Optics as scattering

The art of deriving information about an object from the radiation it scatters, once limited to visible light, now includes much of modern physical research.

Giuliano Toraldo di Francia

One of the most efficient ways for a physicist to collect information about the outside world is through scattering processes: We aim a suitable beam of particles at the target or object to be investigated and observe the recoil particles or, more generally, the end products of the process as in figure 1. Classically, we use the results of this observation to derive, by theory and computation, some properties of the target that are assumed to be more fundamental than the mere scattering data. Alternatively we could take a very cautious attitude and assume that the scattering matrix, without further elaboration, fully describes our target, so far as that kind of primary particle is concerned. How do we get the most useful scattering matrix? And why, for so many years, did the only beam of "particles" exploited to any extent remain electromagnetic radiation within a certain limited frequency band-why did optics, the physics of visible light, develop first?

The negentropy principle

For us to collect information from a scattering process two conditions must he fulfilled:

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A beam of primary particles must be available with sufficient intensity for the end products to be distinguishable from the background as well as from the particles spontaneously emitted by the target.

The observer must be provided with a detector of suitable sensitivity and resolving power.

Thermodynamically, the first condition implies a nonequilibrium situation and is related to the entropy balance. At thermodynamic equilibrium between beam and background no information can be acquired about the details of the surroundings. To collect information we must have a source of

negative entropy or negentropy

The negentropy principle of information was developed many years ago by Léon Brillouin. The negentropy of an isolated system in a given state may be defined as the difference between the maximum admissible (or equilibrium) entropy of the system and the entropy in that state. Negentropy can be either dissipated through irreversible processes or be converted into informa-Information, in turn, can be transformed back into negentropy. The second law of thermodynamics may be written in the form

 $\Delta(N+I) \le 0$

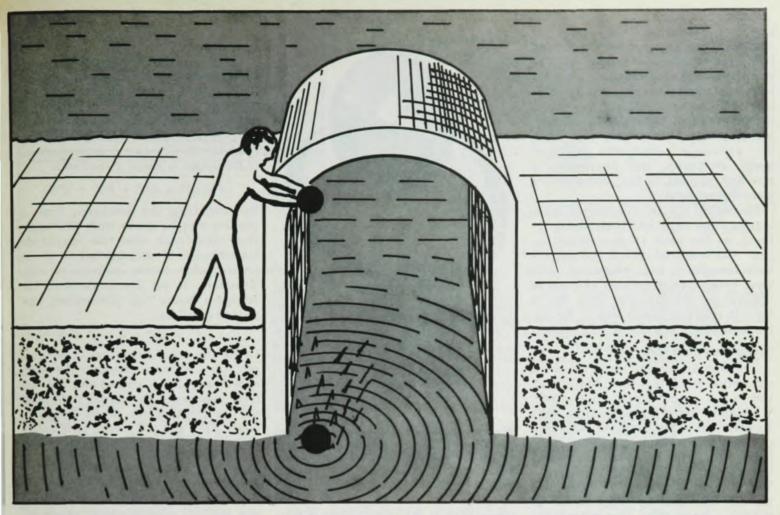
where N is negentropy and I informa-

tion, measured in the same units. One information bit is equivalent to $k \log 2$ thermodynamical units, k being the Boltzmann constant. In other words, one bit of information must be paid for with at least k log 2 units of negentro-

The only important source of negentropy for us is the Sun. That is, the Sun-Earth system has a lot of negentropy to be spent. Negentropy from the Sun is carried by radiation of three main types: neutrinos, electromagnetic radiation and solar wind.

Neutrinos reach the ground freely but cannot be used for acquiring information about terrestrial objects: They are hardly ever scattered by material objects and, at least at present, we have no appropriate detector. As far as the other two types of radiations are concerned, we find ourselves in a very peculiar condition. We are living inside a black box whose walls are represented by the atmosphere. The atmosphere is virtually impenetrable to electromagnetic and particle radiation.

