

letters

tunity of redirecting the emphasis that reviewers of *The Jupiter Effect* have placed on the planetary-alignment hypothesis. The essence of our study is that more seismic activity is likely at times of maximum solar activity; the planetary-influence model is the only one we know of that makes firm predictions about when the next solar maximum will be, and we therefore included one chapter about this for completeness. Even should that model prove incorrect, however, there remains good evidence that the years around the next solar maximum will produce enhanced seismic activity. Since just adding 11 years to the previous maximum suggests that the next peak is due in 1979, the planetary hypothesis might be seen as unduly optimistic, suggesting as it does a further three years of grace. Finally, I fully endorse Bolt's warning that "buildings should be constructed to resist great earthquakes on the assumption that they will occur tomorrow." The presentation of our ideas in the form of *The Jupiter Effect* was exactly to encourage such an attitude among those who have been less aware of the need for caution; I hope that your readers, who clearly do not need to be warned in such dramatic terms, will make appropriate allowance for that presentation and may be interested in the original exposition of these ideas,¹ which drew remarkably little response.

Reference

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Flywheel energy storage

Due to the increased interest in storing energy with flywheels, the reader is reminded of the virial theorem, which predicts the maximum energy that can be stored in the structural mass of a flywheel. This is a "measuring stick" that can be applied to any flywheel design; in general, the theorem predicts the best use one can make of a given material.

Originally derived by Clausius in 1870,¹ the virial theorem applies to gases, plasmas and all mechanical devices including flywheels. A similar theorem applies to electromagnetic systems including superconductive energy storage magnets.^{2,3,4,5}

The virial theorem relates the time-average kinetic energy of a system of particles to the time-average forces acting on those particles, and a little manipulation from the standard form of the theorem yields, for rotating bodies, the result

$$M_s \geq 2\rho_s T/\sigma \quad (\text{uniaxial stress})$$

$$M_s \geq \rho_s T/\sigma \quad (\text{biaxial stress})$$

where M_s is the structural mass and the volume of the load-carrying material is M_s/ρ_s . The equalities in the equation can be approached in certain specialized cases. A thin spinning cylinder has minimum mass for uniaxially stressed structures, while a disc that is thick near its center⁶ can approach the biaxial limit.

Anyone considering the economic feasibility of flywheel energy storage should measure their design against this theorem. Depending on the structural materials used, it places an absolute minimum on the cost per unit of stored energy.⁵ In MKS units, that is

$$\text{cost/joule} \geq (\text{cost/kg}) 2\rho_s/\sigma \quad (\text{uniaxial stress})$$

$$\text{cost/joule} \geq (\text{cost/kg}) \rho_s/\sigma \quad (\text{biaxial stress})$$

These relationships have long been known^{6,7,8} but seemingly have not been used to pinpoint correctly a major obstacle preventing the use of flywheels. This is, of course, the basic materials cost of flywheels. For example, steel at $6.9 \times 10^8 \text{ N/m}^2$ (100 000 psi) should cost less than 31¢/kg fabricated to be used in a storage unit with a ten-hour discharge time for the electric utility industry. Lighter weight epoxy materials at 100 000 psi should cost less than 63¢/kg fabricated for ten-hour use. In addition, there are the costs of the bearings, clutches, motor-generators and containment housings. The basic materials costs above are independent of flywheel design details. Regardless of other costs, the one derived from the virial theorem may make flywheels prohibitively expensive for applications such as diurnal energy storage with ten hours of energy discharge. Flywheels could become more competitive with the development of new strong inexpensive materials.

References

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