. These discoveries have resulted from the application of high-speed data-recording techniques to astronomical observations and from the utilization of the Fast Fourier Transform algorithm to permit efficient power spectrum analyses of these data. Such methods have been employed with notable success by R. Edward Nather, Edward L. Robinson and their collaborators, at McDonald Observatory; by Brian Warner and his colleagues at Capetown, and by James E. Hesser, Barry M. Lasker and their coworkers at the Cerro Tololo Interamerican Observatory. These studies have shown that the variable white dwarfs are either single, normal degenerate dwarfs of spectral type DA or else members of cataclysmic binary systems. The excitation mechanisms are almost certainly different in the two cases, and one can therefore expect to learn about different properties of degenerate dwarfs from the two types of systems.

The single white-dwarf oscillators, named ZZ Ceti stars after the type-member of the class, undergo luminosity variations that have been shown by power-spectrum analyses to involve several different frequencies excited simultaneously (see figure 7). The periods of these oscillations are of the order of several hundred to a few thousand seconds, much too long for radial pulsations of the white dwarfs. On the other hand, the ZZ Ceti stars are concentrated in a very narrow range of effective temperatures, from about 10 000 to 14 000 K; this is precisely where the instability strip defined by the well-known Cepheid variable stars extrapolates into the domain of the white dwarfs. Perhaps the same excitation mechanism that drives the radial pulsations in the Cepheids gives rise to non-radial oscillations in the surface ionization zones of ZZ Ceti stars. Why the same physical process should excite two very different types of oscillations is currently being studied by several groups.

To elucidate the nature of the oscillations observed in the ZZ Ceti stars and cataclysmic variables, theoretical calculations of the free, non-radical oscillation spectra have recently been carried out for a number of different, realistic white-dwarf models. The most detailed and useful theoretical studies to date are those done by Yoji Osaki and Carl J. Hansen of the Joint Institute of Laboratory Astrophysics, by Anthony Brickhill of Capetown Observatory, and by Wojtek Dziembowski of the Copernicus Astronomy Center in Poland.[16] Stellar oscillations in general, and those of degenerate dwarfs in particular, can be grouped into a number of different classes, of which the most thoroughly studied have been the spheroidal oscillations. For these modes, the pulsation eigenfunction is composed of a spherical harmonic $Y_{lm}(\theta,\varphi)$ multiplied by a function of the radius, in close analogy with quantum-mechanical wavefunctions. Two classes of these modes appear to play a role in white dwarfs. In one of these the pressure is the dominant restoring force (the "p-modes"), as in ordinary sound waves. Gravity is the dominant restoring force in the "g-modes," which have somewhat the character of long ocean waves. Radial pulsations are a special case; they correspond to p-modes with $l = 0$. The velocity distributions for all other modes—the remaining p-modes and all g-modes–have transverse components.

Because the g-modes have the longest periods of any of the possible oscillations of white dwarfs, it is generally believed that the oscillations observed in the ZZ Ceti stars are of this type. However, theoretical calculations have yet to demonstrate the existence of the suspected Cepheid-variable-like driving mechanism for modes that are compatible with the observed pulsations.

In contrast to the ZZ Ceti stars, rapid oscillations are not present at all times in the cataclysmic binaries. These systems consist of an ordinary star and a white dwarf that are so close together that matter can spill over from the normal star onto its degenerate companion. Some of these types of systems, the dwarf novae, undergo frequently recurring outbursts. In these eruptions, the luminosity of the system increases manyfold, rising abruptly and then decaying again in about a week or ten days. It is during the outbursts that oscillations have been detected in the dwarf novae. The oscillation periods are generally about 10 to 35 seconds, and they also appear to be somewhat too long for radial modes. In addition, the periods change gradually with the system luminosity (which seems incompatible with identifications as radial modes), and the oscillations appear and disappear much more rapidly than seems possible for radial pulsations. Rotating stars like the white dwarfs in the cataclysmic variable systems can also sustain toroidal oscillations. John Papaloizu and James E. Pringle of the Institute for Theoretical Astronomy in Cambridge have suggested that these modes, which may have periods as short as the rotation periods of the white dwarfs, may be responsible for the observed oscillations. The mechanism of excitation of the oscillations in the cataclysmic binary systems appears to be related to processes, such as accretion or thermonuclear burning, that are associated with the outburst, rather than processes operating in the surface ionization zones, as in the ZZ-Ceti stars. The details, however, are at present completely unknown.



