

Acoustics as a physical science

Acoustics flourishes, despite the tendency of outsiders to regard "true" acoustics as minute and relatively unimportant, with any significant work considered part of some other discipline.

Robert T. Beyer

In late 1969, the National Academy of Sciences created a committee, chaired by D. Allan Bromley of Yale University, to survey the state of physics. In due course, Bromley organized a set of panels, dividing physics into such recognizable areas as nuclear physics, elementary particles, condensed matter and the like. The first part of the report of this committee has now appeared,¹ and more will follow.

In many respects, the Bromley Committee activity repeats that of the Pake Committee of 1964, whose report has long been gathering dust on the shelves. But that committee largely ignored acoustics as a living part of physics. Its entire account of acoustics ran to four paragraphs. The Bromley Committee gave far more attention to acoustics: In 1970, an Acoustics Panel was created, consisting of Andrew V. Granato, Theodore A. Litovitz, Herman Medwin, Wayne Rudmose and Jozef Zwislöcki. I was panel chairman, and we were charged with drafting a report on the role of acoustics in the field of physics.

The Acoustics Panel was faced with major overlap problems. We all know that physical acoustics forms but a small part of acoustics: The well known wheel of Bruce Lindsay (figure 1) indicates this fact. A perfectly bal-

anced report on acoustics would, then, spend only a small part of its time on physical acoustics.

On the other hand, physical acoustics is only a small part of physics. Such a balanced report would then not contain much that would stimulate interest on the part of the physics community.

What was to be done? As a working principle, we adopted the rule that we would take a broad view of the subject of acoustics, giving as best we could an overview of current developments throughout its wide range of interests. In dealing with manpower and financing aspects, however, we have confined our interest to physical acoustics, broadly understood.

Before we go into our subject, I want to point out an attitude of much of the physics community. It is apparently a truism that when acoustics comes up with a new concept, technique, method or application that is first rate, it quickly becomes part of physics (or other parent science) and is no longer recognized as acoustics. It would sometimes seem that the view of acoustics taken by many physicists is that at the bottom of the page, not the Lindsay picture. In this view, true acoustics is a minute and old-fashioned field of relatively little importance. Modern acoustical research belongs to other fields of study and not to acoustics. Thus, in rating the significance of various sub-subfields² the Survey Committee gave a high mark to turbulence but a low one to noise, high

standing to oceanography and almost zero to underwater sound, although there are major interactions within the two pairings.

The historical view

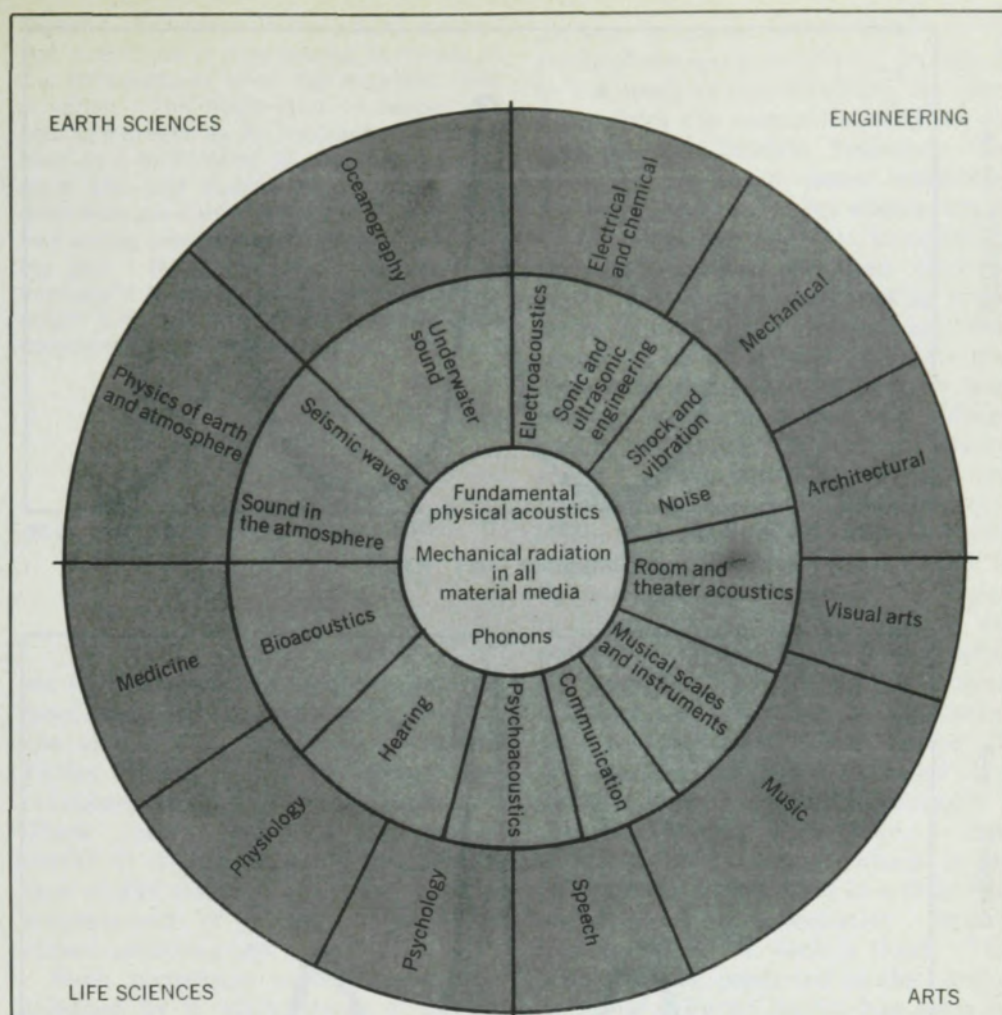
Let us then look at acoustics in terms of its role in physical science. One way of doing this is to compare the position of acoustics at an earlier time with its position today. We shall first go back 60 years, to 1912, just before World War I. What were the concerns of acoustics in physics then?

