NEWS

search & discovery

Superfluid experiments support Kosterlitz–Thouless theory

Five years ago Michael Kosterlitz and David Thouless predicted that in the x–y or planar model of two-dimensional substances, a new type of phase transition would occur. In a two-dimensional superfluid, for example, at low temperatures the system would be arranged as bound vortex–antivortex pairs whose dissociation leads to the transition. More recently David Nelson and Kosterlitz suggested that the theory could be tested experimentally by looking for a universal jump in the superfluid density at the Kosterlitz–Thouless phase transition.

Recently two independent experiments—at UCLA and Cornell University—have determined the discontinuity in the superfluid density as a system passes through the phase transition. Both experiments are in good agreement with the predictions of the Kosterlitz–Thouless theory.

The excitations considered by theorists working on such phase transitions are highly analogous to the topological excitations now popular with particle theorists. The type of problem represented by bound pairs of vortices may offer clues to how one might produce quark confinement.

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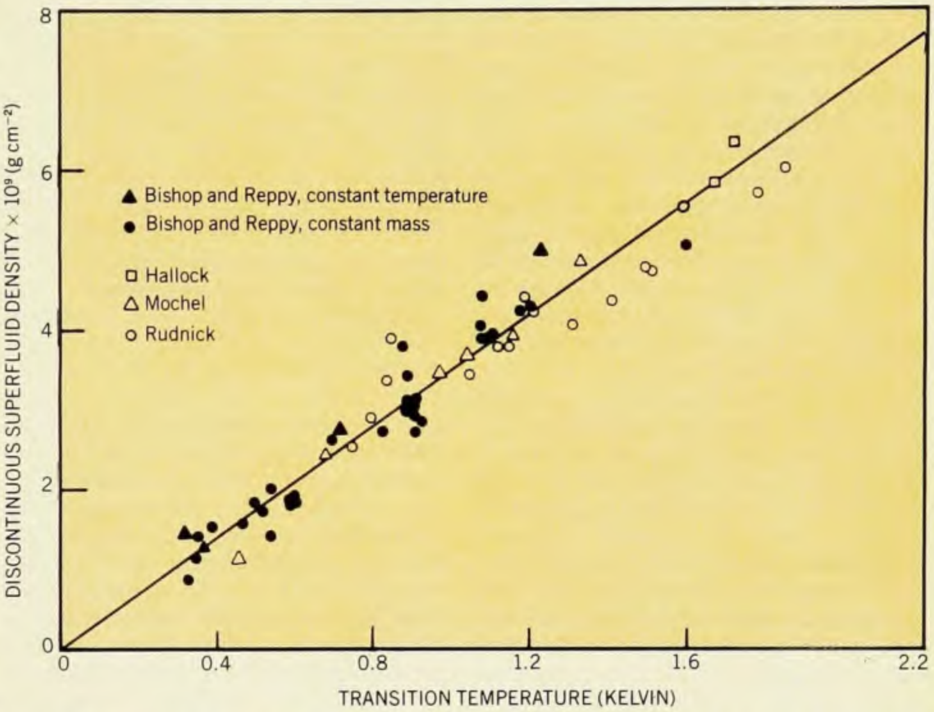

Superfluid transition in two-dimensional He^4 films. Discontinuous superfluid density at onset, ρ_s, is plotted vs. transition temperature. The data by Bishop and Reppy were taken in two ways: constant-mass measurements (using the calculation in reference 6) and constant-temperature measurements. The other data come from third-sound measurements by Rudnick, by Hallock and by Mochel. The line is that predicted by the Kosterlitz–Thouless static theory. (From ref. 8)

New data on two galaxies are consistent with black holes

How is energy supplied in the active centers of radio galaxies and quasars? This intriguing question lies at the heart of various overlapping and competing astrophysical theories, including those that propose the existence of a black hole. Now two separate sets of observations—of an apparent supermassive object[1,2] at the center of galaxy M 87 and of a small narrow radio jet parallel to a much larger jet[3] in NGC 6251—are being welcomed as a chance to test some of these ideas.

The elliptical galaxy M 87, part of the Virgo cluster, was observed by two optical techniques. One set of experiments, done at the Hale Observatories, involves photometry with sensitive TV-type detectors; the other set are spectroscopic measurements at the 4-meter Kitt Peak telescope, done with a different TV image-amplification technique newly applied to astronomy. The combined data point to a supermassive object, possibly a black hole, as a probable explanation for the excess mass-to-light ratio at the center of M 87. In the other experiment, the Caltech radioastronomers who observed the center of NGC 6251 used very long baseline interferometry, with radio dishes at the Owens Valley Radio Observatory (Big Pine, California), the National Radio Astronomy Observatory (Green Bank, West Virginia) and the Haystack Observatory (Westford, Mass). They found an unresolved core and a small (1.7-parsec) jet, pointing in the same direction as a previously discovered 200-kiloparsec jet. Their results appear to support the "beam" theory of energy supply, and are also consistent with the existence of a black hole at the galactic center.

