search & discovery

Superfluid experiments support Kosterlitz–Thouless theory

Five years ago Michael Kosterlitz and David Thouless predicted that in the x–y or planar model of two-dimensional substances, a new type of phase transition would occur. In a two-dimensional superfluid, for example, at low temperatures the system would be arranged as bound vortex–antivortex pairs whose dissociation leads to the transition. More recently David Nelson and Kosterlitz suggested that the theory could be tested experimentally by looking for a universal jump in the superfluid density at the Kosterlitz–Thouless phase transition.

Recently two independent experiments—at UCLA and Cornell University—have determined the discontinuity in the superfluid density as a system passes through the phase transition. Both experiments are in good agreement with the predictions of the Kosterlitz–Thouless theory.

The excitations considered by theorists working on such phase transitions are highly analogous to the topological excitations now popular with particle theorists. The type of problem represented by bound pairs of vortices may offer clues to how one might produce quark confinement.

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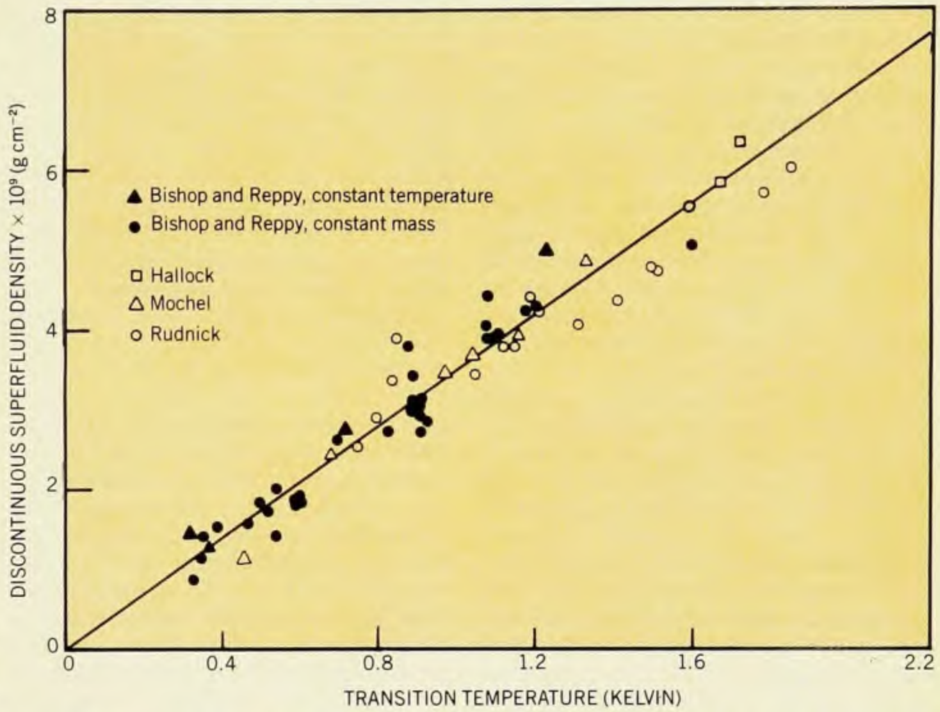

Superfluid transition in two-dimensional He^4 films. Discontinuous superfluid density at onset, ρ_s, is plotted vs. transition temperature. The data by Bishop and Reppy were taken in two ways: constant-mass measurements (using the calculation in reference 6) and constant-temperature measurements. The other data come from third-sound measurements by Rudnick, by Hallock and by Mochel. The line is that predicted by the Kosterlitz–Thouless static theory. (From ref. 8)

New data on two galaxies are consistent with black holes

How is energy supplied in the active centers of radio galaxies and quasars? This intriguing question lies at the heart of various overlapping and competing astrophysical theories, including those that propose the existence of a black hole. Now two separate sets of observations—of an apparent supermassive object[1,2] at the center of galaxy M 87 and of a small narrow radio jet parallel to a much larger jet[3] in NGC 6251—are being welcomed as a chance to test some of these ideas.

