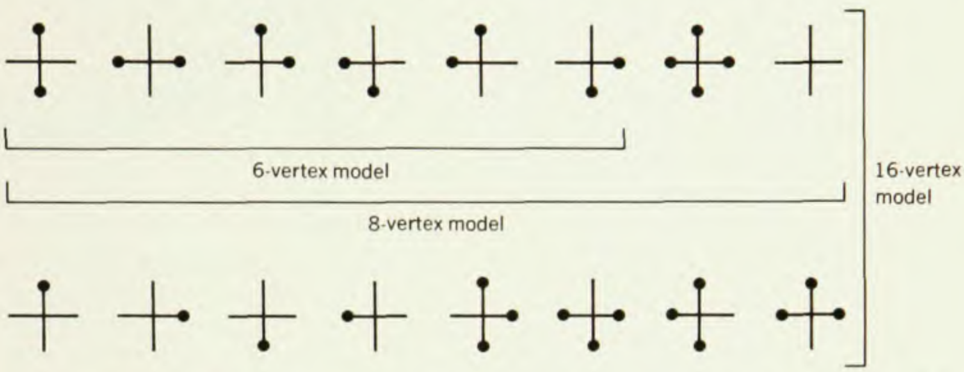
search & discovery

Solution of eight-vertex model excites critical-point theorists

A difficult problem in two-dimensional statistical mechanics has been solved by R. J. Baxter of the Australian National University in Canberra (*Phys. Rev. Lett.* **26**, 832, 1971). The so-called "eight-vertex" model that he solved contains as special cases the square lattice Ising, dimer, ice, F and KDP (potassium dihydrogen phosphate) models. Baxter's elegant and beautiful solution has challenged one of the widely held precepts of critical-point theory, namely that the character of the critical behavior is independent of the substance provided the symmetry properties of the Hamiltonian are the same.



Sixteen-vertex problem involves all the ways you can arrange zero through four dots around a point. The eight-vertex problem allows only even numbers of dots. The six-vertex problem does not allow zero and four dots. R. J. Baxter has solved the eight-vertex model, which contains as special cases the square lattice Ising, dimer, ice, F and KDP models.

In a companion paper,[1] Baxter develops a solution of the general case of the Heisenberg linear chain of spins, a chain with unequal couplings in the x, y and z directions.

The ice problem uses a six-vertex model, in which you think of each oxygen vertex of the ice lattice as having four nearest-neighbor oxygen atoms. There are six ways of arranging two hydrogen atoms around each oxygen atom. In the figure the dots represent the hydrogen atoms. In the ice problem all the hydrogen configurations or vertices have the same energy. On the other hand, in the problem for ferroelectrics or antiferroelectrics, the different configurations have different values for energy.

The eight-vertex problem allows zero and four dots. Besides representing hydrogen bonds, these dots can stand for any physical quantity, such as spin, that exists at the interstices of a lattice. An even more general problem is the 16-vertex problem, which involves all the ways you can arrange zero through four dots around a point. In the eight-vertex case you allow only an even number of dots around each vertex. The dots are arranged at the corners of a square, N-by-N lattice. One assigns energies $\epsilon_1, \ldots \epsilon_8$ to the vertex configurations and the problem is to evaluate the partition function

$$Z = \sum \exp\left(-\sum_{j=1}^{8} N_j \epsilon_j / k_B T\right)$$

where the summation is over all allowed configurations of dots on the lattice, N_j is the number of vertices of type j, k_B is the Boltzmann constant and T is absolute temperature.

History. Most crystals essentially do not have much entropy left. But when the entropy of ice was first measured, it was found that it had a lot of residual entropy left, that is, a lot of residual disorder. Linus Pauling proposed a model for ice where the hydrogen atoms essentially formed the bonds between the oxygens in the water molecules. Each oxygen would have two nearby hydrogens and two far-away hydrogens. You then have a combinatorial puzzle, the number of ways of doing the arranging, which measures the entropy. Then John F. Nagle (Carnegie-Mellon

continued on page 19

ISR experiment shows scaling behavior

One of the first experiments to be done at the new CERN Intersecting Storage Rings has verified certain scaling laws developed by Richard Feynman (Cal Tech) and by C. N. Yang (State University of New York at Stony Brook) and his collaborators. The experiment was just reported by Lazarus G. Ratner (Argonne), Richard J. Ellis and Gianni Vannini (University of Bologna), Bryce A. Babcock, Alan D. Krisch and Jabus B. Roberts (University of Michigan) in the 5 July issue of *Physical Review Letters.*

