Fluid dynamics of electron gases

Our understanding of the behavior of electron fluids is one offshoot of Otto Laporte's deep interest in shock-wave phenomena.

Richard G. Fowler

Although fluid dynamics may be regarded by many physicists as a quiet and wholly classical backwater, those of us in the field know it to be complex, dynamic and exciting. The resurgence of this field was due in no small way to the work of Otto Laporte, who died on 28 March 1971 after an extraordinarily full career dating from the 1920's. Laporte's interest in fluid dynamics, stemming from his early association with Arnold Sommerfeld in Munich, grew after he took over the Shock Tube Laboratory at Michigan in 1946, where he developed many of the now well known results and methods in shock-wave technique.

Shock-tube work at the University of Oklahoma will be the main subject of this article, but our efforts were always so closely followed and aided by Laporte that I have felt it wholly appropriate to include some insights into the life of this genial and extraordinarily versatile man, while dealing with this topic that his interest nurtured. It came as a surprise to us when shock-wave concepts and the equations of fluid dynamics turned up in dealing with electrons, but we have found that behind terms as various as "precursors," "leaders," and "potential waves," in laboratory apparatus, in lightning discharges, in solar flares and even as the result of hydrogen-bomb explosions, electrons lurk as a remarkable fluid substance amenable to treatment, even if only for a brief instant, by the equations of fluid dynamics—in fact, they seem to *demand* such treatment.

Otto Laporte with Patricia and Gerald Fowler and one of his favorite plants (a *Euphorbia lactea*) at the Mexico City APS meeting in 1966. Figure 1

The earlier years

Laporte was born in Mainz, 23 July 1902. He and two sisters, Luise and Marianne, were the children of Wilhelm Laporte, a colonel of artillery in the Imperial Army who was assigned to direct the defense of the Mosel River. Otto attended grammar school in Mainz and the Gymnasium in Frankfurt. After the war the Laporte family moved to Munich so that he could study under Arnold Sommerfeld. Having a budding physicist in the family proved an unexpected asset to Colonel Laporte, who needed a housekeeper, since Otto's mother had died during the war years. Fraulein Käthe Fuchs, the former housekeeper for the Röntgen family, was living in Munich, and she consented to join the Laportes solely because of her admiration for physicists. Otto found her a source of solace and friendship till late in life.

At Munich the youthful student found many kindred minds. Werner Heisenberg in his reminiscences "Physics and Beyond" describes how he, Laporte and Wolfgang Pauli engaged in many intense conversations that set the stage for the wave mechanics. One particularly memorable series took place on a long bicycle trip through southern Germany. Heisenberg says "The talks we began during that tour, and continued in Munich, were to have a lasting effect."

Sommerfeld was away in the US during the last year of Laporte's doctoral program. For his dissertation Laporte solved the problem of the diffraction of electromagnetic waves around a sphere, discovering in the process those extraordinary singular solutions later rediscovered in nuclear physics as Regge poles. But this was really the lesser of two works accomplished by him during the year of his majority, 1923. Spectroscopists of that time had come to believe that the problem of the three-electron spectrum of things like iron, vanadium and chromium would never be solved because of its complexity. Laporte unraveled and published first the vanadium spectrum and then the iron spectrum during that year, enunciating as he did so the fundamental principle known among spectroscopists as the Laporte rule.

The Laporte rule is that with more than two electrons, one must recognize two kinds of energy states (which he called simply "odd" and "even"), and that it is impossible for radiative transitions to occur between unlike states. Although this was based on the right and left handedness of a three-vector system in the old Sommerfeld theory of quantization, it was in fact the enunciation of the principle of *conservation of parity*, as Herman Weyl pointed out in his 1928 book on group theory. Upon his return from the US, however, Sommerfeld regarded this resolution of the iron spectrum problem as too unsophisticated to be a doctoral topic, and insisted that Laporte should submit the spherical-diffraction problem. Nevertheless, with this rule, Laporte joined that select group of people whose generalizations have been passed over, but an exception to whose generalizations has drawn a Nobel prize. Fate continued to play her ironical jokes when his election to the National Academy of Sciences was made before his death, but could not by the rules be told to him until a date after which he had left us. His friends take consola-

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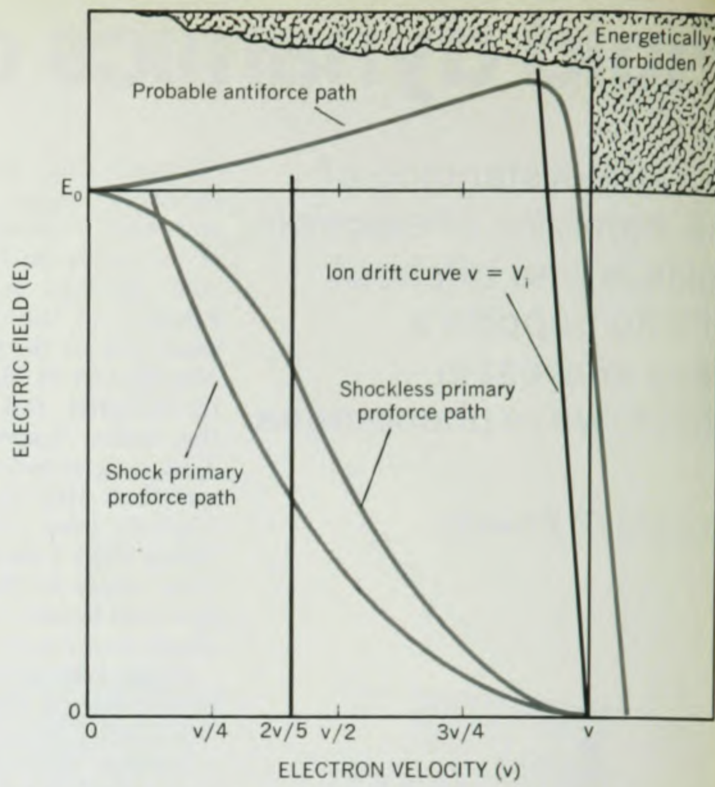
With Alvin Nielsen (left), Laporte talks to Harrison Randall, past president of the APS, at a University of Michigan reception in honor of Randall's 95th birthday. Figure 2



