Why the sky is dark at night

In a universe uniformly filled with stars we would expect the sky to be ablaze with light from all directions, according to a 250-year-old paradox we are just beginning to understand.

Edward R. Harrison

Let us imagine that stars similar to the Sun are uniformly distributed in an infinite and static universe. Edmund Halley[1] in 1720, J. P. Loys de Chéseaux[2] in 1744, and H. Wilhelm M. Olbers[3] in 1823 showed this seemingly reasonable 16th and 17th century model of the universe leads to a remarkable paradox. For whatever direction we look in the sky our line of sight eventually intercepts a star, and the whole sky should therefore be ablaze with light as bright as the Sun. This startling disagreement between theory and observation is nowadays referred to as "Olbers's paradox."

This problem, "Why is the sky dark at night?" has a fascinating history[4-6] and to this day remains a subject of absorbing interest in cosmology. Hermann Bondi[7] remarks that it represents the "first discovery of a link connecting us to distant regions of the universe" and the arguments that disclose the paradox mark the birth of scientific cosmology. Unfortunately, we shall probably never know who originally discovered the paradox. In the first recorded discussion[1] Halley acknowledged the discovery with the words: "I have heard urged . . ." In the interests of historical accuracy and in token honor of the unknown founder of scientific cosmology it would be better if we all agreed to refer to the subject as "the bright-sky paradox."

Various solutions of the paradox have been proposed. Chéseaux[2] and Olbers[3] attributed the darkness of the night sky to the absorption of light while traversing space. Other proposals[6]—that space is non-Euclidean, that the universe is finite in size, that it has hierarchical structure, is young, or is expanding—all attempt to resolve the paradox by modifying the basic assumptions of the original model. It is currently thought[7-14] that a sufficient condition for resolving the paradox is the existence of the extragalactic redshift. The radiation reaching us from distant receding sources is redshifted, and the sky is dark at night, according to the usual explanation, because of this redshift effect.

In my opinion the redshift effect in fact is not important, and *the night sky is dark because the time required for the radiation field to reach thermodynamic equilibrium is large compared with all other time scales of interest.* With our twentieth-century knowledge of physics it is possible to show, as I shall do here, that quite simple arguments are capable of resolving the bright-sky paradox.

Why is the sky dark at night?

Halley, de Chéseaux and Olbers did not perform detailed calculations to establish the paradox; indeed, the conclusion is so obvious that calculation seems superfluous. When standing in a forest, for instance, our view in any horizontal direction is eventually blocked by tree trunks; and if all tree trunks were white we would be surrounded by a wall of uniform whiteness. Although the result is obvious we shall nonetheless perform the calculation because it provides the main clue to solving the 250-year-old problem of why the sky is dark at night.

Wherever possible we conform to the original model and assume that stars are distributed uniformly, with a density n per unit volume, and have an average luminosity L (that is, each radiates on the average L ergs per second). About an arbitrary point O in space a spherical shell of radius r and thickness dr contains $4\pi nr^2dr$ stars, and has a luminosity $4\pi nLr^2dr$. On dividing the luminosity of the shell by $4\pi cr^2$ we obtain the contribution

$$du = (nL/c)dr$$

to the radiation energy density u at O. When integrated, this equation gives $u = \infty$ in an infinite universe. Stars, however, are not point sources (of infinite surface temperature), and consequently they intercept radiation from more distant stars as it travels toward O. Taking into account this absorption, we find that the contribution from any shell becomes

$$du = (nL/c)e^{-r/\lambda}dr \qquad (1)$$

where λ is the photon mean free path

$$\lambda = 1/n\sigma$$

and σ is the cross section of a star of surface area 4σ. The total radiation density from all shells in an infinite universe is now

$$u = (nL/c)\int_0^\infty e^{-r/\lambda}dr = nL\lambda/c$$

or, simply, $u = L/\sigma c$. But the luminosity of a star is $L = u^*\sigma c$, where u^* is the surface radiation density, and it follows that

$$u = u^* \qquad (2)$$

The origin O is chosen arbitrarily, and the radiation density at any point in space is therefore equal to the radia-

Edward R. Harrison is a professor in the department of physics and astronomy at the University of Massachusetts in Amherst and a member of the Five College Astronomy Department.

tion density at the surface of stars. This is the bright-sky paradox, and the problem is to discover what is wrong with this apparently logical calculation.

