The optical model at high energies

Recent study of field-theory models is bringing the optical model back into fashion for high-energy particle scattering. With it we may obtain new insight to nucleon substructure.

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For more than two decades physicists have used the optical model to describe the high-energy scattering of nucleons by nuclei. The motivation underlying the model both in nuclear scattering and in particle scattering is to represent the complex projectile–target interaction by a two-body complex potential. This makes the optical-model description somewhat analogous to the propagation of light through a refracting and absorbing medium. The success of such a description in high-energy nucleon–nucleus scattering is now well established.

In particle physics, however, the optical model fell into disrepute early in its development, because V. N. Gribov showed in 1961 that the usual optical-model amplitude conflicted with unitarity in the crossed channel. This difficulty soon came to be known as the "Gribov disease," and the optical model was left, so to speak, stricken with this disease, while theorists pursued spiritedly the Regge-pole ideas.

By 1967 it was clearly established that aside from poles (Regge poles) there are branch cuts (Regge cuts) in the angular-momentum plane. This indicated that the Gribov disease does not really occur in Nature, because such cuts prevent the analytic continuation to the crossed channel assumed by Gribov. So the optical model regained its viability in high-energy particle scattering. Of course, during this period the model was kept alive by Robert Serber's phenomenological observation that it described remarkably well the precipitous fall of high-energy large-angle proton–proton scattering (figure 1), by the work of C. N. Yang and his collaborators, and by the efforts of those who found the natural emergence from the optical model of a geometrical picture of high-energy scattering physically appealing.

In this article we first outline how field-theory models can lead to the optical-model description and consider a number of examples to illustrate various predictions. Next we make some observations regarding the Regge optical model. This is followed by the droplet model, in which hadron scattering is pictured as one Lorentz-contracted "pancake" passing through another. After that we discuss the nucleon substructure as indicated by optical-model descriptions, and finally we take a look into the crystal ball.

Field-theoretic models

In the last few years many studies have been done on the high-energy behavior of infinite sets of Feynman diagrams in quantum electrodynamics and in a number of field-theory models.[1] These investigations have shown that, at high energy, the elastic-scattering amplitude $T(s,t)$, which is taken as the sum of an infinite set of Feynman diagrams, can assume the form

$$T(s,t) = \frac{is}{4\pi}\int d^2b e^{i\mathbf{q}\cdot\mathbf{b}}\left[1-e^{i\chi(b)}\right] \quad (1)$$

where b is the impact parameter and $-t = \mathbf{q}^2$ is the square of the momentum transfer. Here, "high energy" means $s \to \infty$, where s is the square of the center-of-mass energy. The function $\chi(b)$, which is called the "eikonal," is determined by some basic amplitude, the iteration of which generates the corresponding infinite set of Feynman diagrams. The relation between this basic amplitude (henceforth denoted by $T_1(s,t)$ and called the "optical Born amplitude") and the eikonal is

$$\chi(b) = \frac{1}{\pi s}\int d^2q e^{-i\mathbf{q}\cdot\mathbf{b}}T_1(s,t) \quad (2)$$

Different field-theory models differ essentially in their input amplitude $T_1(s,t)$, which, through equation 1, determines the corresponding elastic-scattering amplitude, the total cross section, the elastic and inelastic cross sections and the behavior of the diffraction peak. For a purely imaginary eikonal ($\chi = i\chi_I$) the formulas for the cross sections take the following simple forms

$$\sigma_{\text{total}} = 4\pi\int_0^\infty bdb\left[1-e^{-\chi_I(b)}\right] \quad (3a)$$

$$\sigma_{\text{elastic}} = 2\pi\int_0^\infty bdb\left[1-e^{-\chi_I(b)}\right]^2 \quad (3b)$$

$$\sigma_{\text{inelastic}} = 2\pi\int_0^\infty bdb\left[1-e^{-2\chi_I(b)}\right] \quad (3c)$$

Equation 1 has been well known to physicists since the original work of Sidney Fernbach, Robert Serber and Theodore Taylor[2] on the scattering of fast neutrons by nuclei. Physically it corresponds to the scattering of a particle by a complex energy-dependent optical potential $V(s,r)$, which determines $\chi(b)$ through the relation

$$\chi(b) = -\frac{1}{s}\int_{-\infty}^{\infty} dz V\left(s, \sqrt{b^2+z^2}\right) \quad (4)$$