Luckily enough, the builder of the box, through a small oversight, has inadvertently left a tiny crack in the wall, which lets in visible light, plus a bigger hole for microwaves. Nature has been conscientious in exploiting the possibilities offered by the radiation coming through the tiny crack.



Classic scattering experiment. Here the "beam of particles" is a single heavy rock and the end products observed are splashes of water and waves.

Figure 1

An overwhelming majority of living creatures is provided with detectors for visible light, and very often such devices are extremely refined and sophisticated.

Why only visible light?

This wonderful display of efficiency makes us ask the following question: Why did nature disregard the microwave radiation coming through the bigger hole? There are indeed very good reasons. An obvious reason is seen in figure 2.

But there is something else. The average number n of photons per degree of freedom contained in electromagnetic radiation at temperature T is represented by

$$n = \frac{1}{\exp(h\nu / kT) - 1}$$

In the case of visible light, $n \simeq 0.05$ for solar radiation and $n \simeq 10^{-26}$ for terrestrial radiation. The scattered solar photons, then, can be perfectly distinguished from the very few photons emitted by the target, even when solar light does not arrive directly at the target but is previously scattered one or more times by other objects (atmosphere, moon, walls of a room). Incidentally, because background noise is absent, a detector of the highest sensitivity, of the order of one photon, is

useful. The human eye has such a sensitivity.

The situation is different for microwaves. In this case we have $h\nu \ll kT$ both for solar and for terrestrial radiation, so that $n \simeq kT/h\nu \gg 1$. Since the Sun's surface is about 20 times as hot as the Earth's surface, the value of n for solar radiation is only 20 times greater than the value for terrestrial radiation. Even for direct illumination and very small absorption, a target that scatters within an angle of more than 2.5 deg sends out more noise than scattered radiation. Nature has apparently decided not to take the trouble to provide animals with such an inefficient system for collecting information.

This is probably the reason why, of all possible scattering processes that can be used to obtain information about the physical world, light scattering has for so many thousand years been virtually the only one known to human beings. Consequently optics, as is revealed by its Greek etymology, has been thought to be necessarily related to the eye and vision.

Now we can ask ourselves: Is there any sensible reason to go on and preserve as a separate science the science that deals with the collection of information by means of light scattering? The limitation to a very small band of

electromagnetic radiation appears to be due rather to an accidental condition of the Sun-Earth system than to fundamental or conceptual reasons.

Of course, the hardware used to deal with visible light is different from that used for other radiation. Consequently, the technology of optical instruments may still represent a separate body of knowledge. And we must not forget that very peculiar and intriguing optical instrument—the human eye. This system is still the object of profound investigation.

However, the emphasis of present-day research is placed rather on a number of methods and devices that do not belong specifically or necessarily in the visible range of electromagnetic radiation. The main problem and leit-motiv of the research is to derive information about an object from the radiation scattered by it or sent out spontaneously. On this account one may even be tempted to say that optics encompasses a great part of modern physics.

Man realized very early that when solar radiation was not available (at night, or inside a cave) he could easily produce more or less the same radiation by means of fire. From that discovery he proceeded to the construction of a host of artificial sources such as oil and gas lamps, candles, torches

and so on. He did not know that some animals, such as bats, were provided by nature with sources and detectors of a different type of radiation, namely ultrasonic radiation, and that the system could be very efficient for gathering information, especially for measuring distances. No wonder that when an extremely serviceable physical agent, electricity, was discovered, it was first applied to the production of conventional light.

Meanwhile the art of gathering information from light had been highly refined, on the basis of geometrical optics. Conventional optical instruments were brought to near perfection, with wave optics considered only as a sort of unfortunate disturbance, that set a limit to resolving power, due to the appearance of the Airy disc of diffraction.