The absorption of radiation by matter distributed in space, proposed as a resolution by de Chéseaux and Olbers, fails to avert the radiation catastrophe. As Bondi showed[8], the absorbing matter heats up and then re-radiates incident radiation. The gathering together of stars into galactic systems, where they are partly obscured from view by gas and dust, also fails to resolve the paradox for the same reason.

Resolution of the Paradox

The conclusion that the whole sky should be as bright as the Sun is based on pre-twentieth-century physics. The laws of nature as understood by Halley, de Chéseaux and Olbers were quite incapable of resolving the paradox. To discuss the paradox we need the following three time scales:

- Either the age t of the universe, or the expansion time T (the inverse Hubble constant) which is approximately 10^{10} years. In most big-bang models $t \approx T$, but in static and steady-state models the age ($t = \infty$) is not a useful time scale.
- The luminous lifetime t^* of the stars. Most light at present (at least in our own Galaxy) comes from stars brighter than the Sun[15] that have a main-sequence lifetime of order 10^8 years. Each unit of mass may be cycled many times through stars and could therefore have a much longer luminous lifetime. If all matter releases energy at an average rate $\epsilon = 10$ ergs $gm^{-1}\ sec^{-1}$ (in the Sun, $\epsilon = 2$ erg $gm^{-1}\ sec^{-1}$) and hydrogen is converted

eventually into iron, then $t^* \approx 10^{-2} c^2/\epsilon \rightarrow 10^{10}$ years. The luminous lifetime t^* is an imprecise time scale, partly because in our model it is assumed that the average luminosity L is constant. A value of 10^{10} years for t^*, however, is not grossly unrealistic.

▶ The time $\tau = \lambda/c$ between emission and absorption of photons by luminous sources. If the average density of luminous matter in the universe is ρ, and consists of stars of n per unit volume, of mass M, luminosity L and surface radiation density u^*, then alternative expressions for this time scale are

$$\tau = \frac{M}{\rho\sigma c} = \frac{u^* M}{\rho L} = \frac{u^*}{nL} \qquad (3)$$

and for $\rho \approx 10^{-30}$ gm cm^{-3} it follows that for stars $\tau \approx 10^{24}$ years. We will show below that $\tau \approx 10^{24}$ years is the *thermodynamic* time scale of a static universe.

According to the bright-sky theory, most of the starlight comes from remote regions at distances of order $\lambda = c\tau \approx 10^{24}$ light years. Therefore, implicit in the theory are two very important assumptions: firstly, that the age t of the universe is at least as great as τ (because of the finite speed of light), and, secondly, that the stars have been luminous for a time t^*, which is also at least as great as τ. The first assumption $t \geq \tau$ is automatically satisfied in a static model (but not usually in a big-bang model); but the second assumption, $t^* \geq \tau$ is invalid. The bright-sky theory is overthrown by the simple fact that the luminous lifetime is very short when compared with 10^{24} years.

Another way of showing that radiative equilibrium (i.e. $u = u^*$) is quite impossible is to imagine that all matter in the universe is suddenly converted entirely into blackbody radiation. The energy density ρc^2 then corresponds to a radiation field of temperature 20 K—very much less than the surface temperature of a typical star. Hence, we see that the bright-sky theory violates conservation of energy.

Let us assume that all stars become luminous at an initial epoch labelled $t = 0$. An observer notices that he is surrounded by a sphere of luminous stars of radius ct and this sphere expands at the speed of light. By integrating equation 1 out to a distance ct we obtain

$$u = u^*(1 - e^{-t/\tau}) \qquad (4)$$

for the radiation density. This equation shows that τ is the thermodynamic time scale of a static universe and is the characteristic time required to fill the universe with radiation up to the density at which it is emitted. Because τ is very much larger than any period of time in which we are interested, this last equation is simply