Equations 2 and 4 show that $V(s,r)$ is simply the Fourier transform of the amplitude $T_1(s,t)$. Thus, a field-theory calculation that leads to equation 1 always provides an optical-model descrip-

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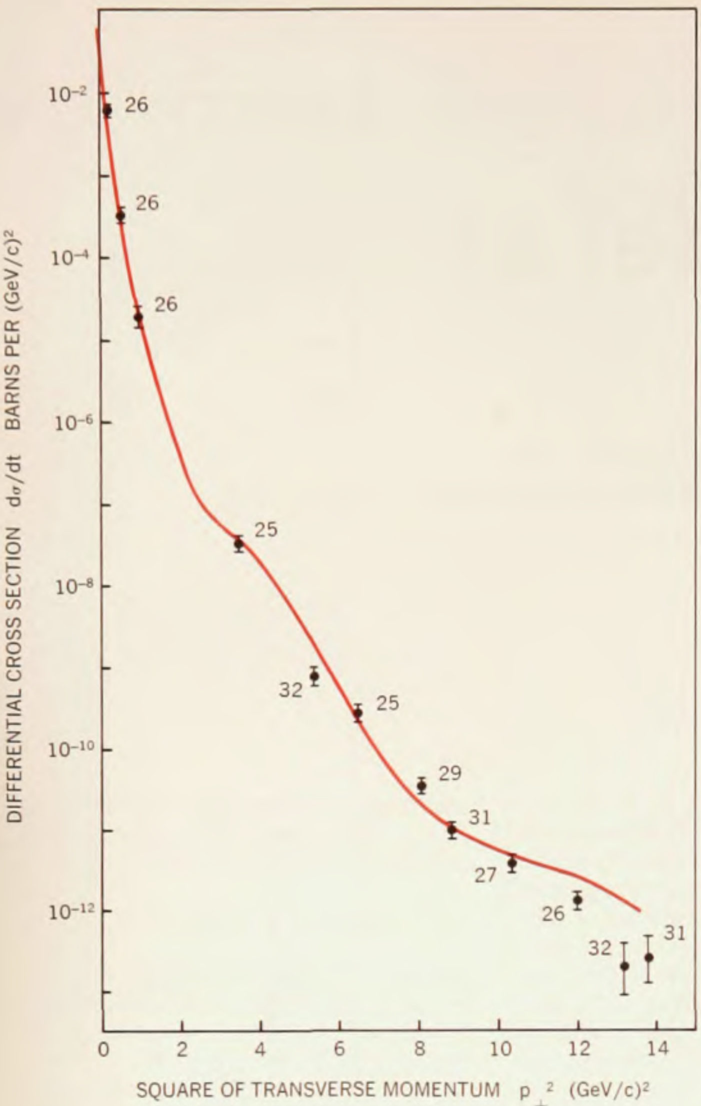
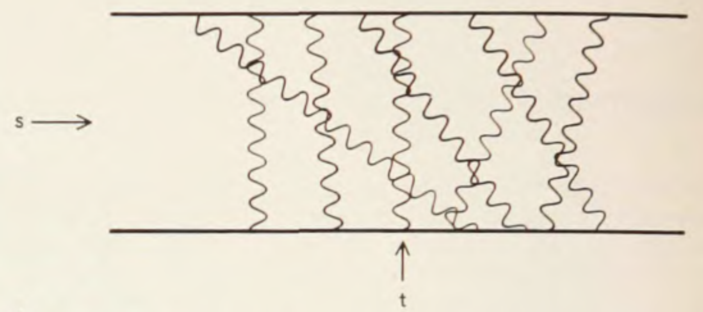


◀ **Proton–proton elastic scattering** at 30 GeV/*c* from an optical-model calculation. This work was done by a student of Robert Serber, Richard Werbin (see his PhD thesis, Columbia University, 1972). Werbin uses a Lorentz-contracted complex optical potential whose radial dependence is derived from the matter distributions of the colliding protons. Following the earlier work of T. T. Chou and C. N. Yang, the hadronic (strongly interacting) matter distribution of a proton is taken to be the same as its electric-charge distribution. The experimental data points, for momenta between 25 and 32 GeV/*c* (as labelled), are from reference 13.
Figure 1



▲
A generalized ladder diagram representing two high-energy nucleons (thick lines) exchanging an arbitrary number of mesons (wavy lines). Such diagrams lead to an optical-model description of high-energy particle scattering. The variables are *s*, the square of the center-of-mass energy, and *-t*, the square of the momentum transfer.
Figure 2

tion. Phenomenologically, equation 1 was used in high-energy particle scattering long before the field-theory models came along. However, what these models have achieved is to provide a relativistic field-theory base for the optical-model description. Furthermore they show that by using the eikonal form, equation 1, for the full amplitude, one is summing over complicated sets of amplitudes iterated to all orders.