A quick perusal of the pages of *The Physical Review* in 1912 turns up only three acoustical articles; one on the singing flame, one on the analysis of complex sound waves and an interesting article on harmonic resonances in vibrating strings written by C. V. Raman. Clearly acoustics was not having a major impact on physics. A year or two before, Rayleigh had published a review of aerial plane waves of finite amplitude, and in 1912 he discussed the problem of sound filtration; but at the time the main interests of Rayleigh would appear to have been elsewhere.

Looking generally over the various fields of acoustics of that day, we see that studies of sound transmission and analysis had about expired for lack of adequate instrumentation. In a few years, electronic and piezoelectric devices would burst the field wide open, but acoustics as a branch of physics was very nearly dead in 1912.

Of course acoustical studies of hear-

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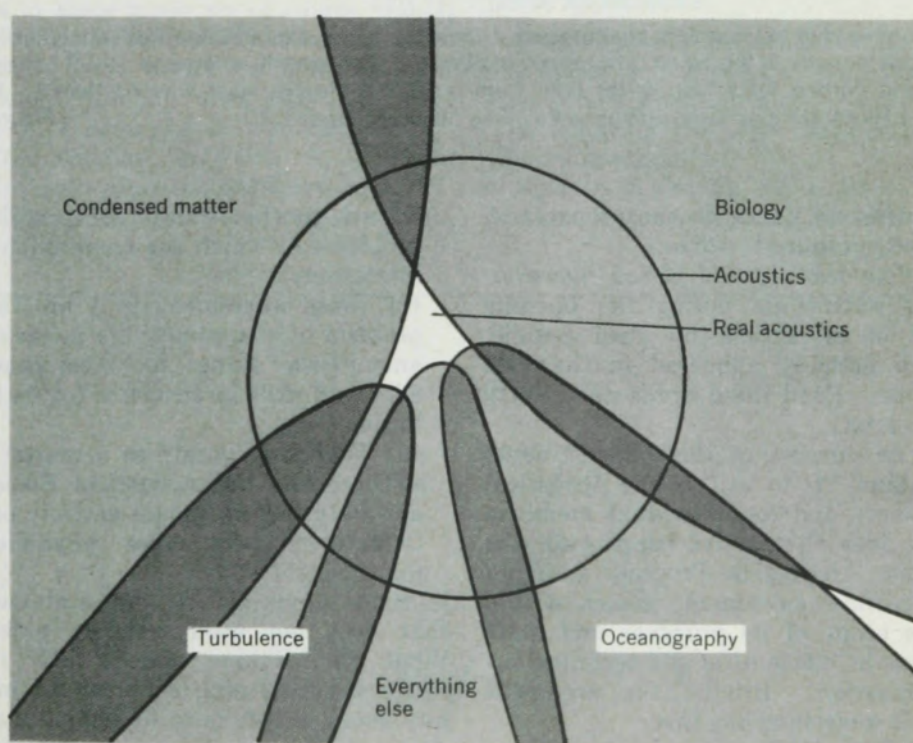
ing and Wallace Sabine's work in architectural acoustics³ made these fields of lively interest, but here again one appeared to be waiting for the new discoveries and instrumentation.

If we now move 30 years forward, to 1942, we find that World War II had already restricted the amount of publishing. It is therefore somewhat more appropriate to look at the year 1940.