Theorists are interested in M 87 because it is an active radio source with compact central components, is fairly close to us and is known to have a jet emitting polarized optical light. The photometric observations of M 87 are reported by Peter Young, James Westphal, Jerome Kristian, Christopher Wilson (all of the Hale Observatories, Pasadena) and Frederick Landauer (Jet Propulsion Laboratory). They used two types of sensitive two-dimensional imaging devices: commercially available silicon-intensified-target tubes and charge-coupled devices (or "CCD's"). The CCD, developed by Texas Instruments for the Jet Propulsion Laboratory, was designed for the Space Telescope and the Jupiter orbital probe. It is, Kristian

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the rate of growth and the kind of interactions that existed.

The double jet, according to Blandford of the Caltech group, is good morphological evidence for the beam theory of energetic double radio sources. Blandford and Martin Rees (Cambridge) are among the main proponents of this theory. Here the larger jet would represent fluid ejected from the center about 5×10^6 years ago, and the small intense jet is the outline of a region of more recently ejected matter. The "remembered" axis of ejection here is the rotation axis of the massive galactic nucleus—perhaps, but not necessarily, a black hole.

In a possible variation of this theory, suggested to us by George Field (Harvard), a dense disc of gas is rotating rapidly at the galactic center, preferentially ejecting matter along the rotation axis. An explosion within this stratified disc would eject matter in a jet, following ordinary hydrodynamic principles. The apparent constancy of orientation over millions of years appears to rule out theories in which multiple-body interaction ejects matter randomly. Blandford also pointed out that it would be interesting to see whether or not NGC 6251 showed superluminal expansion, as do other quasars and active galaxies. This apparent expansion at a rate greater than c is a kinematical effect, due to the time needed for light to travel across the source. —MSR

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Kosterlitz–Thouless

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The Kosterlitz–Thouless theory is believed to apply to a whole group of other two-dimensional problems—the Heisenberg spin-chain model, the one-dimensional electron gas, two-dimensional crystals, the Baxter model, the Ashkin–Teller model, the interface roughening transition, the massive Luttinger model and the Sine–Gordon equation, all of which have a line of critical points.

In the mid-1960's Pierre Hohenberg (Bell Labs) and separately N. David Mermin (Cornell University) and Herbert Wagner (then at Cornell) proved rigorously that there is no long-range order in a two-dimensional superfluid or vector magnet. Meanwhile Thomas Kaplan (University of Michigan) and Eugene Stanley (Boston University), analyzing high-temperature series expansions, found evidence in a number of two-dimensional systems for a phase transition, which they pointed out could exist even if there were no long-range order.

In 1972, at the University of Birmingham, Kosterlitz and Thouless started wondering what could characterize the low-temperature phase of a two-dimensional system (the x–y model) or a two-dimensional crystal. Kosterlitz described his and Thouless's thinking to us recently: Suppose you draw the two-dimensional lattice on a rubber sheet; it can be stretched in one place and compressed elsewhere. But in a small region, the lattice is unchanged. The topology of the lattice could, however, be changed by dislocations.

Kosterlitz and Thouless knew that in three dimensions, dislocations do not occur in equilibrium. And that in two dimensions, the dislocations have a logarithmic energy variation, and so does the entropy balance. They thought that at low temperature, one could not have isolated dislocations because the free energy varies as −constant T log (size of system). If this free energy is positive, free dislocations are not permitted; but if the free energy is negative, dislocations are present in equilibrium. When a system has many dislocations and one applies a small shear stress, the dislocations will move and the system acts as a viscous liquid.

Then they considered superfluids, in which dislocations correspond to vortices. There one can make a similar argument about the energy–entropy balance. Thus the dissociation of bound vortex–antivortex pairs represents a transition from a superfluid state to one in which free vortices cause viscous dissipation.

At that time, the Kondo problem was in vogue. The Birmingham theorists had read the paper of Philip Anderson (Bell Labs), Gideon Yuval (Cambridge University) and Donald Hamann (Bell Labs), often misnamed "The Poor Man's Renormalization Group." (That was the title of a later paper doing the Kondo problem more simply.) Kosterlitz and Thouless noticed that the paper by Anderson and his colleagues had these log-type energies in it, but it only dealt with one dimension. They thought they could modify it to two dimensions, but Kosterlitz recalls that it took him six months of studying the paper before he realized how to do it. Then "I realized our simple-minded ideas were right," Kosterlitz told us. "There actually are hordes of vortices," whose energies are proportional to the log of the separation of the pairs. "And this complicated problem reduces to a simple one."