The elliptical galaxy M 87, part of the Virgo cluster, was observed by two optical techniques. One set of experiments, done at the Hale Observatories, involves photometry with sensitive TV-type detectors; the other set are spectroscopic measurements at the 4-meter Kitt Peak telescope, done with a different TV image-amplification technique newly applied to astronomy. The combined data point to a supermassive object, possibly a black hole, as a probable explanation for the excess mass-to-light ratio at the center of M 87. In the other experiment, the Caltech radioastronomers who observed the center of NGC 6251 used very long baseline interferometry, with radio dishes at the Owens Valley Radio Observatory (Big Pine, California), the National Radio Astronomy Observatory (Green Bank, West Virginia) and the Haystack Observatory (Westford, Mass). They found an unresolved core and a small (1.7-parsec) jet, pointing in the same direction as a previously discovered 200-kiloparsec jet. Their results appear to support the "beam" theory of energy supply, and are also consistent with the existence of a black hole at the galactic center.

Theorists are interested in M 87 because it is an active radio source with compact central components, is fairly close to us and is known to have a jet emitting polarized optical light. The photometric observations of M 87 are reported by Peter Young, James Westphal, Jerome Kristian, Christopher Wilson (all of the Hale Observatories, Pasadena) and Frederick Landauer (Jet Propulsion Laboratory). They used two types of sensitive two-dimensional imaging devices: commercially available silicon-intensified-target tubes and charge-coupled devices (or "CCD's"). The CCD, developed by Texas Instruments for the Jet Propulsion Laboratory, was designed for the Space Telescope and the Jupiter orbital probe. It is, Kristian

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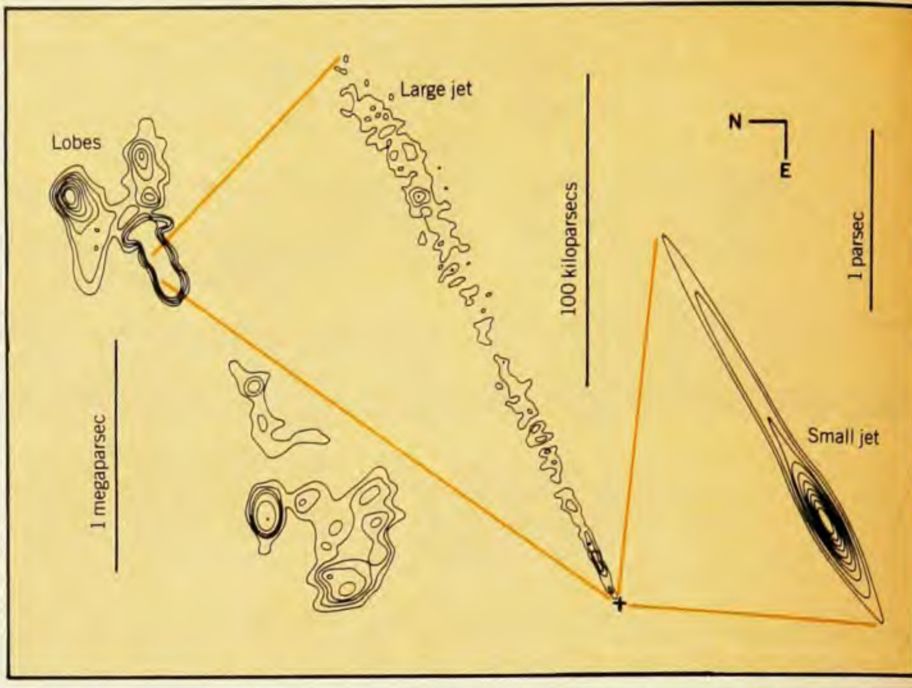
commented to us, in a sense being tested on the ground at Palomar. The performance of the CCD, which has a high photometric accuracy and a very large dynamic range, was carefully checked throughout the observations, and gave the best quality data for the nuclear region of M 87.