The electron velocity–electric field relationship. This sketch shows shock-fronted and non-shock-fronted proforce wave solution paths, and a possible antiforce wave path. Figure 3

tion in the fact that his having honors mattered more to them than to him.

University of Michigan

In 1924 Laporte came to Washington as an International Education Board Fellow in William Meggers's spectroscopy section of the National Bureau of Standards. Two years later he joined the University of Michigan physics department. From 1928 to 1929 Laporte served as guest lecturer at the Imperial University in Kyoto, Japan. While there, he received an urgent request from Sommerfeld asking him to lecture in his stead at Munich during the period of a projected visit to America. Laporte hastened home via the Trans-Siberian Railway, a non-stop trip then of two week's duration. He never tired of describing the rigors of the trip and the monotony of the menus—borscht and scrambled eggs.

Ten years after Laporte joined the physics department at Ann Arbor I began my own all-too-fleeting 35-year association with him. One quickly came to admire Laporte's intellect, but when one later was privileged to perceive his intense humanity, his feeling for nature, for music (he was a creditable pianist and sophisticated amateur of music), for art . . . in short, to understand that Otto was a man who knew how to live life creatively, this admiration quickly deepened into affection. I believe his profoundest feelings were for the beauties of mathematics, of plants, of music, and of young minds. His home was a botanical garden and I was never quite sure whether he attended meetings more for the physics discussed or the arboreta one could visit. His interest in the cultivation of exotic plants was stimulated, he told his friends, by his voluntary wartime labor in the Frankfurt Municipal Gardens. Laporte had his own greenhouse full of cacti and other exotica.

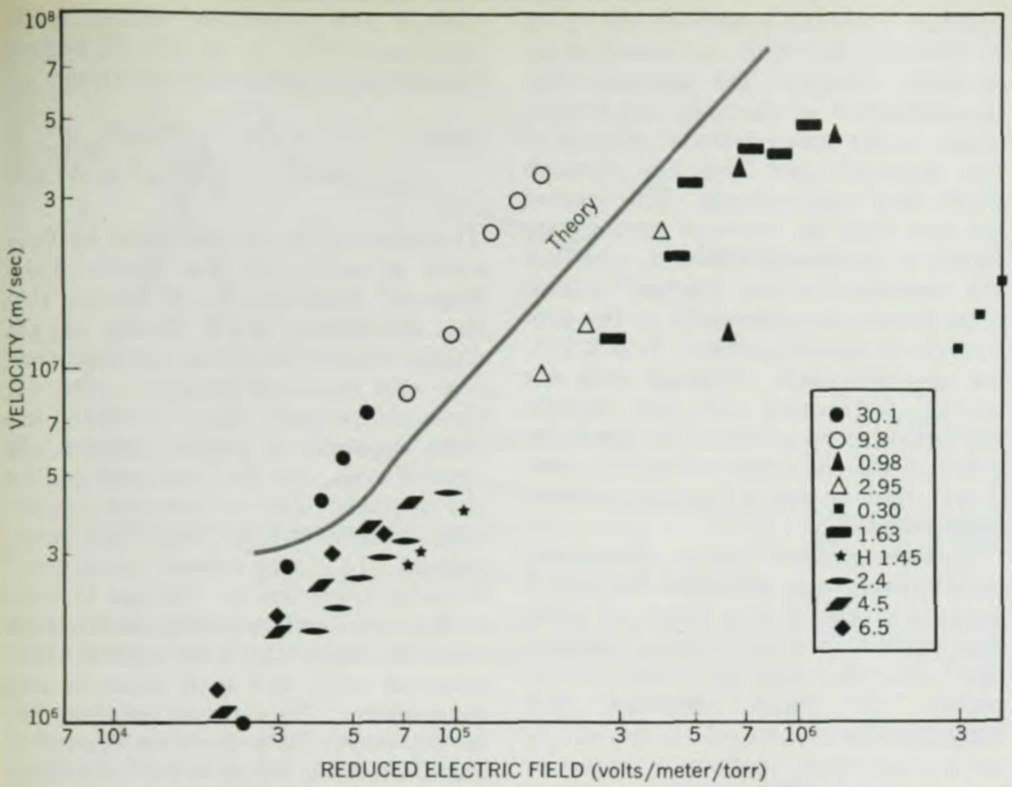
After the last Mexico City meeting, in which the Division of Fluid Dynamics participated, we took a quick excursion to Taxco. In front of a hacienda he spotted an enormous succulent, and in great excitement he called, "That's the largest *Euphorbia lactea* I've ever seen. I must have my picture taken with it!" And so I have the photograph reproduced as figure 1 to assist our remembrance of him. The other photograph reproduced here (figure 2) was taken at a University of Michigan reception.