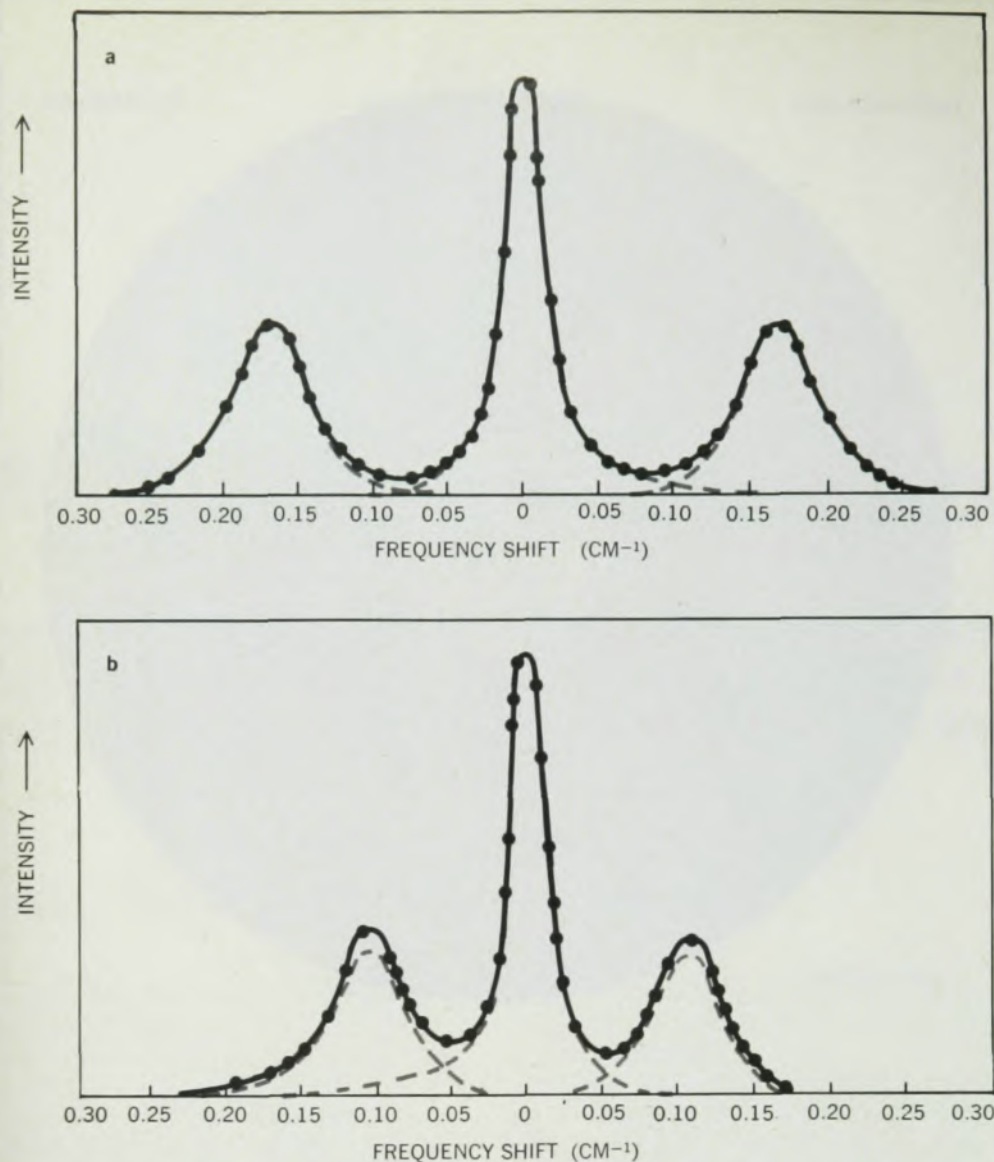
The first thing to be noted is that, with the appearance of the *Journal of the Acoustical Society of America*, acoustics virtually disappeared from the pages of *The Physical Review*. There were in fact only two acoustics articles in that journal for the entire year. I must note that both papers came from the department of physics at Brown University,⁴ reflecting Bruce Lindsay's leadership in physical acoustics.

In *JASA* there was a different story to tell. It was the flowering of architectural acoustics, with papers by (Richard) Bolt, by (Leo) Beranek and by (Robert) Newman, with Vern Knudsen, Paul Boner and Hale Sabine for good measure.

A substantial number of papers appeared on electroacoustics, and the human side of acoustics was well represented by papers on speech, hearing and music. Nevertheless, it must be reported that the number of papers in physical acoustics was still not large. Half of the papers reported ultrasonic propagation in gases, and two or three involved ultrasonic propagation in liquids. Of special research interest to



The science of acoustics divided into its components in Bruce Lindsay's wheel (see reference 2). But, according to many acousticians, the "Bromley committee" view of the subject is much more restricted (bottom), with a good deal of modern acoustical research considered by them to be part of other scientific disciplines. Figure 1



Frequency shift of laser light scattered by a liquid can yield reliable values for sound absorption coefficients in the liquid. In these studies (I. L. Fabelinskii, reference 10) of benzene (a) and carbon tetrachloride (b) light from a 6328-Å He-Ne laser was scattered. The dashed lines are estimated components of the observed curves. Figure 2

me was one paper on the propagation of finite amplitude waves.

Before leaving this period, however, it is worthwhile noting the opening lines of an article by Paul Sabine, which actually appeared in the 1942 volume. Read these words now, thirty years later:

"The purpose of this paper," wrote Sabine, "is to 'sell' to the Acoustical Society and its individual members the idea that active support for the Noise Abatement Program as a nation-wide movement, comes within the scope of its purposes and functions as a scientific and technical organization. Briefly, the argument runs something like this:

- (a) Acoustical science is of more importance to the community at large on the practical than on the theoretical, or purely scientific side;
- (b) The Acoustical Society can, in the long run, command the support of the community at large, for itself and its individual members, only to the extent to which it and they con-

tribute to the solution of practical problems in which the community is interested;

(c) Noise abatement is a practical problem of community life involving, among other things, technical knowledge and skill in acoustics for its solution.