The photometric data showed a "light spike" centered within 0.02 arcsec of the center of M 87. To form a radial luminosity profile—that is, to determine the intensity distribution of visible stars throughout the galaxy—the data were summed in narrow concentric rings about the center. The CCD data were used for the inner region and silicon-intensified-target tube data for the outer. Within the central 10 arcsec the profile did not agree with the type of isothermal sphere models that work for most normal elliptical galaxies. Rather than flattening off as the radius r approaches zero, the profile continues to rise even below $r = 1.7$ arcsec where the luminosity spike begins. Simple alterations of the model either did not improve the overall fit or were inconsistent with the velocity-dispersion results. The profile does, however, fit a black-hole model.

The velocity-dispersion measurements were part of a study of mass distribution in the outer parts of galaxies, made possible by the Image Photon Counting System, a spectroscopic technique used to measure faint light sources. This technique was first developed by Alec Boksenberg (University College London) and applied to astronomy in the past few years by Boksenberg and Wallace Sargent. As Sargent explained to us, it is a four-stage image tube, followed by a TV camera that can record single-photon events.

The present group, consisting of Sargent, Peter Young (both of Hale), Boksenberg, Keith Shortridge (University College), Roger Lynds (Kitt Peak) and David Hartwick (University of Victoria), was able to determine the internal velocity of visible stars at representative points throughout the galaxy. With a specially designed Fourier method, they analyzed the broadening of the spectral lines due to relative motions of the stars, and found that the velocity dispersion σ_v varies with radius, from 350 km per sec at $r = 1.5$ arcsec to 220 km per sec at 72 arcsec. Within the core, σ_v rises rapidly. They were unable to explain the mass distribution within M 87 with any simple theoretical model, but with fairly straightforward statistical mechanical methods, independent of any model, they found the total mass M within any given r. Combining this value of M with the photometrically determined luminosity L, they found the M/L ratio within a radius of 1.5 arcsec to be about 60 solar units, compared with a value of about 6 solar units for r equal to 10 arcsec or more, strongly suggesting the presence of a large, dark central mass in the nucleus of M 87. That is, the photometric and spectroscopic data combined show a normal galaxy, down to about 1 arcsec; within this core radius, however, M/L increases rapidly as r approaches zero, reaching a central value of at least 60, despite the observed constancy of the stellar population. To explain this anomaly, the observers propose a central dark mass of about 5×10^9 solar masses, within a radius of no more than 110 parsec: Image distortion within Earth's atmosphere places this limit on the "seeing disc."



Radio source in NGC 6251. Diagrams on varying scales show total source (left) as two lobes, one of which includes a large narrow jet (center). A Caltech group has now found a much smaller jet parallel to the larger jet and has roughly modeled its contours (right). From reference 3.

The double jet in NGC 6251 has aroused excitement because it shows the existence of the same phenomenon to two very different scales. About two years ago, Cambridge radioastronomers had looked at the galaxy and found a 200-kpc jet aligned with two large radio lobes, perpendicular to the galactic major axis. Last July, the Caltech astronomers—Anthony Readhead, Marshall Cohen and Roger Blandford—were able to look at the faint center of NGC 6251. As Cohen explained to us, these observations were possible only because two of the dishes had new cryogenic amplifiers. They observed at 2.82 cm and analyzed their data with closure phase techniques to compensate for the lack of phase calibration in very-long-baseline interferometry.

Although they could use data from only three telescopes, they were able to model the source and found an unresolved nucleus containing—to their astonishment—a narrow jet, aligned with the larger jet. Cohen stressed to us their confidence in the reported general shape and alignment of the jet, describing the exhaustive search to ensure that no error in sign had crept into either the data, the analysis or the hardware. They do not see any jet in the other direction, but this may be because the opposing jet is even harder to detect, and they were working at the limit of instrumental sensitivity.