The taste of the Orient that Laporte had acquired in 1928 became an enduring interest in his life, leading him to return to lecture in Tokyo in 1933. He studied the Japanese language, Japanese art, and literature as assiduously as he cultivated plants. He became an expert in calligraphy and in forms of Japanese poetry. In a national competition there, he submitted a haiku that won recognition. The lure of Japan attracted him twice, later, to accept the appointment of Scientific Advisor to the American Ambassador in Tokyo. We Michigan graduate students suffered under this passion for things Japanese, for when Laporte ran out of Greek and Latin symbols for physical quantities he never hesitated to employ Chinese characters.

The US State Department credits Laporte, during his second tour of duty in Japan, with having secured the landmark atomic-energy agreement between the two countries. Many of us will remember the Division meeting in Hawaii when he served as a genial master of ceremonies, welcoming the Japanese delegation in Japanese and the American contingent in English. One of the regrets of his life was that our International Shock Tube Symposium never took place in Japan.

Why fluid dynamics?

Otto Laporte was one of the charter members of the American Physical Society's division of fluid dynamics. It marked a second career for him. Regarding theoretical atomic physics as a completed exercise and having little interest in the nucleus, he was casting about for something interesting to do. When I naively asked him in 1950, "How on earth did you get involved in fluid dynamics?" he said "Anyone who had known Sommerfeld would not have asked that question." Sommerfeld insisted that all his students should have a thorough grounding in the subject. In any event, during the second World War, Laporte tackled difficult problems of the shape and design of air foils, finding an exact solution to the lift of a wing of elliptical outline. Then in 1946 the departure of Lincoln Smith, who had initiated the



Experimental data for proforce wave velocity as a function of potential at the wave front compared with theory. For the curve the geometrical factor relating measured voltage to electrostatic field has been (simplistically) chosen at the best value of 0.5. Figure 4

Shock Tube Laboratory at the University of Michigan, left Michigan possessed of a richly endowed set of shock apparatus. Laporte saw this as his awaited opportunity, and took up the responsibilities of the laboratory. Fluid dynamicists are familiar with his developments of the shock tube as a supersonic wind tunnel for model studies, and his subsequent application of the high temperatures predicted behind the reflected shock to problems of a spectroscopic nature, providing *f*-values for remarkable materials ranging from mercury to chromium. We know of his work with converging shock waves, with boundary layers, with bifurcated shock reflections, and finally with cryogenic shock tubes. John Yoder (his last student) remarked that it must have done his heart good to complete his efforts in fluid dynamics with a shock tube situation dominated by a quantum effect.

In 1952 I too joined the Division of Fluid Dynamics, having recognized, with a large measure of Laporte's help, that the peculiar phenomena my group were observing in the expansion of sudden electrical discharges were caused by shock waves, and so were amenable to relatively simple analysis.

Precursors

When it had been established, in 1960, that the expansion of the entire gas of impulsive electrical discharges was the result of the extraordinary electron pressures generated within the gas (atmospheres, in fact) there remained a phenomenon we had regarded as minor, and had set aside for study in the remote future, if at all. This peculiarity was the detectable ionization that always seemed to precede the ordinary shock front and to arrive at the point of observation instantaneously when viewed with apparatus having microsecond resolving power. Fortunately many able researchers were more interested in this than we, and gave this problem and similar problems serious attention. They called these things "precursors." I had a special opportunity to observe the work of Richard Hales and Vernal Josephson closely, and it became clear that in the *electrical* shock tube, they were seeing a wave phenomenon traveling at speeds about one hundredth of the speed of light. This was, however, much faster than the gas could be expected to move.

I should explain why I regarded the phenomenon as trivial. I was reminded from the first of the extensive work in the 1930's by Jesse Beams and his co-workers in Virginia, where waves of this speed and even higher speeds were seen to travel back and forth between the electrodes of discharge tubes in the early stages before the gas broke down into a steady state of conduction. The classical explanation of these waves, given by a large number of distinguished physicists in that field, such as Leonard Loeb and John Meek, was that they were simply a manifestation of individual, in-line, electron accelerations with cumulative ionization from the accompanying inelastic collisions, and this explanation was quite acceptable to me. Moreover, there was strong feeling that the radiation emitted by the gas, under these electron impacts, was the fundamental element in transferring the ionization from one place to another at these high speeds. Radiation probably *does* play a role, but fluid dynamics certainly plays a greater one.

In November 1961 I had the pleasure of listening to a lecture on the plasma oscillations by Oscar Buneman. In discussing these small-amplitude oscillations of the plasma he pointed out that they were really part of a whole family of electron acoustical oscillations in which electron mean thermal speed served as a dimensionalizing factor. It appeared immediately obvious that if these small-amplitude waves had the electron acoustical velocity as a characteristic speed, there might also be large amplitude nonlinear waves for which this same velocity acted to determine a Mach number.

We were at that time engaged in writing the final report to the office of Naval Research of our project on electric shock tubes, and it seemed fitting that we should examine this thought briefly. So Ward Paxton, who had had a long association with the ONR project, undertook to demonstrate that the three-component fluid equations for electrons, ions and neutrals had a solution of shock-wave form in which an electron gas, in the presence of an electric field, permeated and slithered through the neutral gas (plus heavy ions) without accelerating them appreciably, something like a fluid in a porous medium. Paxton found that the reason the solution had been overlooked in previous attempts was that one tended to say that the mass of the ions, differing only as it does from the mass of the neutral molecules by the mass of one mere electron, could be set equal to the mass of the neutral molecule in the initial equations *before* manipulation. Paxton's solution showed reasonable agreement with the experimental evidence available at the time, but showed conclusively that one could not expect to make any progress on the theory of these fast waves without considerably improved data.