(d) Therefore, purely as a matter of self-interest, the Acoustical Society and its members should actively participate in the noise abatement movement."⁵

Is there anything in that statement that does not need resaying today? What Paul Sabine said in 1942 the entire scientific and technical community could well take to its heart today, especially the sentiments of paragraph (b), for much of our support in the long run will come from the public's appreciation of what physics—or science, or technology—can do for mankind.

Ultrasonic absorption studies

Let me now return to my principal theme. One important development of

the physical acoustics of the 1940's was the study of ultrasonic absorption in liquids. Such studies continue to the present day, but, like all researches in physics, they have tended to spread out to the extremities of the parameters involved. Thus the discovery of relaxations in electrolytes in the 100-kHz region,⁶ which supplied the basis for the Nobel-prize researches of Manfred Eigen,⁷ has been followed by the delineation of an even lower frequency relaxation—near 1 kHz in both seawater and fresh water—a discovery that has sent David Browning to virtually all the seas and great lakes of the world for the gathering of evidence.⁸ Identification of this relaxation process is still under vigorous discussion.

This study of absorption has been paralleled by an even more significant breakthrough at the high-frequency end, a breakthrough made possible by the development of the laser. This development forms an excellent case study for the interaction of optics and acoustics and is worth considering briefly.

In 1922, Léon Brillouin developed a theory for the scattering of light by the density fluctuations in liquids.⁹ The scattered light differed in frequency from that of the source by an amount equal to the frequency of the scattered thermal phonon. The measurement of the frequency shift was in effect a measurement of the sound velocity at the frequency of the scattered phonon.

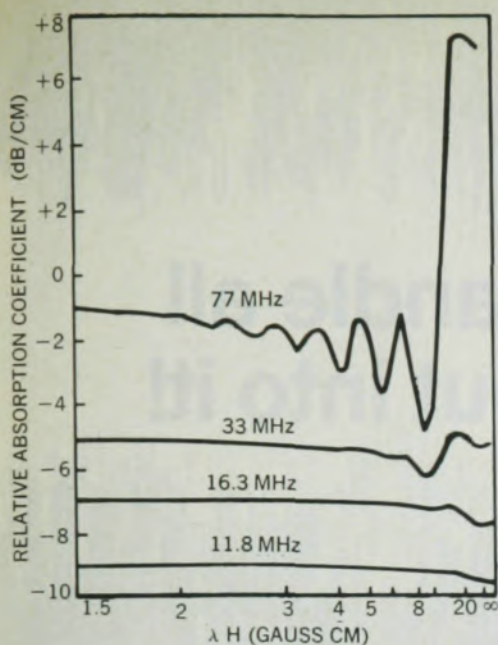
The shift in frequency $\Delta\nu$ for light of incident frequency ν scattered through an angle θ is given by

$$\Delta\nu = \pm 2\nu \left[\frac{c_s}{c_l} \right] \sin \frac{\theta}{2}$$

where c_s is the speed of sound and c_l the speed of light in the test medium. An example of this process is shown in figure 2. If the width of the scattered line could be attributed solely to this process and measured, the sound absorption coefficient could also be determined. Since the acoustic frequencies involved were in the gigahertz range, an upward extension of frequency by a factor of ten or more was possible.

Development of this method was retarded, however, because the line-widths of the best optical sources available at the time were too great to yield any reliable acoustic measurements other than some rough dispersion data. The advent of the laser, however, changed the picture completely. As a result, reliable sound absorption and velocity measurements can now be made in liquids up to 6 GHz, and even higher frequencies may be possible.¹⁰ Thus, acoustics and nonlinear optics have combined to increase our knowledge of the properties of matter.

In the history of absorption measure-



Gigantic oscillations in the sound absorption coefficient α may appear in metals at low temperatures when the magnetic field is varied. The oscillations in copper appear to increase as the frequency increases from 11.8 to 77 MHz (R. W. Morse, reference 13), and studies such as this help determine the Fermi surface of the metal as well as the anisotropy of the superconducting gap. (Note that the magnetic field intensity H is multiplied by the sound wavelength λ so that all the curves can be seen conveniently on a single plot.) Figure 3

ments, it was natural that such phenomena be studied first in gases, since the absorption coefficients of a given sound frequency are much greater there, and next in liquids, and, last of all, in solids. The exploration of sound propagation in solids has taken a number of forms. In the 1950's, H. E. Bömmel¹¹ discovered that the sound absorption in a superconducting medium was much less than in the normal state. (See also reference 12.) This result was soon connected with the size of the superconducting gap. By use of the Bardeen-Cooper-Schrieffer theory of superconductivity, it can be shown that the ratio of the absorption coefficient in the superconductor α_s to that in the normal state is given by

$$\frac{\alpha_s}{\alpha_n} = \frac{2}{e^{\Delta(T)/kT}}$$

where $\Delta(T)$ is the size of the gap in the density of states.