Some examples

To obtain an insight into the field-theory models, let us first consider the generalized ladder diagram shown in figure 2, where two spinless "nucleons" are exchanging an arbitrary number of scalar mesons. The optical Born amplitude corresponds to a single meson exchange and leads to the eikonal

$$\chi(b) = \frac{g^2}{4\pi s} K_0(\mu b)$$

$K_0(\mu b)$ is the modified Bessel function that behaves as $e^{-\mu b}$ for large b, and g is the coupling constant. This result is precisely the one obtained from equation 4 if the optical potential is a Yukawa potential $-(g^2/8\pi)e^{-\mu r}/r$. For large s in this case, $\chi(b) \to 0$, $e^{i\chi(b)}-1 \approx i\chi(b)$ and $T(s,t) \approx T_1(s,t)$. This means that the full amplitude approaches the Born amplitude, and as the latter is energy-independent the differential-scattering cross section vanishes at asymptotic energy.

If we now consider a more realistic situation by replacing the spinless "nucleons" with spin-1/2 nucleons and the scalar meson with a massive neutral vector meson, then we gain an extra factor of s in the Born amplitude $T_1(s,t)$ arising from the spin of the meson. This leads to

$$\chi(b) = -\frac{g^2}{2\pi} K_0(\mu b)$$

that is, an eikonal independent of energy. The elastic amplitude, as seen from equation 1, becomes proportional to s. Because the optical theorem gives

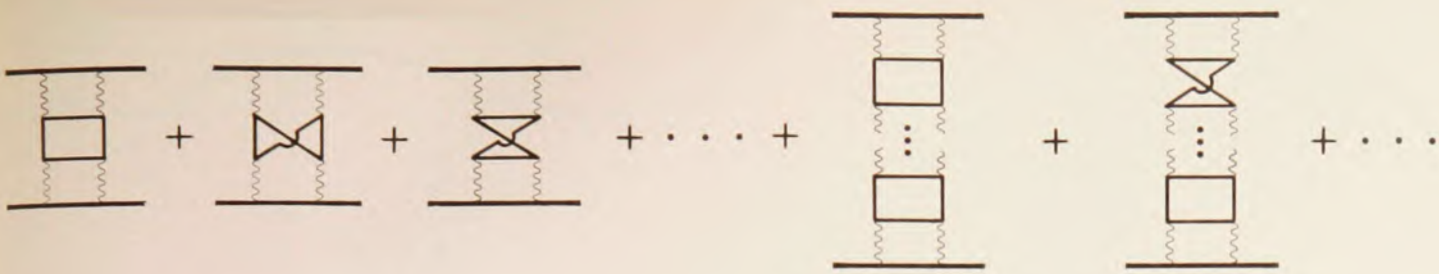
$$\sigma_{\text{total}} = \frac{8\pi}{s}\ \text{Im}\ T(s,0)$$

we obtain a constant total cross section. The inelastic cross section vanishes, since the eikonal is purely real. One point worth noticing in the present case is that, since $\chi(b)$ is energy independent, all the Feynman diagrams corresponding to higher orders in $\chi(b)$ have the same asymptotic behavior. This perhaps reflects an important feature expected of strong interactions; namely, diagrams of higher orders in coupling constants can be equally important in determining asymptotic behavior.