In the decade since that time, our own and others' interest in the phenomenon has grown apace. Albert Haberstich and Jan Burgers, in 1963, at the University of Maryland, began an investigation on the electric shock-tube precursor. They quickly changed it into an investigation of the Beams breakdown wave. Haberstich remarked that the precursor appeared to him to be nothing else than a breakdown wave, but this observation was somehow lost in the impact of the results they achieved. Burgers took the Paxton theory and greatly improved the formulation so that the structure of an electron fluid wave might now be approached.

At Cornell, Evan Pugh in 1961 and Moshe Lubin and Edwin Resler in 1966 conducted some remarkable and beautiful measurements and observations on the electric shock-tube precursor *per se*, and Lubin offered a theory that the precursor was a plasma wave coupled to an electromagnetic wave travelling along a loaded line, something like a self-extending antenna. Although he found excellent agreement between his theory and his experiments, there are self-destructive drawbacks to the Lubin theory. First, the magnetic field does not couple with the electric field in the wave at this speed within a factor of 10^6. Second, he chose a phenomenological equation of electron production containing just the requisite factors to make it fit the experimental results, rather than taking a fundamental collision-production equation. Thus the agreement begs the question.

The exciting progress made by the Cornell and Maryland groups and their apparent lack of intention of continuing with further research stimulated us to renew our research program, on both theoretical and experimental fronts.

Electrons as a fluid

If the electron gas can be treated as a fluid, it is certainly a very *remarkable* fluid. On one side of an electron shock front, for example, it does not exist, but immediately on the other side it has come into existence by being liberated from the neutral atoms. Fields from elsewhere interact powerfully with it, so quickly altering the electron's kinetic energy and temperature. The ability of the electron gas to reproduce itself is proportional to the amount to which it already exists in any given volume. When present in small numbers, say, below a concentration of approximately 10^9 electrons/cm^3, it behaves as a group of individuals going about their own business in that fashion classically presumed by people in the field of gaseous electronics. However, in excess of this concentration, it behaves as a fluid for a very brief instant, during which its kinetic energy cannot be neglected. At the same time, its pressure cannot be neglected, because one must explain the fundamental fact that the wave propagates equally well in a forward direction whether the electric field is *with* the electrons or *against* them.

We call those waves in which the electric force is in the direction of the wave motion *proforce* waves, and those in which the two directions are opposed *antiforce* waves. George Shelton began theoretical research on this problem with the equations laid down by Burgers, but with one essential alteration: Burgers had assumed that the production of electrons was proportional to the mean relative velocity of the electrons and the gas through which they were moving. This ignores the fact that the electron motions are flights of accelerated motion, and that the electron random thermal motion contributes preponderantly to the production of ionization also. It is a better approximation, although still not perfect, to assume that the electron production is proportional to the mean speed of the electrons and with a coefficient that is also a function of their temperature.

Shelton assumed that a one-dimensional theory was adequate for such a wave, and also that a frame of reference existed in which a time-independent solution could be found for its profile. He began, therefore, with these four equations:

$$\frac{dnv}{dz} = \beta n$$

$$\frac{d}{dz}(mnv^2 + nkT_e) = -enE + K_1mn(v - V) + \beta mnV$$

$$\frac{d}{dz}\left(mnv^2 + M_iN_iV_i^2 + MNV^2 + nkT_e + Nkt - \frac{\epsilon_0E^2}{2}\right) = 0$$

$$\frac{d}{dz}(mnv^3 + M_iN_iV_i^3 + MNV^3 + 5nvkT_e + 5NVkT + 2e\phi_inv) = 0$$

The first is for the particle balance, the rate of change of particle current being proportional to the number of electrons present. The ionizing frequency β is a complex function of the temperature. The second is for momentum balance and includes a viscous term and a term for the momentum that must be supplied to the newly created electrons. The third equation is for the global momentum balance: ions, electrons and neutrals. The fourth is for global energy balance. The global equations of energy and momentum respectively can be integrated across the shock discontinuity and the constants can be evaluated at some known place such as in the region of rest in front of the wave. Troublesome terms remain that represent the small amounts of energy and momentum acquired by the ions, measured by $V_i - V$, and the small amounts of pressure and thermal energy acquired by the heavy particles, measured by $T - T_0$. The electron gas is continually losing energy against the neutral atoms, and the ions are being accelerated by the field. One term is usually greater than the other, and which loss mode dominates is controlled by the relative sizes of the electric field and the electron temperature. In his analysis of steady proforce waves, Shelton assumed that ion motion is the significant factor, so he eliminated $V_i - V$, set $T = T_0$ and obtained the following third equation:

$$mnv(v - V)^2 + (5v - 2V)nkT_e + 2e\phi_inv + \frac{\epsilon_0}{2}(E^2 - E_0^2)V = 0$$

The equation set is completed by Poisson's equation for the electric field. Poisson's equation is, of course, that the divergence of E relates to the charge density *difference* between positive and negative charges. Now the three-component fluid present has three equations of particle balance, one for electrons, one for ions, and one for the neutrals. Out of these one can obtain immediately an important result termed the "zero-current condition." Because there are no charges in front of the wave and since the electric field there is static, the total current there must be zero, and so it must be zero everywhere. This result implies that in the wave the convection current of the electrons is equal to the convection current of ions; in other words, $nv = N_iV_i$. Substituting this result to eliminate N_i from the equation of Poisson, one has:

$$\frac{dE}{dz} = \frac{en}{\epsilon_0}\left(\frac{v}{V_i} - 1\right)$$

To a first approximation, adequate for proforce waves, the ion velocity V_i cannot be distinguished from the neutral velocity, which in turn is equal to the wave velocity.