$$u = u^* \frac{t}{\tau} \qquad (5)$$

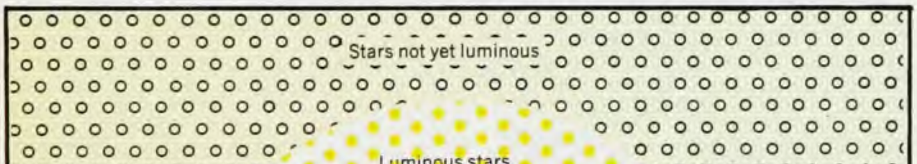
At about the time $t = t^*$ the stars in the neighborhood of the observer begin to die out, and thereafter the observer is surrounded by an expanding sphere of burnt-out stars beyond which lies a shell of luminous stars of thickness ct^*, as shown in figures 1 and 2a. The radiation from this shell is now

$$u = u^* \frac{t^*}{\tau} \qquad (6)$$

as long as $t \ll \tau$. This is the maximum possible radiation density, and therefore *in the static model the night sky is permanently dark because the luminous lifetime t^* is short compared with the thermodynamic time τ.*

It is interesting to note that by distributing stars on an observer's backward lightcone, as we show in figure 2b, it is possible to achieve the bright-sky condition and also satisfy the conservation-of-energy principle. This drastic modification, however, sacrifices homogeneity and requires that an observer possesses special location in space and time.[16]

Our conclusion is that the bright-sky paradox is resolved, within the context of the model in which it was discovered, by realizing that the thermodynamic time scale is extremely large. To resolve the paradox it is not necessary to resort to the fact that the universe is expanding.[17] Indeed, we can say even more: In all models in which the radiation density is less than that given by equation 6, because of expansion and absorption, we have shown that *the sufficient condition for a dark night sky is that the luminous lifetime t^* is less than the time τ required in a static model to attain a bright sky.*