Information from radiation

The first important steps toward a conceptual revolution were due to Max von Laue and Ernest Rutherford. Working at about the same time, they showed that other kinds of radiation, different from visible light, could be applied to gather essential information about crystals and atoms respectively. The inverse scattering problem was making its decisive appearance in physics. Since then, the method has been applied with all conceivable sorts of radiation and with enormous success.

It took some time for optics researchers to realize that what they had long been doing was nothing but a particular case of what a greater and greater number of physicists were now doing with different kinds of radiation. The eye, like the other optical instruments of classical type, is an analog computer that elaborates the information carried by scattered light and presents it in a convenient form to the mind of the observer. However, information can also be displayed in different and sometimes more convenient ways, and much of the information carried by the scattered light is missing in the image formed by a conventional instrument.

Two main factors are responsible for the missing information:

- ▶ Some *real* and all evanescent waves scattered by the object are not collected by the instrument; consequently their information is lost.
- ▶ Classical instruments do not measure the *phase* of scattered light, and the corresponding information is therefore also lost.

As is well known, the latter shortcoming was removed between the first and second world wars by Frits Zernike² with the invention of the phase-contrast method, and in general with the method of the coherent background. The idea is to introduce a proper



Why only visible light? If we were equipped instead with, say, microwave detectors, a device with suitable resolving power would be too big for convenience. Figure 2

phase shift in the light scattered by a small object and to make it interfere with the light of a coherent background. Mere phase differences are thus converted into amplitude differences.

Today, this idea appears so simple and natural that one may miss its historical significance and wonder why it had such an influence on all later developments. For the first time after the pioneering efforts of Ernst Abbe,3 coherent light entered the scene as an important tool for investigating the structure of the visual world. turned out to be so useful that soon after World War II many refined methods and devices were developed to take full advantage of the possibilities it offered. Note that this happened long before coherent light could be produced with substantial efficiency by means of lasers.

Zernike's phase contrast is very efficient but applicable only to particular microscopic objects. The large-scale utilization of interference with a coherent background came with the introduction of holography. The first ideas about holography occurred to Dennis Gabor in 1948,4 when he was thinking about electron microscopy, particularly the difficulty of correcting the spherical aberration of electron lenses, and he thought it possible to compensate for spherical aberration by wavefront reconstruction. However his method has never found a practical application in electron microscopy but has instead

come into prominence for the use with laser light.

Let S represent the signal, the complex amplitude of a beam of coherent light scattered by the object, and R the coherent background, or reference beam as in figure 3a. When they impinge simultaneously on a photographic plate, the exposure E is proportional to the square modulus of their sum:

$$E = SS^* + RR^* + RS^* + R^*S$$

By controlling the developing time, a positive can be made from this negative plate such that the amplitude transmission is proportional to E (in photographic terms, with $\gamma = -2$). Therefore, if we illuminate with the reference beam, the transmitted amplitude A will be

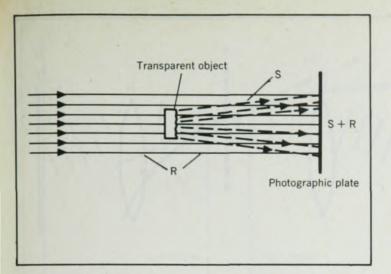
 $A = (SS^* + RR^*)R + R^2S^* + RR^*S$ Under the usual conditions of holography, the first term represents the reference beam almost unmodified, the last term represents the signal multiplied by the constant (or nearly constant) factor RR^* , and the second term represents the complex conjugate of the signal, or "twin image."