If a shock-like interface exists between the neutral gas and the ionized gas, as proposed by Paxton, then two Rankine–Hugoniot type shock conditions must be met:

$$n_1\left[v_1(v_1 - V) + \frac{k(T_e)_1}{m}\right] = 0$$

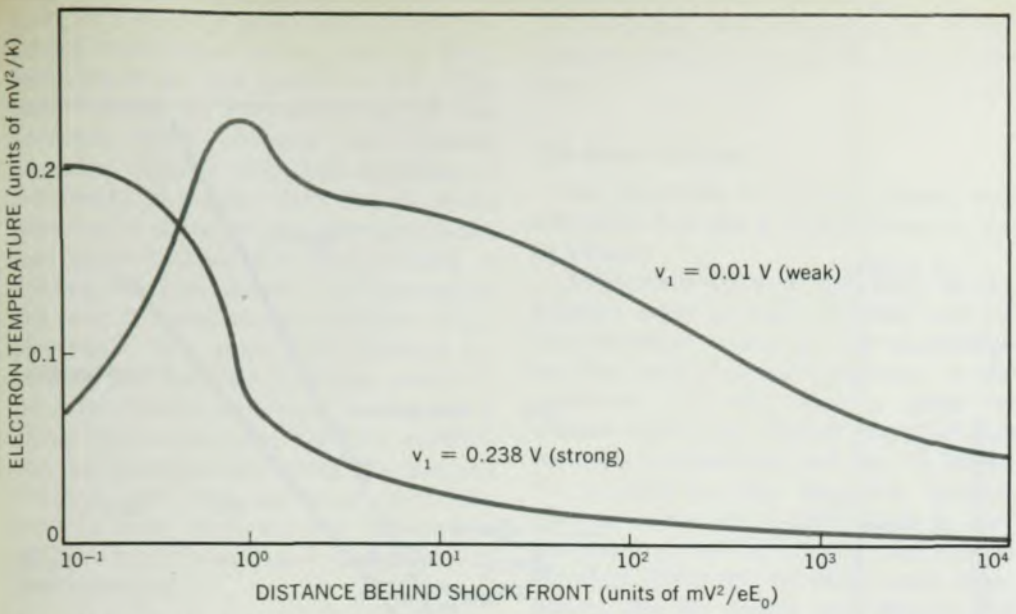
$$n_1v_1\left[v_1^2 - V^2 + \frac{5k(T_e)_1}{m} + \frac{2e\phi_i}{m}\right] = 0$$

There are three ways by which these conditions can be met. They are: First, a conditional solution in which the electron concentration n_1 behind the front and the electron velocity v_1 behind the front are not zero. Then

$$v_1 = \frac{5V}{8} - \frac{[9V^2 + 16(2e\phi_i/m)]^{1/2}}{8}$$

Second, there are two unconditional solutions, one in which the electron concentration is zero with a discontinuity in velocity and temperature permissible, and another in which the electron velocity is zero and the electron temperature is also zero, but there may be a discontinuity in electron density.

Paxton had arrived at similar equations, but had omitted the term $2e\phi_i$ for the energy of ionization of the ionized states. This term appears unequivocally if there is *no* ionization ahead of the



Temperature profiles for both a strong shock and a weak shock in a proforce wave, shown here as samples of the calculations that have been performed. Figure 5

front. However, if there *is* ionization ahead of the front, this term is absent. When the ionization term is present, and the conditional solution implying existence of a shock is in force, one has the additional fact that not all solutions are possible. The electron velocity v_1 can never be directed toward the front but must always be directed away from it. This limitation produces the requirement that $mV^2/2$, the energy of an electron moving at the velocity of the wave itself, must be greater than $e\phi_i$, the ionization energy, a result first remarked on by Shelton and which we term the "Shelton minimum condition."

Solution technique

The essential element of Shelton's solution of these equations was to observe that the four dependent variables, n, T, E, and v are not all equally important in the four equations. Field and velocity are closely coupled and undergo marked variations with respect to each other and with respect to the distance. Temperature and density undergo much slower variations that take place over a much larger span of distance. In fact, the distance variation over which n and T vary is approximately 1000 times that over which the electric field E and electron velocity v vary. This difference is not surprising considering the agility with which an electron responds to an electric field.