Possible explanations. If the center of M 87 is a massive black hole, much of what we know about the galaxy falls into place. For example, the observed x-ray emission, about 5×10^{42} ergs per sec, is consistent with a model that attributes the x-ray luminosity to gas accretion in a disc about a black hole, where gas is furnished by the debris of highly evolved stars in the core, and by stars disrupted by tidal effects as they pass too near the black hole. How to form a black hole of this size, given the age of the Universe and the calculated accretion rate, is not so clear. One possibility is that of "globular cluster cannibalism," originally proposed for processes on a smaller scale within our galaxy. Jeremiah Ostriker (Princeton) points out that a similar, even smaller light spike, found in the center of our companion galaxy, Andromeda in 1975, has not been taken as evidence for a black hole.

The Space Telescope, which is scheduled for 1983 and will see down to 0.1 arcsec, may answer some of these questions. The various proposed alternatives to a black hole—dead neutron stars, dead white dwarfs, a collection of smaller nonluminous stars—could be ruled in or out. If the galactic center is a black hole, the density profile within the 1 arcsec will show something of the history; the rate of change of density with radius is related to the rate of growth and the kind of interactions that existed.

The double jet, according to Blandford of the Caltech group, is good morphological evidence for the beam theory of energetic double radio sources. Blandford and Martin Rees (Cambridge) are among the main proponents of this theory. Here the larger jet would represent fluid ejected from the center about 5×10^6 years ago, and the small intense jet is the outline of a region of more recently ejected matter. The "remembered" axis of ejection here is the rotation axis of the massive galactic nucleus—perhaps, but not necessarily, a black hole.

In a possible variation of this theory, suggested to us by George Field (Harvard), a dense disc of gas is rotating rapidly at the galactic center, preferentially ejecting matter along the rotation axis. An explosion within this stratified disc would eject matter in a jet, following ordinary hydrodynamic principles. The apparent constancy of orientation over millions of years appears to rule out theories in which multiple-body interaction ejects matter randomly. Blandford also pointed out that it would be interesting to see whether or not NGC 6251 showed superluminal expansion, as do other quasars and active galaxies. This apparent expansion at a rate greater than c is a kinematical effect, due to the time needed for light to travel across the source. —MSR

References

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2. W. L. W. Sargent, P. J. Young, A. Boksenberg, K. Shortridge, C. R. Lynds, F. D. A. Hartwick, Astrophys. J. **221**, 731 (1978).
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Kosterlitz–Thouless

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The Kosterlitz–Thouless theory is believed to apply to a whole group of other two-dimensional problems—the Heisenberg spin-chain model, the one-dimensional electron gas, two-dimensional crystals, the Baxter model, the Ashkin–Teller model, the interface roughening transition, the massive Luttinger model and the Sine–Gordon equation, all of which have a line of critical points.

In the mid-1960's Pierre Hohenberg (Bell Labs) and separately N. David Mermin (Cornell University) and Herbert Wagner (then at Cornell) proved rigorously that there is no long-range order in a two-dimensional superfluid or vector magnet. Meanwhile Thomas Kaplan (University of Michigan) and Eugene Stanley (Boston University), analyzing high-temperature series expansions, found evidence in a number of two-dimensional systems for a phase transition, which they pointed out could exist even if there were no long-range order.

In 1972, at the University of Birmingham, Kosterlitz and Thouless started wondering what could characterize the low-temperature phase of a two-dimensional system (the x–y model) or a two-dimensional crystal. Kosterlitz described his and Thouless's thinking to us recently: Suppose you draw the two-dimensional lattice on a rubber sheet; it can be stretched in one place and compressed elsewhere. But in a small region, the lattice is unchanged. The topology of the lattice could, however, be changed by dislocations.

Kosterlitz and Thouless knew that in three dimensions, dislocations do not occur in equilibrium. And that in two dimensions, the dislocations have a logarithmic energy variation, and so does the entropy balance. They thought that at low temperature, one could not have isolated dislocations because the free energy varies as −constant T log (size of system). If this free energy is positive, free dislocations are not permitted; but if the free energy is negative, dislocations are present in equilibrium. When a system has many dislocations and one applies a small shear stress, the dislocations will move and the system acts as a viscous liquid.