So far we have discussed two of the simplest models—one leading to a vanishing σ_{total}, the other to a constant σ_{total} and a constant diffraction peak. We can generate a wide variety of models by taking more complicated optical Born amplitudes. For example, H. Cheng and T. T. Wu[3] have taken the set of diagrams in figure 3 (which are called "tower" diagrams) as their input amplitude $T_1(s,t)$, while others have taken the ladder diagrams shown in figure 4 as their input amplitude. Cheng and Wu obtain a completely absorptive black-disc picture of high-energy scattering. Consequences of this model for σ_{total}, σ_{elastic} and the diffraction peak can be easily seen from equation 1, with cut-off $b = R$ and $e^{i\chi(b)} \approx 0$. We obtain $\sigma_{\text{total}} = 2\pi R^2$, $\sigma_{\text{elastic}} = \pi R^2$, and

$$\frac{d\sigma}{dt} = \pi R^2 \left[\frac{J_1(R\sqrt{-t})}{\sqrt{-t}}\right]^2 \qquad (5)$$

where J_1 is the Bessel function. Furthermore, if Γ is the value of $|t|$



where the first minimum of the diffraction scattering occurs, then, from equation 5, we obtain $\Gamma\sigma_{\text{total}} = 2\pi\beta^2$; β is the first xero of $J_1(x)$. We observe that if the radius R of the black disc increases like log s, then σ_{total} increases like (log $s)^2$. This is the maximum rate theoretically allowed for the increase of the total cross section with energy. (High-energy physicists among my readers will recognize this as the Froissart bound.) The total and the elastic cross sections in this case are going to be infinitely large asymptotically. Models in which the input amplitude is that given by figure 4 correspond to the optical Born amplitude being taken equal to the single Regge-pole amplitude. If we take this to be the conventional Pomeranchon pole ($\alpha(0) = 1$ and $\alpha' \approx 1(\text{GeV}/c)^{-2}$), then

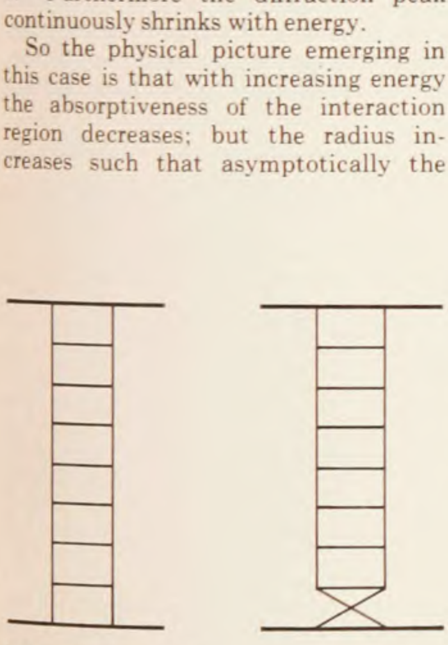
$$T_1(s,t) \approx isCe^{\alpha' t \log s}, \quad (C>0) \qquad (6)$$

and

$$i\chi(b) = -\frac{C}{\alpha' \log s}\, e^{-b^2/(4\alpha' \log s)} \qquad (7)$$

Equation 7 shows that this is a pure absorptive model where the absorptiveness (Im $\chi(b)$) decreases with increase in energy. For s asymptotic, $\chi(b)$ is small, and we obtain from the leading term on the right-hand-side of equation 1 the result σ_{total} = constant, $\sigma_{\text{elastic}} \propto 1/\log s$. Furthermore the diffraction peak continuously shrinks with energy.

So the physical picture emerging in this case is that with increasing energy the absorptiveness of the interaction region decreases; but the radius increases such that asymptotically the total cross section remains finite while the elastic cross section vanishes. Obviously, by using a different input amplitude, we have obtained results totally different from the previous case.



Ladder diagrams. These represent a single Regge-pole exchange. The number of rungs, summed, is infinite.
Figure 4

Recent experimental results from the CERN Intersecting Storage Rings have shown that even at 500 GeV/c lab momentum the proton–proton cross section is not very much different from its value around 25 GeV/c lab momentum.[4] This result argues against the Cheng–Wu model, where the total cross section increases like (log $s)^2$. Also, measurements of proton–proton small-angle elastic scattering show no shrinkage of the diffraction peak at lab moment 500 GeV/c and 1100 GeV/c,[5] whereas the lower-energy data from Serpukhov[6] indicated such a shrinkage. Thus we see that an input amplitude given by the Pomeranchon pole alone predicting continuous shrinkage is not satisfactory, either.