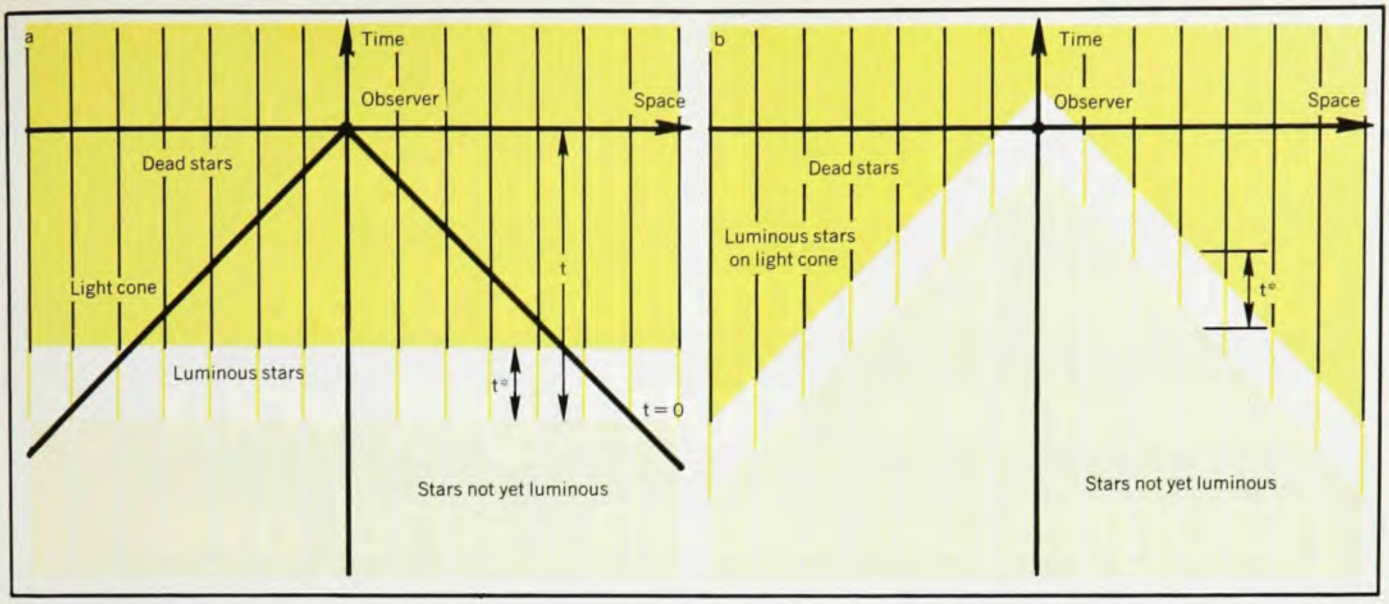


Stars in a dynamically static universe. Assume that the stars commence shining everywhere at time $t = 0$; the observer then sees a luminous sphere of stars expanding radially at the speed of light. After a period of time lasting t^* the stars in the observer's neighborhood begin to die out. The observer is then surrounded by an expanding sphere of dead and guttering stars (black in this figure), beyond which lies a shell of luminous stars (colored) of thickness ct^*. Outside this shell lie stars that have not yet become luminous (shown here as open circles on light color background). Figure 1

Cosmology in a nutshell

If the night sky is dark because the universe is expanding, as many have supposed, then it is natural to wonder how fast the expansion has to be to resolve the paradox. It would also be of interest to know what happens in a collapsing universe when the extragalactic redshift changes over into a blueshift.

In a homogeneous and isotropic universe the radiation field could be evaluated by a time-retarded integration of the redshifted stellar contributions. In the integration we would have to take account of evolutionary and absorption effects and the increased density of sources in the past; also we would have to take care not to fall into the trap of



Space-time diagrams. Part **a** shows a static homogeneous universe; as in figure 1, the stars commence shining everywhere at the same instant $t = 0$. After a period of time t^* the observer's backward lightcone is intersected by a space section of thickness t^* that contains luminous stars (colored). The observer is then surrounded by dead stars out to a distance $c(t - t^*)$, followed by a luminous shell of thickness ct^*. Part **b** shows the bright-sky universe. This is an inhomogeneous model in which all stars are luminous on the observer's lightcone. The sky is now ablaze with light, as in the bright-sky theory. Figure 2

integrating without regard to the finite energy supply of the sources. In 1964 while considering this problem I realized that there is a very much simpler way of looking at the whole subject.[17]

We start by supposing that a relatively small region of the universe, of comoving volume V and containing average conditions, is enclosed by imaginary walls that are perfectly reflecting on both the inside and outside (see figure 3). An observer outside V notices that the radiation field contains contributions from redshifted sources and sets up, in the usual way, an integral equation. He is particularly careful about the possibility of photons circumnavigating many times around a closed model, and he is perhaps surprised to find that the curvature of space plays no direct role in his equation. An observer inside V, however, notices that the radiation field contains only contributions from neighboring sources that are redshifted because of the continual reflection of light off the expanding walls. He therefore uses the differential equations of classical thermodynamics, and because V is relatively small he is not surprised that these equations are independent of the overall curvature of space. As the reflecting walls cannot in any way disturb the radiation field, conditions inside and outside V at any instant are identical. It follows that the integral- and differential-equation methods are equivalent ways of treating the same problem.

As before, u is the energy density of isotropic radiation, and in volume V

$$d(uV) + pdV = dQ$$

where $p = u/3$ is radiation pressure and dQ is the increase in radiation energy as a result of a uniform distribution of sources and sinks. The gain from sources (assuming they are stars only) in time dt is

$$nVLdt = u^* \frac{V}{\tau} dt$$

where u^* is the surface radiation density and τ is given by equation 3. The loss in time dt due to interception of radiation by sources is $u(V/\tau)dt$. By assembling together these results we obtain a differential equation

$$\frac{d}{dt}(uV^{4/3}) = \frac{V^{4/3}}{\tau}(u^* - u) \qquad (7)$$

(A more general equation can be constructed for arbitrary emission and absorption in each frequency interval.[18]) As a simple check, let us solve this equation for the static model; in this case V is constant and the solution is

$$u = u^*(1 - e^{-t/\tau}) + u_0 e^{-t/\tau}$$

where $u = u_0$ at time $t = 0$ when the sources become luminous. This result is the same as equation 4 obtained by the integral-equation method with one important exception: When integrating the differential equation (7) we do not overlook the possibility of an initial radiation field.