Then they considered superfluids, in which dislocations correspond to vortices. There one can make a similar argument about the energy–entropy balance. Thus the dissociation of bound vortex–antivortex pairs represents a transition from a superfluid state to one in which free vortices cause viscous dissipation.

At that time, the Kondo problem was in vogue. The Birmingham theorists had read the paper of Philip Anderson (Bell Labs), Gideon Yuval (Cambridge University) and Donald Hamann (Bell Labs), often misnamed "The Poor Man's Renormalization Group." (That was the title of a later paper doing the Kondo problem more simply.) Kosterlitz and Thouless noticed that the paper by Anderson and his colleagues had these log-type energies in it, but it only dealt with one dimension. They thought they could modify it to two dimensions, but Kosterlitz recalls that it took him six months of studying the paper before he realized how to do it. Then "I realized our simple-minded ideas were right," Kosterlitz told us. "There actually are hordes of vortices," whose energies are proportional to the log of the separation of the pairs. "And this complicated problem reduces to a simple one."

In 1973 Kosterlitz and Thouless presented an approximate calculation[1] of the two-dimensional transition. The following year Kosterlitz did a more quantitative renormalization-group calculation.[2]

Binding vortices. In the x–y model, one imagines a system with classical two-component vectors, whose lengths are fixed; thus $S_x^2 + S_y^2 = 1$. The fundamental excitation is a swirly pattern that resembles the flow of water going down a drain. Kosterlitz and Thouless suggested that the phase transition in the planar model was connected to the binding and unbinding of these vortices. A close mathematical analogy exists between two of the vortices with opposite swirliness and two lines of opposite charge.

At high temperature, the swirls are dissociated, and the system resembles a plasma; particles of opposite charge or opposite swirliness can move freely in all directions. At low temperature the system is more like atoms in which the positive and negative charges are bound together or the swirls are bound. Although the x–y system is mathematically very similar to the Kondo problem, the physics is different. In the Kondo problem, one has a continuous behavior. In the x–y model, there is a Kosterlitz–Thouless critical temperature at which the swirls become unbound.

The low-temperature phase of the x–y model had been discussed earlier by Franz Wegner (University of Heidelberg), by V. L. Berezinskii (Landau Institute near Moscow) and more recently by Johannes Zittartz (University of Cologne) using arguments borrowed from conventional spin-wave theory.

Initially, the reaction to the Kosterlitz–Thouless theory was not terribly enthusiastic. Several competing theories of the phase transition in the x–y model had been proposed, which differed in their physics and in their prediction of the critical index, η. This critical index occurs when one takes the product of correlations at large distances:

$$\langle S_x(r) S_x(0) \rangle = 1/r$$

Kosterlitz and Thouless said that at the transition $\eta = 1/4$. Zittartz said $\eta = \infty$. Alan Luther (NORDITA) and Douglas Scalapino (University of California at Santa Barbara) argued that η was in general nonuniversal and in particular found that $\eta = 1/\sqrt{8}$ in a particular model.

In 1975 Jacques Villain (Grenoble) and in 1977, J. V. Jose (Brown University), Leo P. Kadanoff (then at Brown, now en route to the University of Chicago), Scott Kirkpatrick (IBM) and David R. Nelson (Harvard University) showed[3] that there was an exact way of formulating the Kosterlitz–Thouless discussion in terms of vortices and in terms of a Coulomb gas, by using a lattice, Jose *et al* were led to conclude that $\eta = 1/4$ was a universal result, in contrast to the Luther–Scalapino conclusion. Nelson and Kosterlitz then observed[4] that an exact relationship between η and the superfluid density per unit area, ρ_s, in other words,

$$\eta = 2\pi m^2 k_B T / h^2 \rho_s$$