Shelton, therefore, divided his solution into three regions: a preshock neutral region, a postshock sheath region in which v and E were to undergo prompt and extensive variation, and a *quasineutral* or *thermal* region in which the principal adjustments of temperature and electron density would take place. In a one-dimensional wave, if there is no current in front of the wave it is also not possible (without horrendous singularities) to have a current behind the wave. Therefore, the electric field must go to zero behind the wave. Shelton chose the point at which the field goes to zero as defining the interface between the sheath and the thermal region. He also assumed that the responsiveness of the electron flow velocity to electric field at this point would be lost and therefore that the flow velocity of the electrons would become equal to the velocity of the wave, or, in other words, in the laboratory system the electrons would come to rest. This leads to the result:

$$n(2e\phi_i + 3kT_e) = \epsilon_0 E_0^2$$

which is all that remains of the energy equation when mechanical motion ceases. Since the temperature of the electrons must ultimately drop to that of the neutral gas in the absence of the field, $kT_e/e\phi_i$ will approach a small value, and therefore the solution at infinity will be $n_f = \epsilon_0 E_0^2/2e\phi_i$. Expressed in terms of the physical variables this result means that all of the electrostatic energy at the front has become ionization or, in other words, that the mixture of electric-field-plus-gas in advance of the wave has been detonated and has been left in the form of ionized plasma. This is an idealized result that ignores excitation and elastic losses. With the above result, the continuity equation can now be solved in such a way as to find the relation between position and temperature:

$$z = \frac{V}{\beta_0}\left[f\left(\frac{e\phi_i}{kT}\right) - f\left(\frac{e\phi_i}{kT_2}\right)\right]$$

The function $f(u)$ is merely the result of integrating the velocity dependence of the cross section for ion production over the Maxwell distribution of velocities. β_0 is an ionization frequency constant, which in helium is 5.2×10^8 p sec^{-1}, with pressure p expressed in torr. T_2 is the value of temperature at the interface between the sheath and thermal region and serves as a boundary condition on that side for solutions in the sheath region, to which we now turn. The sheath is the site of real fluid behavior of the electrons.

The technique of solution in the sheath region is to eliminate position z as an independent variable and to replace it with either the electric field E, or the electron current $j = nv$, as occasion demands. The requirements of the various solutions are then best understood in terms of the geometry of the (v,E) plane, which is shown in figure 3. The initial position of the undisturbed gas is at $v = V$, $E = E_0$. After passage through the shock the gas finds itself at the point $v = v_1$, $E = E_0$ where v_1 lies between zero and $V/4$ because of the Shelton minimum condition. Next the gas must undergo transition to the point $v = V$, $E = 0$, which is the terminus of the sheath. Proforce waves make this transition along a descending path, usually monotonic. Comparison of several such paths and a computer solution has shown only moderate differences. At the terminus, we must fulfill the conditions that since $v = V$, $E = 0$ and $dE/dx = 0$, the spatial approach to the terminus must be parabolic, and it is especially easy then to assume that the whole path is parabolic:

$$E = E_0\left(1 - \frac{v}{V}\right)^2$$

A solution for shock-fronted proforce waves can now be obtained. One divides the continuity equation by the Poisson equation, to obtain a simple

Symbols

a	tube radius
b	ion mobility
β	ionization frequency
E	electric field
E_0	initial electric field
e	electron charge
ϵ_0	permittivity of free space
k	Boltzmann's constant
m	electron mass
M	heavy particle mass
M_i	positive ion mass
n	electron number density
N	neutral particle number density
N_i	positive ion number density
p	gas density (in standard torr)
ϕ	experimentally applied voltage
ϕ_i	ionization potential
T	heavy particle temperature
v	electron fluid velocity
V	heavy particle fluid velocity
V_i	positive ion fluid velocity

relation between j, E and v. This can be integrated under the assumed relation between E and v, to obtain

$$j = nv = \frac{2\beta\epsilon_0}{e\left(\frac{v_1}{V} - 1\right)}(EE_0)^{1/2} + n_2V$$

Returning now to the energy equation we have an opportunity to determine some of these constants, because the critical point $v = 2V/5$ lies within the domain between $v = V$ and $v = v_1$. Consequently, at this point the coefficient of T_e goes to zero. This would create a singularity in T_e if the remaining terms did not also go to zero. Therefore, one can fix a value of j at this point; let us call it j^*.

$$j^* = \frac{1 - 81/625\left(1 - \frac{v_1}{V}\right)^4}{1 + 9mV^2/50\,e\phi_i}$$

The four equations for the problem really contain five, rather than four, dependent variables. The fifth dependent variable is the wave velocity V. The evaluation of V is done by taking the momentum equation and dividing it by the continuity equation, whereupon it can be integrated using the path relation. This procedure leads to the expression

$$\frac{K_1m}{eE_0}\cdot V = \frac{2}{3\left(1 - \frac{v_1}{V}\right)} + \frac{n_2kT_e}{\epsilon_0E_0^2}$$

The general agreement with experiment that wave profiles of this type provide is shown in figure 4.

Finally, using these solutions, one can invert the continuity equation and integrate z itself to find the position as a function of the variable j. The symbol B stands for the incomplete beta function.

$$\frac{\beta}{V}\cdot z = \log n_2V/nv + \left(1 - \frac{v_1}{V}\right)\left(\frac{n_1v_1}{n_2V - n_1v_1}\right)B_{\left(\frac{1-nv}{n_2V}\right)}(2,0)$$

The technique here is very much like that regularly used in shock-structure theory. It is novel in that the self-consistent field E can be brought under the same umbrella. Out of this expression one can calculate all the necessary parameters and eventually connect the position z to the variation of the other variables E, v and T_e.

As a sample of the profile calculations we have given the temperature profile for both a strong shock and a weak shock over both the sheath and thermal regions in figure 5.