Expanding universe

Let us consider briefly radiation in an expanding universe and for illustrative purposes assume that expansion obeys the simple law $V \propto t^{3\alpha}$, where t is the age of the universe and α is a constant. The deceleration parameter[19] (see figure 3) is $q = \alpha^{-1} - 1$, and in general our universe can be represented by a model for which α is of order unity. For example, in Dirac's model $\alpha = 1/3$; in the Einstein–de Sitter model[20], $\alpha = 2/3$, and in Milne's model $\alpha = 1$.

Now let us assume that stars of constant luminosity commence radiating at time t_0, when the radiation density is u_0, and solve equation 7 for epochs $t \gg t_0$. Bearing in mind that $n \propto V^{-1}$, we find

$$u = u_0\left(\frac{t_0}{t}\right)^{4\alpha} + \frac{u^*}{1 + \alpha}\frac{t}{\tau}$$

on neglecting the small absorption loss. The primordial contribution u_0 is at present small (from the bright-sky point of view) and therefore

$$u = \frac{u^*}{1 + \alpha}\frac{t}{\tau} \qquad (8)$$

At the end of the luminous phase we have

$$u = \frac{u^*}{1 + \alpha}\frac{t^*}{\tau} \qquad (9)$$

and thereafter u diminishes as $V^{-4/3}$.

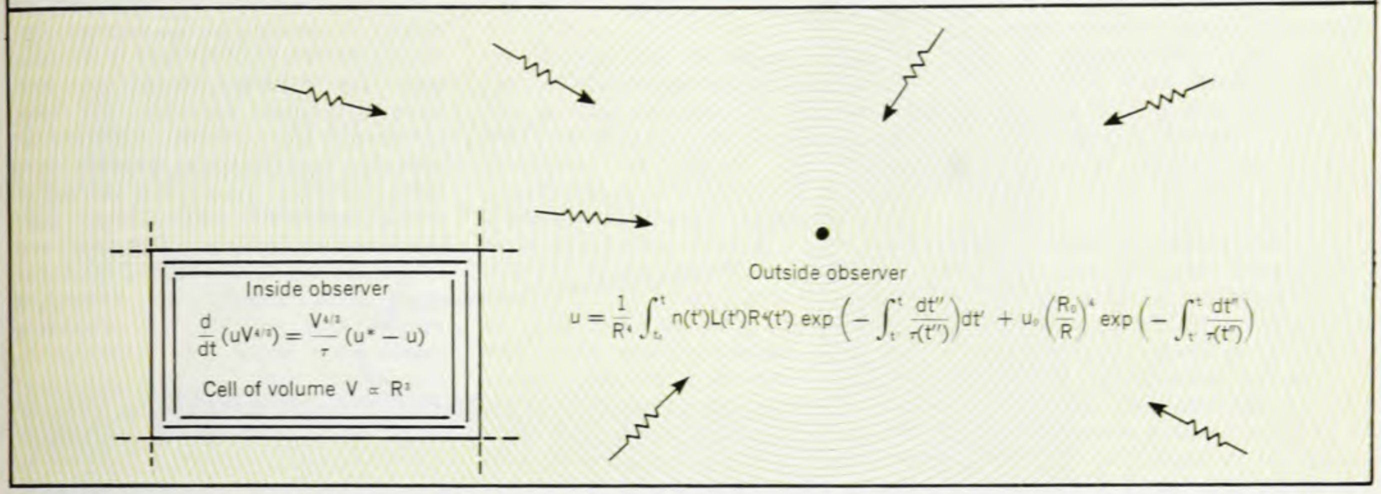
Several important remarks can be made concerning the last two equations.

We notice that the radiation density is less than in the corresponding equations of the static model (equations 5 and 6) by a factor $(1 + \alpha)^{-1}$. And,[19] since $\alpha \approx 1$, the radiation densities in static and expanding models are of the same order of magnitude. The night sky in an expanding universe is therefore dark for the same reason that it is dark in a static universe of the same age, and expansion plays a relatively unimportant role.