Another feature of ultrasonic behav-

ior in metals at low temperatures has been the effect of a magnetic field on the size of the absorption coefficient. Under certain conditions, gigantic oscillations can occur in α , figure 3.¹³ These latter measurements proved useful in determining the Fermi surface of the metal in question, and the measurement of the anisotropy of the superconducting gap.

Such pioneering studies have been followed by a whole series of discoveries of the usefulness of acoustic waves—phonons—in spin-lattice interactions, acoustic nuclear magnetic resonance, acoustic electron spin resonance and even more.¹⁴ The researchers in these areas are of course SOLID STATE PHYSICISTS: Many of them would never admit to being acousticians. But the work they do is the work of acoustics in the modern day. They are like Molière's M. Jourdain, who spoke prose without knowing it.

The link between acoustics and

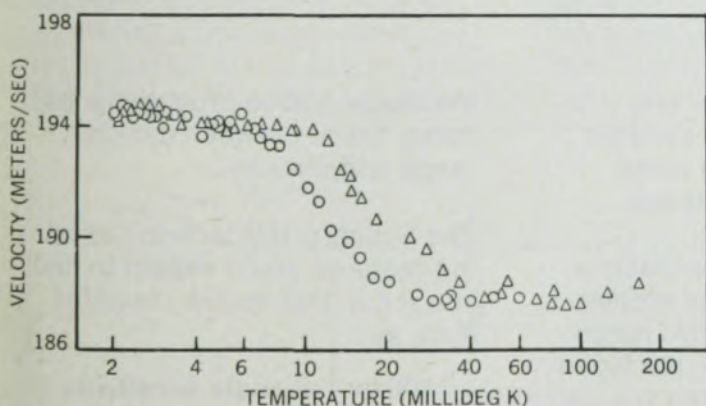
physics is particularly strong in the study of the quantum liquid, helium II. In the study of liquid helium, the transition from the normal fluid to the superfluid—the lambda transition—has been the object of many researches. An early study of sound absorption by C. E. Chase reveals this transition.¹⁵ Because of what has been observed since, this conventional type of sound wave is commonly known as "first sound." In the 1940's, Landau predicted the existence of a periodic temperature wave in superfluid helium, which has become known as "second sound." More recently, we have had "third sound"—the longitudinal oscillation of the superfluid component of a thin helium film and "fourth sound"—the compressional wave of the superfluid component moving through the pores of a finely dispersed solid when the pores are filled with liquid helium. The study of all of these phenomena provides important opportunities for proving the physical character and quantum behavior of liquid helium.¹⁶

Before leaving this topic, I shall mention "zero sound," which is the passage of a sound wave in a fluid that obeys Fermi-Dirac statistics. Helium III appears to be such a fluid. This type of sound, predicted by the Landau model of a Fermi liquid, has been extensively observed¹⁷ (see figure 4). And recently, the appearance of zero sound has also been reported in crystalline quartz.¹⁸

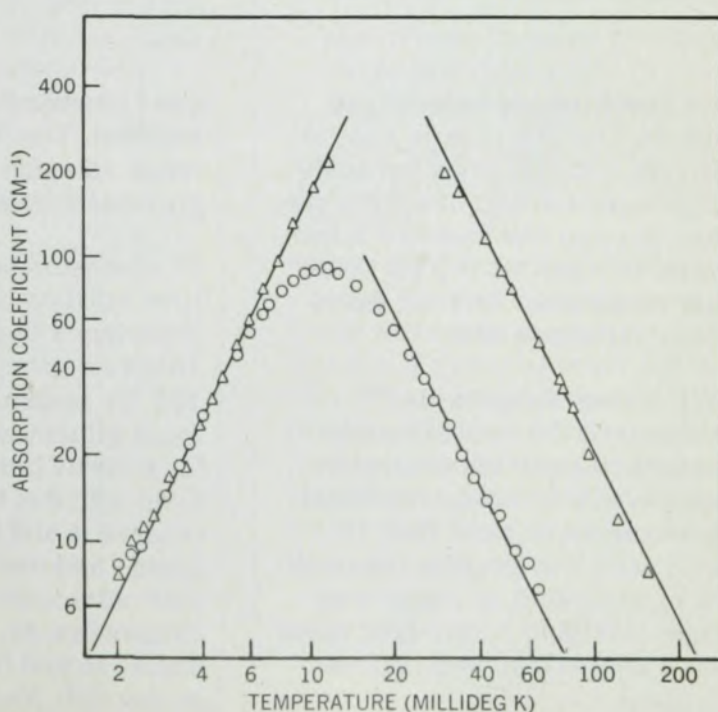
Other applications

There are so many other instances of the role of acoustics in modern physics that it is difficult to make a selection. The following are a few such applications of acoustics.