In 1973 Kosterlitz and Thouless presented an approximate calculation[1] of the two-dimensional transition. The following year Kosterlitz did a more quantitative renormalization-group calculation.[2]

Binding vortices. In the x–y model, one imagines a system with classical two-component vectors, whose lengths are fixed; thus $S_x^2 + S_y^2 = 1$. The fundamental excitation is a swirly pattern that resembles the flow of water going down a drain. Kosterlitz and Thouless suggested that the phase transition in the planar model was connected to the binding and unbinding of these vortices. A close mathematical analogy exists between two of the vortices with opposite swirliness and two lines of opposite charge.

At high temperature, the swirls are dissociated, and the system resembles a plasma; particles of opposite charge or opposite swirliness can move freely in all directions. At low temperature the system is more like atoms in which the positive and negative charges are bound together or the swirls are bound. Although the x–y system is mathematically very similar to the Kondo problem, the physics is different. In the Kondo problem, one has a continuous behavior. In the x–y model, there is a Kosterlitz–Thouless critical temperature at which the swirls become unbound.

The low-temperature phase of the x–y model had been discussed earlier by Franz Wegner (University of Heidelberg), by V. L. Berezinskii (Landau Institute near Moscow) and more recently by Johannes Zittartz (University of Cologne) using arguments borrowed from conventional spin-wave theory.

Initially, the reaction to the Kosterlitz–Thouless theory was not terribly enthusiastic. Several competing theories of the phase transition in the x–y model had been proposed, which differed in their physics and in their prediction of the critical index, η. This critical index occurs when one takes the product of correlations at large distances:

$$\langle S_x(r) S_x(0) \rangle = 1/r$$

Kosterlitz and Thouless said that at the transition $\eta = 1/4$. Zittartz said $\eta = \infty$. Alan Luther (NORDITA) and Douglas Scalapino (University of California at Santa Barbara) argued that η was in general nonuniversal and in particular found that $\eta = 1/\sqrt{8}$ in a particular model.

In 1975 Jacques Villain (Grenoble) and in 1977, J. V. Jose (Brown University), Leo P. Kadanoff (then at Brown, now en route to the University of Chicago), Scott Kirkpatrick (IBM) and David R. Nelson (Harvard University) showed[3] that there was an exact way of formulating the Kosterlitz–Thouless discussion in terms of vortices and in terms of a Coulomb gas, by using a lattice, Jose *et al* were led to conclude that $\eta = 1/4$ was a universal result, in contrast to the Luther–Scalapino conclusion. Nelson and Kosterlitz then observed[4] that an exact relationship between η and the superfluid density per unit area, ρ_s, in other words,

$$\eta = 2\pi m^2 k_B T / h^2 \rho_s$$

would allow this controversy to be resolved experimentally. If η were indeed $^3/_4$ at T_c, this would mean that the jump in superfluid density divided by the critical temperature was calculable in terms of fundamental constants—$8\pi k_B(m/h)^2$ where m is the mass of the He^4 atom, h is Planck's constant and k_B is Boltzmann's constant.

Kosterlitz and Thouless found a continuous specific heat at the transition. In particular, there is no latent heat. So the transition is not of first order, despite the jump in superfluid density.

The Kosterlitz–Thouless theory is a static calculation; that is, it applies only for zero frequency and zero wave vector. But experiments are usually done at finite frequency. The extension of the theory to nonzero superfluid velocity was done by two groups: by Bernardo Huberman (Xerox Palo Alto Research Center), Robert J. Myerson (Institute for Advanced Study) and Sebastian Doniach (Stanford University)[5] and by Vinay Ambegaokar (Cornell), Bertrand Halperin (Harvard), Nelson and Eric Siggia (Cornell).[6] The latter group extended the theory to finite frequency.

Experiments. Measurements of third-sound propagation in He^4 films were done by Isadore Rudnick (UCLA) and his collaborators, in 1969 and subsequently. At the time, it was expected that ρ_s would vanish when the film was sufficiently thin. Instead, they found the superfluid density was large and finite at the transition. Unfortunately, the value of the van der Waals constant in use at that time was incorrect, and the film thickness used by Rudnick and his collaborators was accordingly $^1/_3$ higher than it was actually.

Recently, Rudnick reanalyzed[7] his third-sound data, and found that $\rho_s/T_c = (3.30 \pm 0.21) \times 10^{-9}\,\mathrm{g\,cm^{-2}}T^{-1}$ where T_c is critical temperature; the value obtained by Nelson and Kosterlitz was 3.49×10^{-9}.