Regge model

A few additional remarks are worthwhile regarding the "Regge optical model" (in which the Regge-pole amplitude determines the optical potential). This model has been given wide phenomenological consideration.[7] Richard Arnold conjectured from Ronald Torgerson's field-theoretic work that the Regge-pole amplitude may be used as the optical potential. Soon it was realized that if a Regge-pole amplitude is taken as the optical Born amplitude, then the higher-order terms in the multiple scattering series can be interpreted as Regge cuts arising from Regge-pole exchanges. These cuts have the same branch-point positions and the same asymptotic form as deduced from Feynman-diagram models and from the continuation of the multiparticle unitarity equation[8] in the complex j-plane.

To get some feeling on this point let us examine the double-scattering term in equation 1. If the single scattering is due to a Regge-pole amplitude, then $T_1(s,t)$ is of the form $\gamma(t)s^{\alpha(t)}$, and the double-scattering term is

$$T_2(s,t) = \frac{i}{2\pi s}\int d^2q_1\gamma(-\mathbf{q}_1{}^2)\gamma(-|\mathbf{q}-\mathbf{q}_1|^2)\, s^{\alpha(-\mathbf{q}_1{}^2)+\alpha(-|\mathbf{q}-\mathbf{q}_1|^2)} \qquad (8)$$

This corresponds precisely to the Mandelstam cut diagram due to two Regge-pole exchange. Steven Frautschi and Bernard Margolis[9] have applied this model for proton–proton elastic scattering, predicting not only the diffraction but also the large-momentum-transfer cross sections at future accelerator energies. One interesting feature of the model is that the multiple-scattering terms add up to give an exponential dependence of the form exp $(-b\sqrt{-t})$ to the differential-scattering cross section for large $-t$. From a pure Regge theory, of course, one can prove neither why one should generate Regge cuts using the eikonal representation, nor how these cuts correspond to the nonplanar diagrams (not lying in a plane; the lines cross without intersecting) that asymptotically become dominant. However, field-theoretic calculations indicate that the simple eikonal model may provide the full asymptotic amplitude.[10]

Tower diagrams. This set of diagrams corresponds to the optical Born amplitude in the Cheng–Wu calculation.
Figure 3

Droplet model

From a phenomenological viewpoint we may regard the eikonal amplitude as an adequate high-energy representation of the actual scattering amplitude, which tells us where the physical idea about the interaction should be put in—namely through $T_1(s,t)$ or $V(s,r)$. We may therefore conceive of an optical potential dictated by some physical assumptions, and use the eikonal representation to obtain the corresponding amplitude. Taking this view Yang and his collaborators[11] have regarded hadrons as droplets and envisaged the scattering of two hadrons as the passage of one Lorentz-contracted hadronic disc through another (in the center-of-mass system), and assume that this gives the "opaqueness"

$$-i\chi(b) = K \int d^2b' D(\mathbf{b}')D(\mathbf{b}-\mathbf{b}') \qquad (9)$$

where K is a real constant and $D(\mathbf{b})$ is a two-dimensional density obtained from the three-dimensional hadronic density $\rho(r)$ by integrating along the incident direction. Assuming further that the hadronic matter distribution is the same as the electromagnetic-charge distribu-

Interaction regions of the nucleon as envisaged in three different models of high-energy proton–proton scattering: (a) Diffraction model, where the same dynamical mechanism is responsible for small and large momentum transfer scattering; (b) Serber's model where small momentum transfer scattering is due to diffraction, while large momentum transfer scattering is due to a localized singularity at the center, and (c) Nucellar model, where small momentum transfer scattering is also due to diffraction, but large momentum transfer scattering is due to the existence of two inner cores (nucelli) of the nucleon. Figure 5

tion, T. T. Chou and Yang[11] have obtained the asymptotic proton–proton differential cross section predicted by this model. Because the constant K in equation 9 is energy independent, this model leads to a constant σ_{total}, a constant diffraction peak and a finite σ_{elastic}. For large momentum transfers the model predicts sharp zeros in the differential cross section. While the zeros are filled in at finite energy because of spin effects and the existence of a complex $\chi(b)$ instead of a pure imaginary $\chi(b)$, they should become more prominent as the energy increases. It should be possible to test predictions of this model and those of other large momentum-transfer models in the near future at the 76-GeV Serpukhov proton synchrotron, at the 1500-GeV CERN intersecting storage rings, and at the 500-GeV National Accelerator at Batavia.