$$\text{Average redshift: } \langle z \rangle = R\left(\int_{t_0}^{t} n(t')R^3(t')\,dt' \Big/ \int_{t_0}^{t} n(t')R^4(t')\,dt'\right) - 1,$$

$$= \begin{cases} \alpha, \text{ in big-bang model when } R \propto t^\alpha \\ 1/3, \text{ in steady-state model where } R \propto e^{t/T}, \\ \quad t_0 = -\infty, \text{ and } n \text{ is constant} \end{cases}$$

$$\text{When } R \propto t^\alpha\text{: } T = R/\dot{R} = t/\alpha;\ q = \frac{\ddot{R}R}{\dot{R}^2} = \frac{1}{\alpha} - 1$$



The integral- and differential-equation methods of evaluating the radiation field. The "outside observer" integrates the redshifted contributions from all expanding shells of luminous sources. In this article we assume that the sources are stars of density n per unit volume and average luminosity L, and τ is then the time taken for a single star to fill a volume n^{-1} with radiation up to the density u^* at which it is emitted. The "inside observer" imagines that he is surrounded by perfectly reflecting walls, which expand with the universe. He thus occupies a cell of volume V, proportional to R^3 where $R(t)$ scales all distances in a homogeneous and isotropic universe. The inside observer sets up a differential equation based on very simple thermodynamic principles. The volume V, although small on the cosmic scale, is large enough to contain average conditions, and therefore the radiation field inside the cell is identical in all respects with the radiation field outside. The two methods thus give identical results. Figure 3

If we compare static and expanding models at an instant when their compositions are in identical states, they will both have the same number of photons per unit volume but the photons in the expanding model will have on the average $(1 + \alpha)^{-1}$ times as much energy. The average redshift $\langle z \rangle$ is therefore given by $1 + \langle z \rangle = 1 + \alpha$, and hence photons in an expanding universe have an average redshift $\langle z \rangle = \alpha$. (See also figure 3.) If the extragalactic redshift were indeed the main reason for the darkness of the night sky we would require an average redshift of many orders of magnitude greater than the value of α. Equation 8 also holds for a collapsing universe of $0 > \alpha > -1$, and in this case $-\alpha$ is the average blueshift.

It has been said[8] that the sky is dark in a young universe. This, however, is not always true. Because t/τ in equation 8 is proportional to $t^{1-3\alpha}$, the radiation density increases with time when $\alpha < 1/3$, is constant when $\alpha = 1/3$ and decreases when $\alpha > 1/3$. In those models with $\alpha > 1/3$ (for example, the Einstein–de Sitter model) the radiation density rises as we go back in time, and, according to our idealized assumptions, it is possible at an early epoch (provided t is still greater than t_0) to have a bright sky of $u \approx u^*$.

Many have said that a dark night sky implies a universe of finite size. But the thermodynamic time scale τ is the period of time taken by an average source to fill, in effect, a surrounding region of volume n^{-1} with radiation of intensity equal to that of the source. Therefore τ, and hence the darkness of the night sky, is quite independent of whether a homogeneous universe happens to be finite in size or not.

Equation 8 serves the purpose of demonstrating why the night sky is necessarily dark in an expanding universe. It is, however, only an approximate equation. In most applications a more precise treatment is required, taking into account the important contribution of the primordial fireball, the aggregration of stars in galaxies[21], evolutionary effects (if total luminosity in V varies as t^β, change α to $\alpha + \beta$ in equation 8), and the detailed processes of emission and absorption in each frequency interval of the electromagnetic spectrum.

Steady-state models

The bright-sky paradox has played a prominent role in steady-state theories. This is particularly true of the steady-state idea proposed by William D. MacMillan[22] in 1918. In this theory the universe is static and of infinite age; stars form, evolve, and in the process radiate away their mass. Out in space radiation is slowly converted back into matter, which eventually condenses into new stars, and the cycle is repeated. MacMillan writes that the darkness of the night sky suggested to him the novel process in which atoms are "generated in the depths of space through the agency of radiant energy," thus making possible a steady-state universe. If t^* is the luminous period in the matter→radiation→matter cycle, the condition that the night sky is dark is simply $t^* \ll \tau$, as in the static model.

In the expanding steady-state model of Bondi and Thomas Gold[23] and Fred Hoyle[24], matter is continuously created and maintains a constant density because of the expansion of the universe. Radiation is continuously emitted by stars and also maintains a constant density owing to the expansion. For our discussion the problem is—What are the conditions for a dark night sky? Because of the subtleties of the problem certain misconceptions have arisen.