David J. Bishop and John D. Reppy (Cornell University) have measured[8] the superfluid transition in two-dimensional He^4 films by essentially shaking the fluid, using the Andronikashvili torsional-oscillator technique. A helium film is adsorbed on a Mylar film substrate. A strip of the film is wound like a jelly roll and oscillated at the resonant frequency. When the helium film of the desired thickness (one or two monolayers) has been deposited, the Cornell team measures the period and Q to determine the temperature dependence of the superfluid mass and dissipation. They found a value for ρ_s/T that is in good agreement with the Nelson and Kosterlitz value. The experiment had an accuracy of ±13%.

The results of the Cornell and UCLA groups plus a reanalysis of third-sound measurements by Robert Hallock (University of Massachusetts) and Jack Mochel (University of Illinois) are shown in the figure. The agreement with the Kosterlitz–Thouless static theory (after correction for finite-frequency effects) is very good. Kosterlitz notes that the evidence for $\eta = {}^1/_4$ is much stronger than that for competing theories.

Scalapino believes the Kosterlitz–Thouless picture is probably the appropriate one. He explained why: He and Luther went from a discrete lattice to a continuous lattice and approximated a rigid rotor by keeping only the three lowest states; these approximations may have produced differences. The Kosterlitz–Thouless approach fits nicely with the renormalization idea and contains a physical picture; the theory of Luther and Scalapino is more mathematical.

Two-dimensional films of certain liquid crystals may also provide an experimental test of the Kosterlitz–Thouless theory. This year C. Y. Young, Ronald Pindak, N. A. Clark and Robert B. Meyer (all then at Harvard), studying the stiffness constants of smectic-C liquid crystals, suggested that the transition to the smectic-A phase may be of the Kosterlitz–Thouless type. In 1975 Huberman, Douglas Lublin (Stanford) and Doniach had shown that the Kosterlitz–Thouless theory could apply to the phase transition of smectic liquid crystals.

Related theoretical activity. Recently Siu Tat Chui (State University of New York at Albany) and John D. Weeks (Bell Labs) have applied[9] the Kosterlitz–Thouless and Jose *et al* method to study the dynamics of the surface roughening transition, which occurs in the crystal-growth problem. Chui and Weeks believe their method can also be applied to two-dimensional nematic crystals and to the motion of solitons in some one-dimensional crystals. A similar analysis has been applied to the melting of a two-dimensional crystal by Kosterlitz and Thouless, and this year by Halperin, Nelson and Peter Young (Cornell). Earlier, Jose *et al* had argued that one might see planar-model behavior in magnets with an underlying hexagonal symmetry.

The relationship among many different problems has been emphasized by Luther and by Sidney Coleman (Harvard). And Kadanoff argued, as he says, "persuasively but not convincingly" that the solution to the Kosterlitz–Thouless phase transition also gave solutions to the F model, the four-state Potts model, and that both are connected to the Baxter model—that these models all end at a critical end point.

The connection with particle physics is also attracting attention. Instantons, now a popular subject with particle theorists, are topological excitations that describe how the internal symmetry and the symmetry of space fit together. Kadanoff told us that the Kosterlitz–Thouless phase transition is the first example of a tractable problem in which topological excitations play an essential role. But Anderson said that his paper with Yuval and Hamann on the Kondo problem and one-dimensional Ising model is in fact the first.

In the paper by Jose, Kadanoff, Kirkpatrick and Nelson, the group showed how these topological excitations could be expressed as an Abelian U(1) gauge theory—a gauge theory with fewer dimensions than those popular in particle physics, but with some analogies. Earlier, A. A. Migdal (Landau Institute) had said that four-dimensional gauge theories are analogous to two-dimensional phase transitions.

There also appear to be analogies between the Kosterlitz–Thouless picture and various phenomena in three-dimensional gauge theories. For example, Curtis Callan (Princeton), Roger Dashen (Institute for Advanced Study) and David Gross (Princeton) have argued that the dynamical mechanisms in quantum chromodynamics responsible for quark mass generation and confinement bear many similarities to the Kosterlitz–Thouless phase transition. In their picture, the objects that are analogous to vortices in two dimensions are four-dimensional instantons and the so-called "merons." —GBL

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in brief

TRW has been awarded a $2.64-million contract by DOE to design systems for generating electricity using ocean temperature gradients. It will prepare conceptual and preliminary designs of a commercial-scale ocean thermal-energy conversion power system and of a smaller scaled-down pilot plant. Following completion of designs in September 1978 by TRW and by Westinghouse and Lockheed (who also have signed similar contracts with DOE), DOE will select one or more for actual fabrication of a 5000–12 000-kW ocean-based pilot plant. □