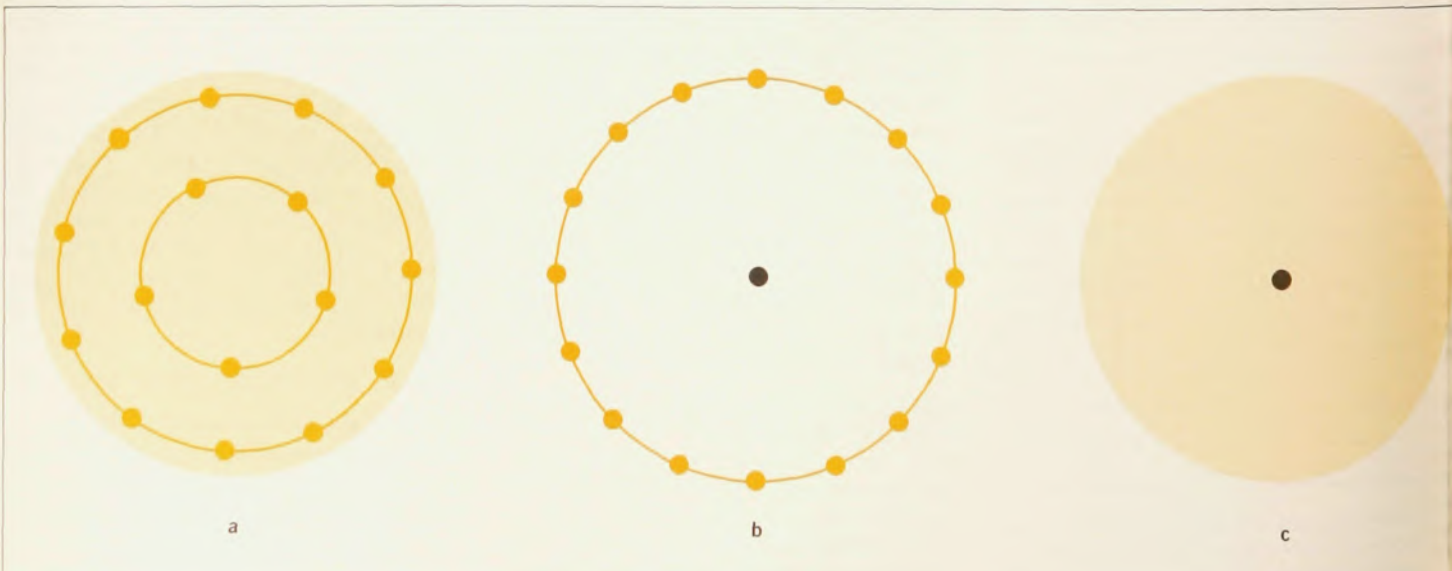
Nucleon substructure

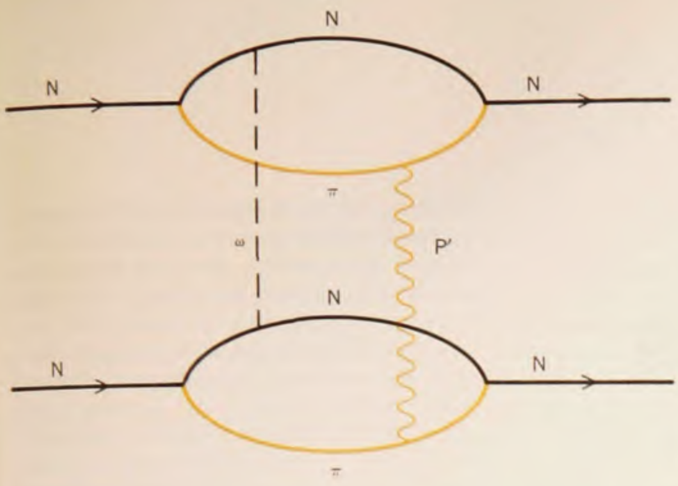
Both the Chou–Yang and Frautschi–Margolis models for elastic proton–proton scattering use the same dynamical input for diffraction and large-momentum-transfer scattering. On the other hand, the original optical model of Serber[12] makes a distinction between the diffraction region and the large-angle scattering region. In this model the diffraction scattering is due to a purely absorptive Gaussian potential (whose strength and range are adjusted to fit the observed diffraction peak), while the large-angle scattering is due to a purely absorptive Yukawa potential. It has been found that the present extensive data[13] on large-momentum-transfer proton–proton scattering cannot be satisfactorily explained in this way.[14] However, it is interesting to note that the interaction region of the proton as envisaged in Serber's model (figure 5b) is different from that of a "diffraction model" (figure 5a), where the same dynamical mechanism is used for diffraction and large-angle scattering.