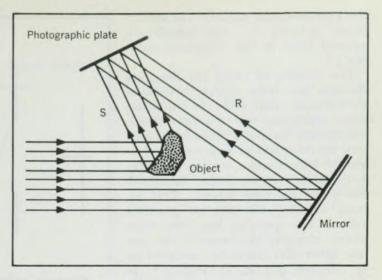
Holography with laser light

In his original work, Gabor had to use the "in-line" method, where the signal and reference beams were approximately in one line. This was because of the limited coherence of the light used (a high-pressure mercury lamp), which did not allow the production of interference at large angles. As a result, the image, the twin image and the reference beam were very inconveniently superimposed. This difficulty was overcome by Emmett Leith and Juris Upatnieks in 1962, when lasers were available, by taking a skew reference beam (see figure 3b).5 Thus in the reconstruction the three beams were angularly separated. The assumption that $\gamma = -2$ is useful for an elementary discussion, but can easily be dropped. If γ does not equal minus two, we get not only the ordinary and twin images but also a series of "harmonic terms" or of angularly separated images. Black-and-white holograms, bleached holograms and phase holograms have become possible.

Black-and-white holograms can be computed and synthesized. By taking advantage of the thickness of the emulsion, one can combine the principle of Lippmann color plates with holography, as suggested by Yu. N. Denisyuk, and get color holograms.

One feature that makes holography very useful is that information about a given wavefront remains "frozen in" in the emulsion and can be added linearly to the information of a different wavefront. This property is used in holographic interferometry, which has





Hologram recording systems. In the early "on-line" system (left) a beam of coherent light is scattered by the object. This beam (S) and the coherent background beam (R) impinge simultaneously on a photographic plate and are recorded. To view the hologram, the plate is illuminated with the reference beam. Holography with a skew reference beam (right) was developed in 1962, when lasers were available. This method overcame the problems of superposition of image, twin image and reference beam (see reference 5).

many industrial applications. Two holograms of a given object, taken at different times, are recorded on the same plate. The two wavefronts obtained in the reconstruction can interfere, and from the interference pattern one can tell whether the object has moved or been deformed between the recordings (see figure 4).

In 1971, looking back to his intended electron-optics application, Gabor recalled: "Why should one bother in light optics, with such a complicated two-stage process, [with] coherent light so weak and uncomfortable to use, when we had such perfect lenses, even achromatic ones? Little did I think at that time that after 24 years the application of holography in electron

microscopy would still be in a primitive state, while the simple optical experiments, which I considered only as model or feasibility experiments, would give rise to a new branch of optics, with some 2000 papers and a dozen books!"⁷

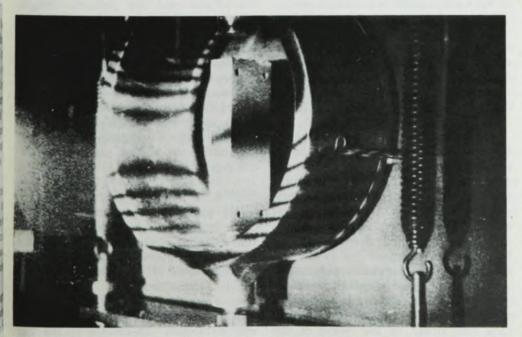
Image formation with coherent light can be described for a typical case of a transparent object. A coherent plane wave illuminates the plane of the object O and is scattered by "inverse interference" into many plane waves with different directions, plus a set of evanescent waves. Each plane wave is brought to a focus by lens L₁ at a point of the focal plane F, conventionally termed the "pupil plane." Lens L₂ transforms the wave back into a plane

wave. All the output plane waves are brought together to direct interference at the plane of the image (see figure 5).

Each one of the scattered plane waves represents a spatial frequency or a component of the Fourier spectrum of the object. This Fourier analysis shows that there is an analogy between an optical signal as a function of space coordinates and an electrical signal as a function of time, so that the ordinary techniques of frequency manipulation can be applied. The only difference is that in optics we have two dimensions instead of one.

Let us first consider the question of resolving power. If the instrument could collect all the scattered waves and bring them back to interference onto plane I, the image would be similar to the object. However, any instrument has a finite aperture, and some real waves as well as all the evanescent waves do not enter the system and are missing in the image. There is a cutoff in the spatial frequencies of the image, and consequently the finest details of the object are lost. This is tantamount to saying that an infinite number of different objects should have one and the same image: The image is ambiguous.