Surface waves on solids have been known since the time of Rayleigh and



Absorption of "zero sound" in He III. This type of sound was predicted by the Landau model of a Fermi liquid and has been extensively observed. Here we see the data of W. R. Abel, A. C. Anderson and J. Wheatley (reference 18) for velocity and absorption of zero sound in He III at 45 (Δ) and 15 MHz (O). Figure 4



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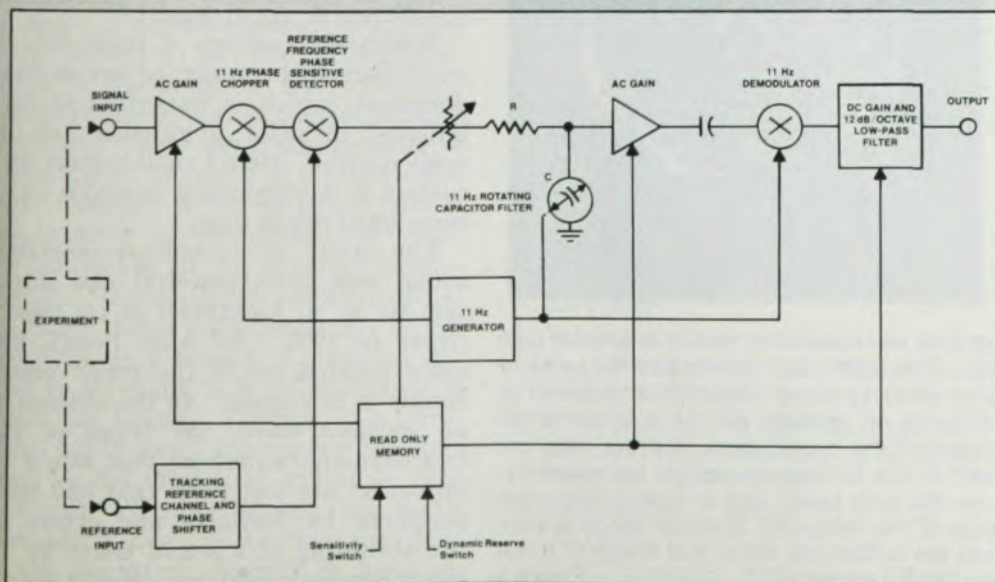
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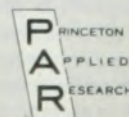


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have been widely applied in geophysics, but for many years they have been virtually forgotten in ultrasonic work. However in the past four years, a spectacular surge of interest has occurred. Bulk elastic waves have long been used in electronics for signal delay, because the speed of sound is so much less than the speed of electromagnetic propagation.

One reason for interest in surface waves has been that transducers could be located at will in the path of propagation to tap off signals or to contribute new excitation at selected intervals following the initial acoustic signal

(See John de Klerk, *PHYSICS TODAY*, November, page 32, and reference 19). Added impetus to such work resulted from the development of crystals of lithium niobate, on which surface waves propagate with particularly low loss. Another development that favored surface-wave exploitation has been the refinement of photolithography for the production of integrated circuits. At the present time, the practical upper limit of frequency for generation of surface waves is about one gigahertz, but other techniques may raise this level.

Surface waves offer special advantages

in information storage and signal filtering. They require only one highly accurate surface and are better suited than bulk waves to piezoelectric amplification. At the present time, a surface-wave device about as large as a 25-cent piece has a storage capacity of several thousand bits. In signal filtering, the advantages of surface waves is that they allow the designer to prescribe the phase characteristic independently of the amplitude characteristic.