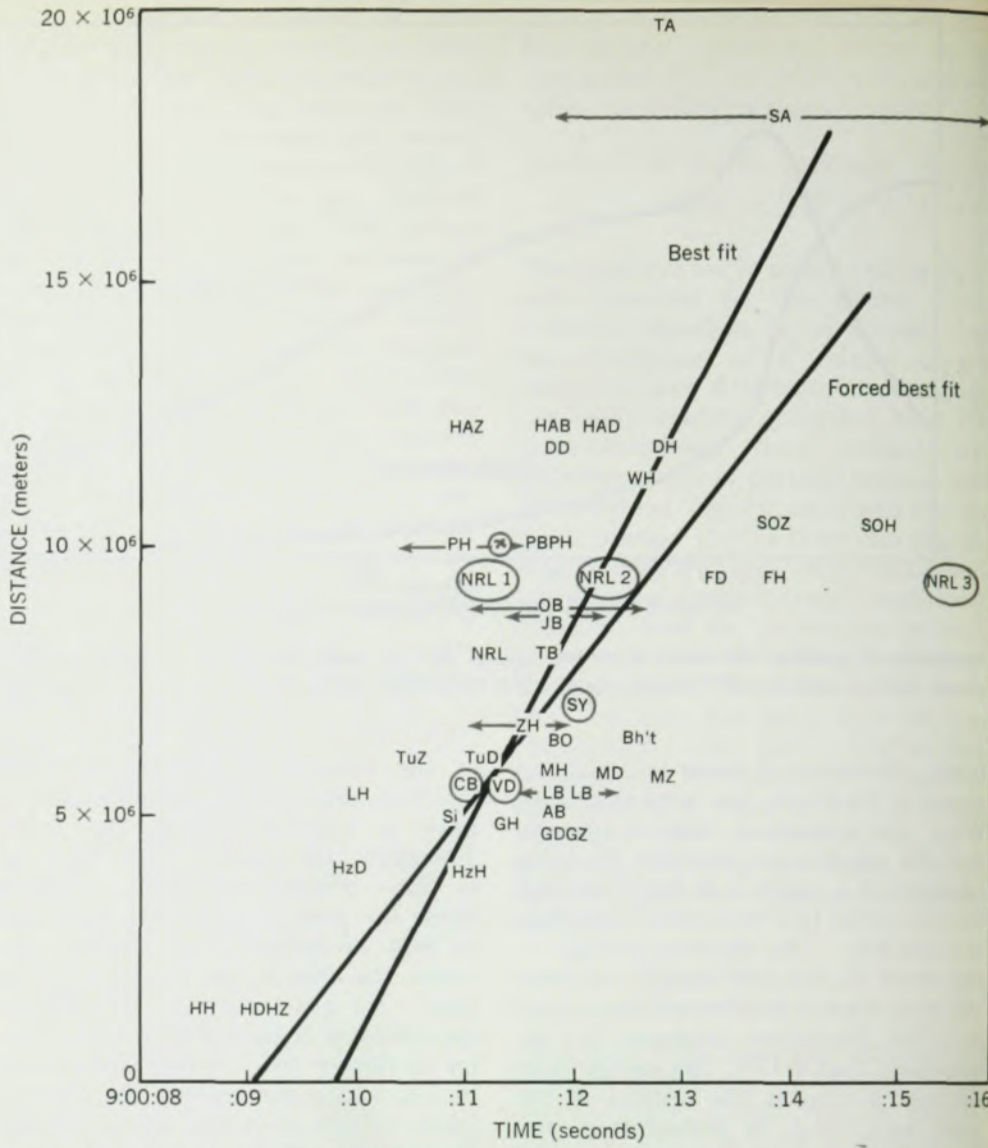
Primary and secondary waves

The proforce wave with a density shock at the leading edge is, however, only one out of four or five waves that are now recognized. We saw, for example, that waves are observed with velocities below the minimum velocity at which a fully shock-fronted wave can exist. Those waves are, in fact, not exactly the same as those above the Shelton minimum, but have now been clearly distinguished experimentally as one of two kinds of proforce waves. They exist, as the data show, primarily at high pressures. In addition to these waves there are the well known antiforce waves, where the wave runs opposite to the electric force on the electrons. If we call those waves in which the medium is not ionized in front of the wave "primary," then there are also "secondary" waves where the medium in front of the wave is ionized but the zero-current condition still applies. And then there is at least one more class of waves in which the zero-current condition is removed also. We will call these "tertiary" waves. In lightning the primary waves are called "leaders," while both the secondary and tertiary waves are called "return strokes."

The primary waves were discovered by Beams, investigated first by Haberstich and recently by Blais. The secondary waves were first seen by J. J. Thomson in 1893, were subsequently investigated by Beams, and most recently by Russell Westburg and by William Winn. The tertiary waves, as far as I know, have only been investigated in the laboratory by John Barach and John Sivinsky.

Let me add a word or two about possible theoretical models for some of these other waves. A group working with Derek Tidman at Maryland has recently dealt with a solution of the equations for the non-shock-fronted case, using a more complex ion-production expression than single electron-impact ionization. They use the electric field as the dependent variable, and find solutions for both proforce and antiforce waves. They assert that the solution for antiforce waves is only possible if the ion production includes photoionization. I am not at present prepared to concede this point, in spite of its popular appeal. The volume of experimental evidence is not great enough to consider it proven, and there still appears to be a distinct possibility that the Shelton method will yield solutions in which ion production is wholly by electron collision. This is not to say that photoprocesses may not have strong secondary effects, however.