The question of how many and what objects have one and the same image is closely related to the problem of finding the number of degrees of freedom of the image. This question dates back at least to Laue's discussion of the degrees of freedom of electromagnetic radiation. In more recent times it has been argued that, due to the finite width of the spatial frequency band of the image, one can apply the sampling theorem: The result is that the number S of degrees of freedom is proportional to the object area times



Holographic interferometry is a technique with many industrial applications. Two holograms of an object are taken at different times and recorded. The interference pattern indicates whether the object has moved or been deformed between recordings.

the entrance solid angle of the instrument, divided by the wavelength squared (this is the "Shannon number").8

The validity of using the sampling theorem has been repeatedly refuted for the reason that when the object has finite extension, the frequency distribution over the pupil plane is an analytic function and as such can be completely known when we know its behavior even over the limited domain of the pupil aperture region. This remark is true, but has no practical value. The question has been completely clarified by means of the prolate spheroidal functions, analyzed by D. Slepian and H. O. Pollack.9 In this approach, an object distribution represented by a (properly scaled) spheroidal function ψ_n has an image similar to the object; that is, an image represented by the same distribution multiplied by a factor λ_n . The ψ_n form a complete set of orthogonal functions for the object, which can therefore be expanded in series of the ψ_n functions. Then each coefficient of the series corresponds to a degree of freedom of the object. It turns out that λ_n equals about one up to values of n approximately equal to S, whereas for ngreater than S the factor λ_n drops practically to zero. The corresponding degree of freedom is lost for the image, no matter what physically conceivable type of detector we use. Therefore S represents the "physical" number of degrees of freedom.

The problem of noise

Of course, the evaluation of S represents only a preliminary stage in the application of information theory to optics. A complete theory can be developed only by taking noise into account. Noise in optics can arise from different sources, but one source that can never be eliminated is represented by the photon nature of light.

The following argument, due to Gabor, 10 is very instructive: A photographic plate collects energy $RR^* \equiv E$ from the reference beam, $SS^* \equiv e$ from the signal, and from their interference, $RS^* + R^*S \equiv 2(Ee)^{1/2}\cos\phi$. The last term, which represents the interference fringes, carries the information and can be made as large as we want by increasing E, no matter how small the signal energy e.

In the limit one could (seemingly) transmit information without energy. Of course this is absurd. The signal cannot be recognized once it is small enough that it is drowned out in the fluctuations of E. Now the mean-square amplitude of the fringes is 2 Ee. We postulate that the signal becomes unrecognizable at some minimum value of e, called ϵ , at which the mean square fluctuation of the reference ener-

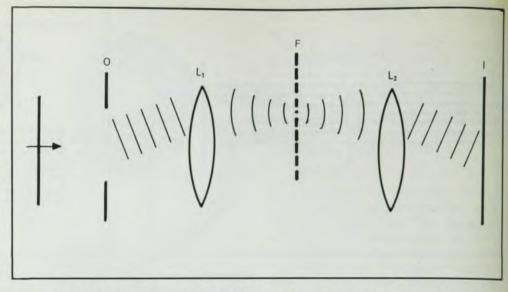


Image formation for a transparent object. A coherent plane wave illuminates the object O and is scattered into many plane waves with varying directions (only one is shown above), plus a set of evanescent waves. Each plane wave is focussed by lens L₁ at a point along the focal plane F (the "pupil plane") and is transformed back into a plane wave by lens L₂. All the output plane waves are again brought together, by direct interference, at the image plane I. Fourier analysis of this process shows an analogy between the optical signal in space and an electrical signal in time, so that the ordinary techniques of frequency manipulation are applicable to optical as well as to electrical waves.

gy E exceeds this by a factor k. Thus

$$\langle \Delta E^2 \rangle = 2kE\epsilon$$

or, if $n \equiv E/\epsilon$

$$\langle \Delta n^2 \rangle = 2kn$$

By putting h = 1/2 we get Poisson's law

$$\langle \Delta N^2 \rangle = n$$

which indicates that the fluctuation of the energy is of the nature of shot noise. We recognize that the result is correct so long as the photons are distributed over a very great number of cells of phase space. Otherwise there is a correction term, because photons are not classical particles. The problem, we see, is similar to one of fluctuations of blackbody radiation in an enclosure.