The steady-state solution of equation 7 is

$$u = \int_{-\infty}^{0} nLe^{4t/T}\,dt = u^*\frac{T}{4\tau} \qquad (10)$$

(neglecting the small absorption effect) where n, L and τ are constant, $T \approx 10^{10}$ years is the expansion (or Hubble) time, and $V \propto \exp(3t/T)$.

It is said[23] that when "a photon is finally absorbed, then it will, as seen by an observer on the absorbing matter, be subject to a Doppler-shift which reduces its frequency by a large factor." We can show (see figure 3), however, that the average redshift of all photons is $\langle z \rangle = 1/3$ and is not the large factor that is necessary if redshift is the cause of a dark night sky.

Expansion of the universe maintains a steady-state radiation field and, according to Bondi and Gold,[23] "it is clear the universe provides in this way a sink for radiative energy, and this sink is in fact available to the majority of photons produced at the surface of stars." This is true, but we must remember that expansion merely ensures a steady-state condition and does not by itself guarantee a radiation field of low intensity. In fact, equation 10 as it stands is misleading because it does not show how the radiation density depends on the luminous time t^*. We can imagine a steady-state model (if it is possible) in which the expansion is so slow that it is barely distinguishable from a static model, or a model in which stars have an extremely short lifetime, and in neither case is expansion an important consideration concerning the brightness of the night sky. Something is therefore wrong with equation 10.

In the static model we imagined all stars shining simultaneously for a time t^*. Let us suppose that only a fraction f are in fact luminous at any instant; the overall luminous lifetime is then increased to t^*/f, but the thermodynamic time scale is also increased to τ/f, and the ratio t^*/τ in equation 6 remains unchanged. Hence, switching stars on sequentially does not alter the eventual intensity of the background radiation. Let us suppose for illustrative purposes that each star in the steady-state model, as in the previous models, shines for a period lasting t^*. The fraction of stars that are luminous at any instant in the steady-state model is

$$f = \int_{-t^*}^{0} e^{3t/T} dt \int_{-\infty}^{0} e^{3t/T} dt = 1 - e^{-3t^*/T}$$

Therefore, to compare this model with the static model we must use nf instead of n in equation 10 for the density of luminous stars. We now obtain

$$u = u^* \frac{T}{4\tau}(1 - e^{-3t^*/T}) \qquad (11)$$

whereupon, when t^*/T is small,

$$u = \frac{3}{4} u^* \frac{t^*}{\tau} \qquad (12)$$

and the night sky is then dark in a steady-state model for the same reason that it is dark in a static model.

The misconception that expansion and redshift guarantee a dark night sky in the steady-state model has been carried over into other expanding models. But we have seen that expansion by itself is not a sufficient reason; it is only sufficient and even then not necessary when the expansion time is much smaller than the thermodynamic time.

Why bother?

When at night we look up into the star-strewn sky we cannot help but pay homage to the unknown person who first wondered why the heavens were dark. This simple yet perplexing question has teased the imagination of many people and shaped the history of cosmology. It raises important issues even nowadays and no doubt will continue to do so in the future.

One might of course argue that the bright-sky paradox is now out of date with the discovery of quasars, pulsars, radio-, infrared- and x-ray sources, the 3K background radiation and other emission processes occurring in the early universe and during the time of galaxy formation. This argument misses the point; the aim of the game is first to determine what is wrong with the reasoning that originally led to the conclusion the universe is in thermodynamic equilibrium. (The extensive literature available on astronomy bookshelves that discusses "Olbers's paradox" is mostly misleading and of little help.) Knowledge gained from understanding the original paradox may then aid us in understanding a universe of more complex constitution.

In this discussion we have kept as closely as possible to the original conditions of the paradox and have supposed the universe consists of a uniform distribution of stars. We have shown that the sufficient condition for resolving the bright-sky paradox in a static universe is that the thermodynamic time scale is much larger than the luminous lifetime of the stars, and in general this is also true of the static steady-state model, the expanding models and possibly the expanding steady-state model. In other words, this means that the luminous emissions from stars are much too feeble to fill in their lifetime the vast empty spaces between stars with radiation of any significant amount.

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