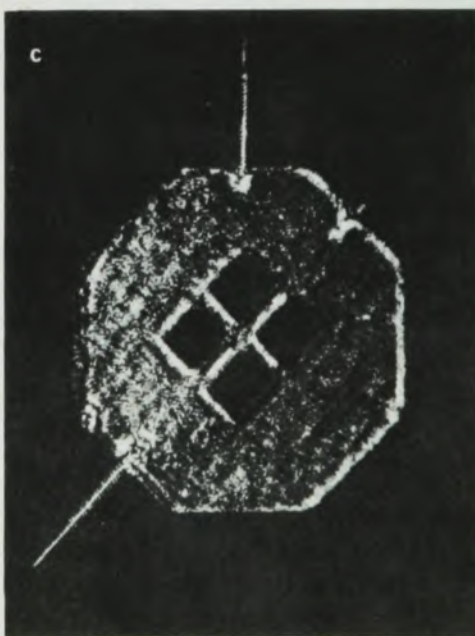
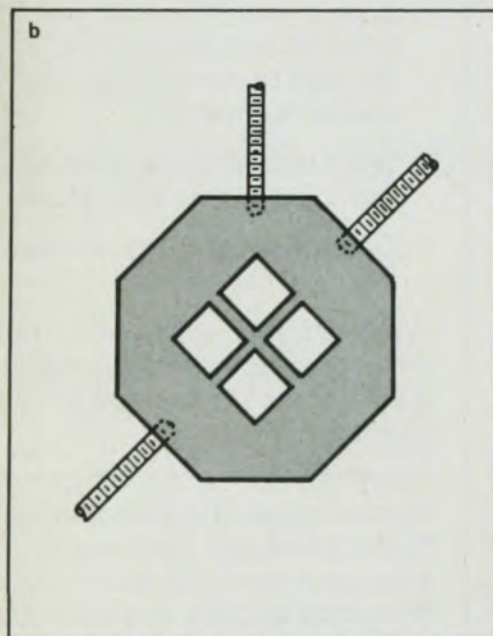
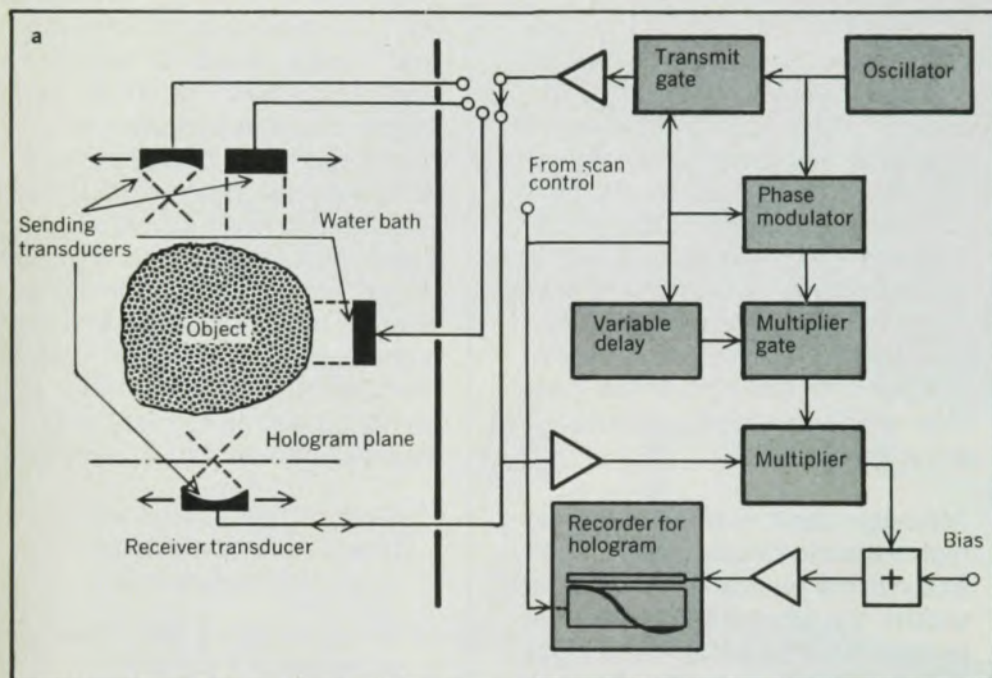
A variety of further investigations is possible. These include the propagation of surface waves on anisotropic materials, the development of waveguide techniques, and the application to the problem of epitaxial growth of a thin crystalline layer on a substrate. The transition of surface-wave studies from basic physical research to technical application has been rapid, but the process is far from complete.

Acoustic holography represents another major development of the last five years. This rapidly expanding field is likely to provide an indispensable approach in widely diverse areas of research, and many related sciences and techniques including acoustic properties of materials, electronic scanning, laser beams and electro-optical coupling make their contribution to it.

The basic feature of acoustic holography, through development of a water-air interface method of acoustic imaging (see figure 5a) allows the transfer of spatial modulation of a sound field onto a light field, which can then be used to produce optical images. At the water-air interface, the surface of the water is deformed by the incident sound pressures. A light beam is then reflected from this deformed water surface to obtain the required spatial modulation. Such acoustic holography offers new possibilities for more accurate imaging of objects within the human body and for nondestructive testing applications generally (see figures 5b and 5c, for example).²⁰

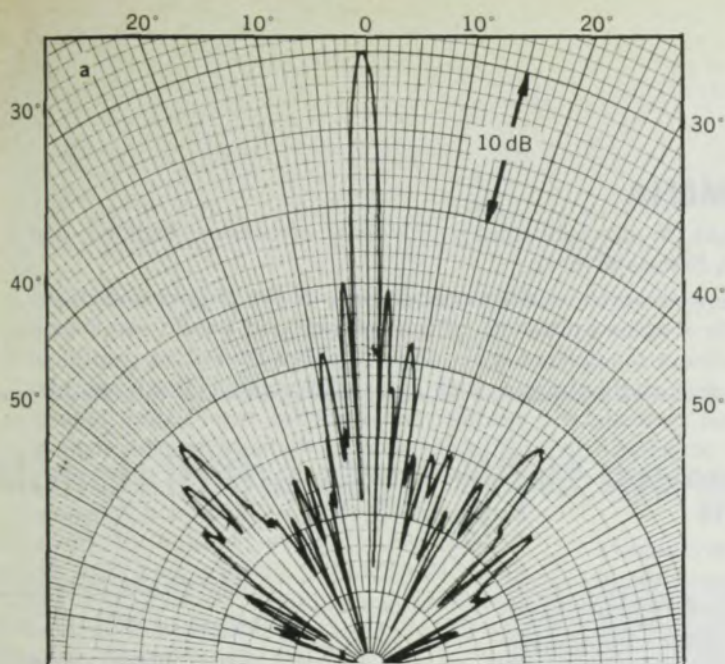
Nonlinear acoustics is itself only a small part of the major subject of fluid dynamics, which is important in turbulence, in noise production and in noise control. Here I shall restrict discussion to developments in which I had some small role to play.