Whatever the source of noise, we can code and process the information so as to reduce its effects. For instance we can filter out unwanted frequencies from the plane of the pupil, a procedure pioneered by André Marechal and Paul Croce.¹¹ To mention a few interesting cases:

The object can be periodic, giving rise to a series of dots on the image plane. By blocking these frequencies with black dots, we can remove the image of the "perfect" object and reveal only its faults. Conversely, if one lets through only the periodic frequencies by means of a diaphragm with holes, one can remove the faults and restore a perfect image of the object.

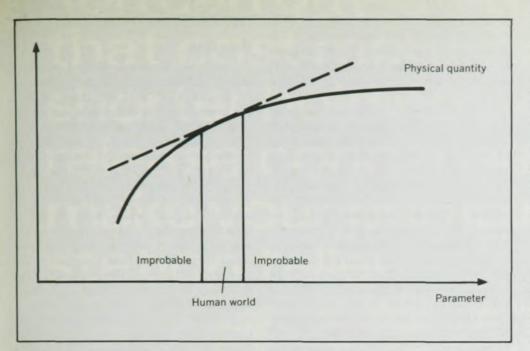
In general the restoration of an image damaged by blurring or other causes requires filters for both amplitude and phase in the plane of the pupil. This was a formidable requirement until holography was applied to it by Anthony van der Lugt and George

Stroke.¹² Holography allows us to build a plate that gives to the impinging wavefront any wanted modulation of amplitude and phase.

One of the first things made possible by this filtering technique is the restoration of images blurred by defocusing. movement or other causes. Another application of great interest is in pattern recognition. If the pattern to be recognised is p, its Fourier spectrum P is formed in the focal plane and holographically recorded. The hologram will contain both P and P*. The latter defines the filter to be used. When P* is multiplied with the Fourier spectrum of the object, one obtains in the image plane the correlation function of the object and the pattern. Bright spots appear in the places where the object contains the pattern to be searched. This procedure suggests the possibility of building a reading machine; however, no practical device appears so far to have been built on this principle. One of the serious problems in applying this technique is that the pattern may have different size and orientation from that used in the recording.

Nature of coherent light

All these developments appear to have been made possible by coherent light. What is coherent light? For a long time people were content with the vague notion of coherent light being highly monochromatic and coming from a very small source. A quantitative analysis was pioneered by Zernike¹³ and developed extensively in the last decades. Let us denote by V(t) the complex wavefunction (or analytic signal) whose square modulus $V(t)V^*(t)$



Nature is linear. A modern rephrasing of *natura non facit saltus* (nature does not jump) might be that any curve representing a physical phenomenon can, within a limited interval, be replaced by a straight line. The important philosophical question here is why the interval of physical interest is usually limited to this straight line portion of the curve.

= I(t) represents the light intensity. For two points P_1 , P_2 we define the mutual coherence function

$$\Gamma_{12}(\tau) = \langle V_1(t+\tau)V_2^*(t) \rangle$$

as a time average taken over a sufficiently long period. The normalized function

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\langle I_1 \rangle \langle I_2 \rangle}}$$

is called the *complex degree of coherence*. It can be shown that the maximum value of $|\gamma_{12}|$ is a measure of the visibility of the fringes formed by light coming from P_1 and P_2 in a Young interferometer.