Returning to figure 3, we see that



Unexpectedly quick magnetic responses received around the world from the July 1962 Johnstone Island hydrogen bomb test above the ionosphere. Does the data best fit a line something like those shown, or a vertical line through 11.5 sec? Figure 6 there is a solution path for primary antiforce waves that starts upward from some point on the line $E = E_0$. The path cannot go downward as in the proforce wave because the Poisson equation coupled with the equation of continuity demands that the slope be positive as it leaves the starting point, and that it shall not change until it crosses the line where the velocity of the ions is equal to the velocity of the electrons. We must now however introduce the ion velocity more carefully into the Poisson equation, because to a closer approximation, the ion velocity *can* be distinguished from the neutral velocity, and then one must introduce for V_i a term involving $b\,E$ (where b is the ion mobility): as is shown in the next equation.

$$\frac{dE}{dz} = \frac{ne}{\epsilon_0}\left(\frac{v}{V + bE} - 1\right)$$

This correction term is exceedingly small and wholly negligible in many cases, but if it were not present, it would not be possible for this equation to change its slope until the path crossed the line $v = V$, and then having crossed that line, the path would be in an energetically forbidden region. Immediately after crossing the $v - V = bE$ line, the path must turn sharply down and must cross both the line $E = E_0$ and the line $v = V$ inside the point (E_0, V), and must then continue on with the electrons over-speeding the wave up to some terminal point. At the present moment there are various options for both the initial and terminal points.

Part of the difficulty in deciding these options at present rests on the fact that the experimental situation of proforce waves is very complicated, and we now know for certain that there are at least two such waves—and we are not sure that there are not more. Whatever is said of these proforce slow waves is probably also true of the antiforce waves, and their starting point from the $E = E_0$ axis is probably determined in the same way. But experimentally, at present, there appears to be only one kind of antiforce wave, and it seems to be observed equally as well below the Shelton minimum as above.

The secondary waves differ from the primary waves only in that the initial shock condition is removed, and that the wave may begin at any point along the $E = E_0$ axis between $v = 0$ and $v = V$. Its terminal point will still be subject to the same requirements as before. It is thus probable that there are secondary antiforce waves as well as proforce waves. The tertiary waves are distinguished by the existence of a terminal line other than the line $v = V$. It is moved to the left by an amount equal to the electron current in the wave and the terminal point of the tertiary wave will be at the foot of this line.

The observations

Let us return to the laboratory, and also take a glance at a phenomenon out in nature.

Breakdown wave studies with an apparatus such as that of Blais will be the ultimate source of our knowledge in this field, but investigation of the precursor (as was recently done by James Mills and Masud Naraghi) has been an interesting problem in itself. To summarize the situation briefly: The precursor is neither what it was supposed to be by myself and James Hood in 1962, nor by Hales and Josephson, nor by Lubin and Resler, nor precisely what it was supposed to be by Haberstich, although his conclusion came closest. The precursor is indeed a breakdown wave; that is to say it cannot exist without a breakdown wave to lead it, but it receives perhaps 90% of its energy via some form of nonradiant transfer from the hot driver plasma of the shock tube. Geoffrey Russell showed that the energy transfer was nonradiant by contrasting a side arm with the main expansion tube. The process is thus either heat conduction or the conduction of potential as in the Thomson effect.

The source of the potential that drives the precursor as a breakdown wave is itself remarkable. It arises by induction from the enormous current-rise rate in the plasma discharge, which is on the order of 10^{11} or 10^{12} amperes per second. It enters, via 10^{-7} to 10^{-9} henry of mutual inductance, into the ground circuit of most apparatus. The outer electrode, which is supposed to be at ground potential, rises above ground potential and serves as a conducting electrode for the breakdown wave, abetted by the wealth of electrons in the driver plasma connected to it. If extreme care is taken to arrange the apparatus so that the outer electrode of the shock tube is connected integrally to the shielding surrounding the entire power supply and discharge circuit, the breakdown wave vanishes completely, and so does the precursor.

Finally, I cannot resist mentioning a phenomenon observed around the world following the explosion on 7 July 1962, of the Johnston Island hydrogen bomb at 400-kilometers altitude. The data in figure 6 indicate magnetic effects experienced at various points over the entire face of the earth. The great argument is whether the clustering of data we see represents a world-wide instant occurring at light speed a constant interval of two seconds after bomb zero, or whether, as I maintain, a progressive phenomenon moved out at 10^6 m/sec from time zero or at 10^7 m/sec after a brief delay during which the fireball expansion made an electrical connection between two ionospheric layers. I like one of the latter explanations, because this wave would have had the speed of an electron fluid wave, and since it is well known that the bomb explosion developed a tremendous vertical potential difference resulting from Compton scattering of the β-rays. It is frequently objected that this propagation could not have occurred because of the magnetic field of the earth, which would have deflected the electrons, and that the wave is therefore some form of magnetohydrodynamic wave. Magnetic fields do not have, however, any substantial effect upon breakdown waves, as we know both from theory and experiment. The breakdown wave represents only the locus at which new electrons are being produced by old electrons and the old electrons are left behind a very short distance from the point at which they are produced. Consequently, the motion is not the advance of an electron beam, which would indeed be deflected by a magnetic field. I feel, therefore, that a breakdown wave travelled through the E layer of the earth outward from a center at Johnston Island to remote parts of the earth.

The foregoing may or may not have convinced the reader that electrons are a remarkable fluid substance requiring treatment by the equations of fluid dynamics, but if it has not, I will have served my purpose if I have called attention to the fact that very recently one of the most widely cultured and finest minds of our age has passed, as quietly and unobtrusively as it lived, beyond the ken of its many friends. Otto Laporte added new dimensions to my life. It is pleasant to feel that the founding of the Laporte lecture series by the Division of Fluid Dynamics will do this for others in years yet to come.

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