Joe Rosen and I have put forward another optical model[15] that makes a distinction between the diffraction region and the large-momentum-transfer region ($|t| > 1\ (\mathrm{GeV}/c)^2$). In our model the diffraction region is parametrized, while the large-angle scattering is considered due to optical potentials whose radial dependences are of smoothed Yukawa form

$$e^{-\mu(r^2+\beta^2)^{1/2}}\big/(r^2+\beta^2)^{1/2}$$

The physical interpretation of the parameter β is that it represents a finite size of the nucleon-matter distribution, as opposed to Serber's $e^{-\mu r}/r$, which corresponds to a δ-function core distribution ($\beta \rightarrow 0$). In fact we found two finite-size core distributions for the proton (figure 5c) necessary to fit the data. These cores were identified as





Mandelstam cut diagram with a Regge pole (P′) and a single particle (ω) exchange. Solid black lines represent the nucleons and the smooth colored curves the pions.
Figure 6

interacting via exchanges of the vector mesons ω and ω'. Essentially what we have here is a model of the nucleon with two inner cores or nucelli and with each core or nucellus being probed by a photon-like hadronic quantum. (*Nucellus*, plural *nucelli*, means the central part of an ovule. I am using it to designate a nucleon core.) Since the mesons ω and ω' couple to a neutron and a proton alike, but do not couple to a pion, the large $|t|$ neutron-proton and proton-proton cross sections will be equal, and appreciably different from those of the pion-proton collision.

A natural theoretical question arises at this point. How does one interpret a phenomenological optical potential of the form

$$g(s)e^{-\mu(r^2 + \beta^2)^{1/2}}/(r^2 + \beta^2)^{1/2}$$

from the viewpoint of S-matrix theory? The answer may have been found only recently.[16] The idea is that such an optical potential can arise from a Mandelstam cut diagram of the type shown in figure 6. This diagram describes the following physical process: A high-energy nucleon breaks up into a "core nucleon" and a "cloud pion"; the core nucleon interacts with the core nucleon of the other incoming nucleon with the exchange of a vector meson (say ω); the cloud pion interacts with the other cloud pion via exchange of a Regge pole (say P′), and finally the nucleons absorb the respective cloud pions and emerge as outgoing nucleons. Apart from providing a Feynman-type diagram for the origin of the optical potential, this interpretation removes another limitation of the optical model—namely, the introduction of an unknown complex energy dependence $g(s)$. A diagram like figure 6 shows that the complex energy dependence is determined by the exchanged Regge pole. From this analysis we see that the S-matrix description provides the detailed mechanism of the process and its asymptotic energy dependence (or, equivalently, j-plane behavior). But we also see that the optical model is revealing the proton substructure by physically relating the observed momentum-transfer dependence with the hadronic core distributions of the proton.

hree models of the atom

. proposed at the turn of the century. In the irst one (a), by J. J. Thomson, the atom is a phere of positive charge with electrons mbedded in it and arranged in rings. The econd one (b) is by the Japanese physicist lantaro Nagaoka, who pictured the atom as onsisting of a central positive charge urrounded by a single ring of electrons. The hird model (c) is the atom of Ernest Rutherford, n which a central point charge is surrounded by pposite charge distributed throughout the emainder of the volume.

If we compare these models of the atom with hose of the nucleon shown in figure 5, we are truck by the close parallelism between the resent search for determining the substructure f the nucleon with the historic establishment of he substructure of the atom. Very likely the nner structure of the nucleon will be established efore the end of the 1970's.

What next?

The present surge in studying field-theory models to gain insight into high-energy scattering should be continued, probably with more emphasis on large-momentum-transfer scattering. However, answers to questions such as: "Is the nucleon granular[17] or nucellar or both?" can only be obtained by probing deeply the nucleon itself. To this end it will perhaps be greatly worthwhile to look experimentally into high-energy large-angle elastic scattering in the new accelerators. After all, it was a foresighted experimentalist who, by looking at the high-energy large-angle scattering of his time, discovered the substructure of the atom,[18] while other physicists in the field were puzzling over cross sections in the forward direction. One wonders—will history repeat itself?

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