There is a beautiful theorem, due to P. H. van Cittert14 and Zernike,13 on the value of γ_{12} for two points illuminated by a planar quasimonochromatic source S: \(\gamma_{12}\) is the normalized complex amplitude that would be produced at P_2 by a spherical wave centered at P_1 and diffracted through an aperture equal to S. This theorem easily explains why Albert Michelson could measure star diameters with his stellar interferometer, despite its shortcomings. Scintillation and the difficulty of maintaining the interferometer arm length constant to within a quarter wavelength make it impossible for the arm length to exceed a few meters.

However, if light can be considered as a random Gaussian process, the fluctuations of *intensity* obey the equation

$$\langle \Delta I_1(t+\tau)\Delta I_2(t)\rangle = I_1I_2|\gamma_{12}(\tau)|^2$$

Intensity fluctuations are much slower than amplitude oscillations and the measurement is much easier; this is the basis of the intensity interferometer that R. Hanbury-Brown and R. Q. Twiss¹⁵ have constructed both for visible light and microwaves. Radiation is collected by two mirrors, at P_1 and P_2 , and sent to two phototubes that reveal intensity. Signals from both phototubes are sent to a multiplier and correlated. In this way one can measure $|\gamma_{12}|$ for two points even hundreds of meters apart. By recording the signals, one can also compare fluctuations at places thousands of kilometers apart.

Phototubes can be sensitive to individual photons, and one may wonder if classical wave theory is adequate to treat these phenomena. However, semiclassical theory shows that, because photons are bosons, fluctuations of photocounts obey substantially the same laws as fluctuations of intensity.

A more important question is whether and when the assumption that light is a Gaussian random process is justified. It is certainly justified when light is generated by many independently excited and spontaneously emitting atoms. But what about lasers?

The question can be clarified only by an appropriate quantum treatment. Pioneering work in this direction has been done by Roy J. Glauber. ¹⁶ As is well known, the electric field E can be represented by an expansion:

$$E = \Sigma [a_k u_k(r) e^{-i\omega Kt} + a_k^{\dagger} u_k^{\dagger} (n) e^{i\omega kt}]$$

where a_k , a_k^+ represent the annihilation and creation operators. The annihilation and creation parts of E will be represented by E_+ and E_- respectively.

If by x_1, \ldots, x_{2N} we denote 2N points of space-time and by ρ the density matrix of the states of the field, we can

introduce the correlation function of Nth degree by

$$G^{(N)}(x_1 \dots x_{2N}) =$$

$$\Sigma[\rho E_{-}(x_1)...E_{-}(x_N)E_{+}(x_{N+1})...E_x(x_{2N})]$$

It turns out that $G^{(1)}$ $(x_1 \ x_1)$ is proportional to the probability P_{x1} of having a photoelectron produced at x_1 ; that is, to the intensity at x_1 . Similarly $G^{(N)}$ $(x_1, \ldots, x_N, x_1, \ldots, x_N)$ is proportional to the probability P_{x1}, \ldots, x_N of revealing one photoelectron at each point x_1, \ldots, x_N (N-fold coincidence). Let us introduce a normalized correlation function

$$g^{(1)}(x_1x_2) \ = \frac{G^{(1)}(x_1,x_2)}{\sqrt{G^{(1)}(x_1,x_1)G^{(1)}(x_2,x_2)}}$$

and similar expressions $g^{(N)}(x_1, \dots x_{2N})$ for the higher-order correlation functions. A field will be said to be coherent in the first order if $g^{(1)} = 1$, in the Nth order if $g^{(N)} = 1$ everywhere.

If $g^{(N)} = 1$, then the probability $P_{x_1x_2...x_N}$ of a N-fold coincidence turns out to be equal to the product

$$P_{x_1x_2...x_N} = P_{x_1}P_{x_2}...P_{x_N}$$

of the probabilities of revealing one photon at each point x_1, \ldots, x_N . Therefore such probabilities are independent and there is no correlation in the photocounts. Coherent light obtained with ordinary sources (that is, filtered Gaussian light) is coherent only to the first order and therefore shows twofold correlations, whereas the field radiated by a classical antenna is coherent to any order. Laser light lies in between. It is coherent to a high, but not infinite, order.