The study of turbulence-generated sound was given powerful new direction by M. J. Lighthill²¹ in his classic paper in 1952. As Alan Powell has since pointed out,²² the result seems simple in retrospect: In the absence of an incident wave, the terms of the hydrodynamic equations that would be important are just those that had been neglected by Rayleigh and others in the scattering of a sound beam by an inhomogeneous medium. But it didn't

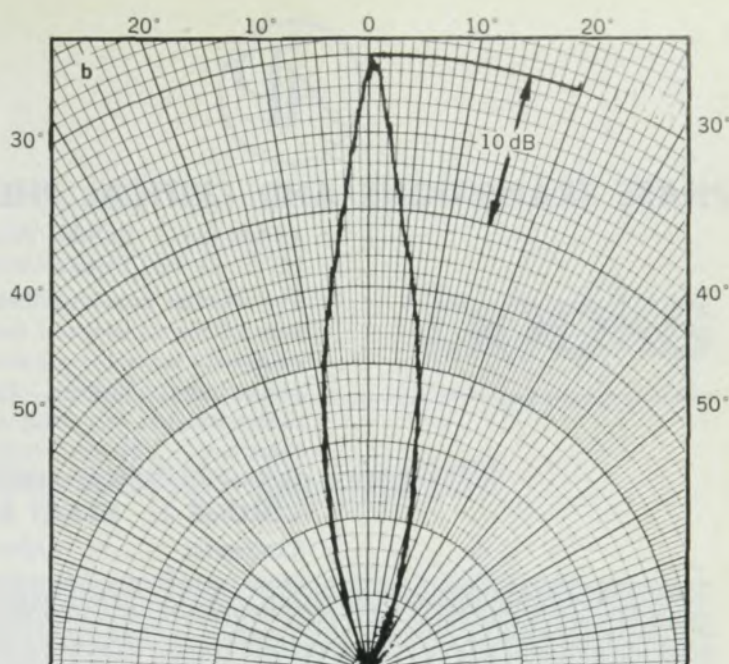


Acoustic hologram recording system. The object to be recorded is placed in a water bath and illuminated by a set of sending transducers. The signals are received by the same or different transducers, and the object is detected either by direct reflection or by direct or indirect transmission. This acoustical system yields an intensity plot on a facsimile recorder that is synchronized with the scan system of the transducers, and the resultant acoustic hologram is photographed and reduced in size to make an optical transparency. The optical reproduction is illuminated with a He-Ne laser beam with an optical stop used to remove the real image and direct transmission of the hologram; a virtual image is then obtained. In parts b and c, for example, we see the outline and the virtual image of a test object, which is roughly three centimeters across (from reference 20).

Figure 5



Difference-frequency generation with "parametric array sonar." A high-intensity, high-frequency sound beam (a) is mixed with a second, collinear beam to give a low-frequency difference beam



(b). Directional characteristics for the difference frequency are much narrower than for the primary frequency. Data are from Raytheon Submarine Signal Division (reference 26). Figure 6

seem simple until after Lighthill worked it out. In 1960, Peter Westervelt²³ applied the Lighthill theory to the passage of two collinear high-intensity sound beams of different frequency and showed that the result ought to be rather narrow in its directivity.

In 1962, my student Jack Bellin detected this difference-frequency component experimentally,²⁴ and found that it was actually even narrower than expected by the simple theory.

These observations have now reached the practical stage, and a so-called "parametric-array sonar" has been developed.²⁵ Figure 6 shows first the directional characteristics of the primary frequency of such an instrument developed by the Raytheon Company,²⁶ and then the same plot for the difference frequency. Note the absence of side lobes.

There is much more that might have been said. There is the work in infrasonics, with possibilities of major advances in our knowledge of the large-scale behavior of the atmosphere. There is the study of wave propagation in plasmas, and of magnetoacoustic waves. There are measurements of the modes of vibration of the earth and more recently, of the moon. There is the problem of the physics of noise. And there are all the advances in biological, technological and cultural acoustics, but I have made my point. Acoustics as a branch of physical science is not dead; it is flourishing as it never has before. Perhaps it is fortuitous, but it seems appropriate that this year, for the third time in history, The American Physical Society has as its president a man who was once president of the Acoustical Society of America.

In Harvey Fletcher, Dayton Miller and now Philip Morse, we have three outstanding reminders that acoustics and physics are continuing today in an interrelation that has existed since the days of Aristotle.

In regard to both the panel report and to our discussion here, I am aware of the weaknesses and omissions, and can only recall by way of defense the answer reportedly given by the pianist dePachmann to an admirer who, after a concert, had complimented him for not missing a single note. "Madam," said dePachmann, "with the notes I missed, I could have given another concert."

* * *

This article is adapted from an invited paper, given in 1971 at the 83rd meeting of the Acoustical Society of America in Buffalo, New York. The full report of the acoustics panel will appear as a part of the Survey Committee report. The author expresses his thanks to the panel, to George Wood of NAS and to all the other acousticians who helped prepare the report.

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