Oscillations of the rapid variable white dwarf G29-38. Part a shows a portion of the light curve of this star taken during one observing run. (The data are continuous from the right part of the upper graph to the left of the lower.) Part **b** shows the power spectrum derived from the full data from this run. Some of the prominent frequencies giving rise to the regularities seen in part **a** are marked with colored dots and several of the beat frequencies that result from their combinations are marked with black dots. (Adapted from reference 15). Figure 7

New challenges

The discoveries about degenerate dwarfs that have occurred within the past decade have created new problems and raised new challenges. Observations from space have already brought about changes in our understanding of these stars, and additional observational data at the short-wavelength end of the spectrum will soon become available from the International Ultraviolet Explorer satellites. Such observations provide essential data for questions about the origins of the white dwarfs. In addition, because the degenerate dwarfs are low-luminosity stars, detailed observations of the faintest ones can be carried out only from above the background of scattered light due to the terrestrial atmosphere. This will become possible with the advent of the Space Telescope to be orbited aboard the Space Shuttle early in the 1980's. Such observations will permit investigations of the very faintest—and thus oldest and coolest—of the degenerate dwarfs. Some of these stars are expected to date back to the earliest periods in the evolution of our Galaxy and may provide new insights into events that happened then. In addition, observations of the faintest degenerate dwarfs will sharpen the comparison with theoretical cooling models and may help to constrain the internal composition of these stars. This would in turn provide information about the processes taking place in the final phases of stellar evolution preceding the white dwarf stage.

Research in the theory of degenerate dwarfs is currently confronted with a variety of problems of considerable interest. For example, we do not at present understand the reasons even for the existence of the non-DA (hydrogen-deficient)

white dwarfs. Related to this is the problem of the spectral evolution of these stars: How do gravitational settling, convection, accretion, and perhaps other processes interact to produce the observed types of white dwarf spectra? Another class of problems concerns the physical properties of matter under extreme conditions: How can these properties be calculated for partially ionized plasmas in the complex physical regime intermediate between rarefied gases and near-solid densities? The answer to this question, in particular, is essential not only for understanding the nature of white-dwarf surface layers and the excitation mechanisms for white-dwarf oscillations, but also for analyses of various layer-produced plasmas and certain types of thermonuclear fusion experiments. Yet additional problems concern the effects of rotation, the origins of white-dwarf magnetic fields, and the role of phase separation during core crystallization in degenerate dwarfs.

Finally, a subject about which I have said little in this article, but which increasingly involves knowledge of the structure and properties of degenerate dwarfs, is that of the cataclysmic variables. Despite recent successes in theoretical modelling of the common nova outburst,[17] severe problems remain. In particular, why do current thermonuclear explosion models fail to "turn off," as novae are observed to do, shortly after the initial outburst? Furthermore, in the case of the dwarf novae, the mechanism of the outburst itself is rather controversial: Is it an accretion event, as suggested by Geoffrey Bath and his colleagues in England, or is it a thermonuclear outburst—a scaled-down version of a nova? Related to this are a whole host of questions pertaining to accretion disks in cataclysmic binaries, including those relating to the structure and properties of the disks themselves and to the nature of the disk–white-dwarf interface. In addition, within the past few years a completely new class of cataclysmic variables has been discovered—the AM Herculis systems. These appear to resemble the dwarf novae, except that the degenerate dwarfs are believed to possess magnetic fields of order 10^8 gauss, which prevent the formation of accretion disks in these systems. The study of these systems has evidently just begun.

A brief overview of white-dwarf research such as I have given here cannot hope to do justice to the many fascinating problems that arise in this area of astrophysical investigation. The variety of physical conditions present in these stars and their intrinsic faintness combine to make observational and theoretical investigations both challenging and rewarding. We thus anticipate the continuing development of interest in the study of degenerate dwarfs as investigations extend the frontiers of research.

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