The invention of the laser is just one more step in a process that has been going on since the beginning of mankind: At first, Man played only a role analogous to that of Maxwell's demon, selecting from the random noise of all natural objects and phenomena those improbable cases that were advantageous for his survival. Then, little by little, he learned to build the improbable and useful things right away, without having recourse to noise. Thus coherent light from a natural or conventional source is only filtered noise. Making a laser is much more clever.

Noise, however, can be used in a very subtle way. We are accustomed to thinking that random fluctuations mar the signal sent out by an object and diminish obtainable information. Nevertheless random fluctuations carry some valuable and quantitative information about the object. This was first shown by Einstein with his theory of Brownian motion. Another good example is Rayleigh scattering of light by the sky. A more recent and striking case is correlation interferometry. 15

Nonlinear optics

Departing from filtered noise and natural phenomena occurring around



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us very often brings us to discovering nonlinearities. When a young student first encounters the mathematical expressions of elementary physical laws, he may be tempted to conclude that nature is linear. Is this conclusion right?

A more correct statement appears today to be that nature is analytic, which in turn is a modern rephrasing of the old statement natura non facit saltus, or nature does not make jumps. Any curve representing a physical phenomenon has a continuous tangent and within a limited interval can be replaced by a straight line (figure 6). The important philosophical question to answer is why the interval of interest is in most cases limited to the straight portion of the curve. Why is the usual departure of the parameters from their equilibrium values so small? Is this an essential feature of the human world? We live in a degenerate world, very close to absolute zero, the world of molecular forces. But even if we include the Sun in our system, we remain in the realm of modest energies as compared with the high energies that we know to be physically possible. Near equilibrium only the lower states tend to be filled, and the more we want to depart from equilibrium, the more ingenuity and effort we must spend in order to make a very improbable case to occur. Laser light can easily be made a million times hotter than the Sun's surface. No wonder therefore that the straight-line limits are exceeded and optics may become nonlinear.

Classical optics is based on the circumstance that in an ordinary material the electric polarization is proportional to the electric field; waves scattered by the atoms have the same frequency as the incident wave. However, when E becomes sufficiently large, we discover that P is a nonlinear function of E:

$$P = a_1 E + a_2 E^2 + a_3 E^3 + \dots$$

The second term of the expansion gives rise to scattered waves with double frequency; the third term gives rise to a third harmonic, and so on. In this way one can multiply the frequency of optical radiation, as was first shown by Peter A. Franken.¹⁷ Because two or more photons of the incident radiation give rise to one photon of the harmonic, there is of course a condition of momentum conservation: $\hbar K_1 + \hbar K_1 =$ $\hbar K_2$. Since K defined as 2π is proportional to the refractive index, this is equivalent to an index-matching condition $n_1 = n_2$. This condition can be met in an anisotropic medium in a particular direction.

Nonlinear optics has opened a huge field of prospective applications in communications systems, where many well known radiofrequency techniques such as mixing, heterodyning, modulating and so on can be transferred to the domain of optical frequencies.

More light

Light, the first gift of God to Man. one of the first physical phenomena to be investigated, still continues to supply a wealth of problems for our mind. a wealth of applications for our welfare. This is a gratifying realization. However, no discussion of optics and its latest applications would be complete if we ignored the fact that with all our light we are not yet able to illuminate human brains and to defeat human stupidity. Many people working with lasers were simply horrified when they realized that the most effective application of lasers is today represented by the guidance of deadly missiles. What could have been a monument to man's intelligence has turned into a shame for all mankind. More light! Like Mehr Licht! Goethe on his deathbed, we desperately need more light. More light for the human mind.

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