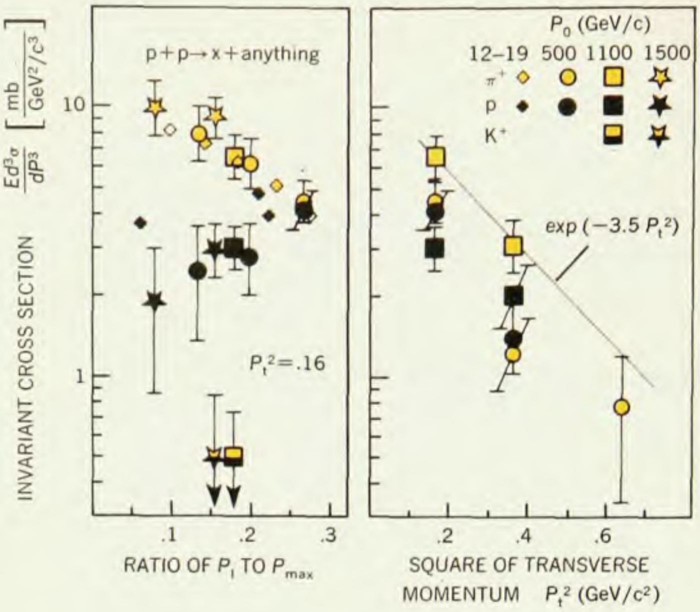
The group studied inelastic inclusive processes, that is reactions in which

$$p + p \rightarrow \pi^+ + \text{anything}$$
$$p + p \rightarrow p + \text{anything}$$
$$p + p \rightarrow K^+ + \text{anything}$$

They varied the incident proton momentum in each ring between 15.3 and 26.5 GeV/c; the equivalent laboratory momentum is 500–1500 GeV/c. Each ring was filled to an intensity of about 4×10^{13} protons, and then the beams were coasted for 6 to 12 hours with a loss rate that was often as low as 0.1% per hour. Vacuum was maintained at 5×10^{-11} torr.

A 45-meter-long spectrometer detected outgoing particles. Two threshold gas Cerenkov counters were used in coincidence and anticoincidence to tag the particles as pions, kaons or protons.

One might guess that the single-particle distribution observed in the ISR experiment would depend on the total energy of the reaction, P_l (the component of the momentum of the produced particle parallel to the initial direction) and P_t (the component of the produced par-



Invariant cross section vs. $X = P_l/P_{max}$ with P_t^2 held fixed, and vs. P_t^2 with X held fixed. In the plot for X each of the three kinds of particles show a universal function of X, independent of energy.

ticle perpendicular to the initial direction).

The idea of scaling promulgated by Feynman[1] and Yang and his collaborators[2] says that the cross section approaches a limiting behavior independent of the energy of the collision and depending only on P_t^2 and on X, the ratio of P_l to the square root of the collision energy, or more precisely, $X = P_l/P_{max}$. What might have been expected to be another parameter, namely the total energy, disappears.

The invariant cross section plotted by the ISR experimenters is $Ed^3\sigma/dP^3$; dP^3/E can be thought of as the differential of the fraction of the energy in the collision that is on the detected particle. The figure includes data from one-particle distribution experiments at Argonne, Brookhaven and CERN,[3-6] which in some cases have been interpolated to match the ISR group's values of P_t and X. For each of the three kinds of particles one can see that there is a universal function of X, which for pions, for example, resembles a parabola bending down. The curve is independent of initial bombarding energy, whether it be 12, 500 or 1500 GeV.

Krisch points out that it is only the parameter $Ed^3\sigma/dP^3$ that is invariant. The ISR data show that at about 1000 GeV the cross section $d^3\sigma/dP^3$ is ten times smaller than at 10 GeV.

The experimenters say that in the P_t^2 plot the pion and proton cross sections both drop roughly as exp $(-3.5\ P_t^2)$.

At each bombarding energy the experimenters are now investigating larger and smaller values of X, to extend the range beyond that reported in *Physical Review Letters.*

What connection does the scaling behavior found at the ISR have with the scaling observed in deep inelastic electron-proton scattering at SLAC? The connection is not very apparent to those theorists with whom we discussed the ISR experiment.

Feynman motivates his description of high-energy scattering behavior by using a particular distribution of partons. Yang says that at high energies when one hadron hits another, how the target or projectile hadron breaks up into a spray of secondaries is independent of what hits it, and is independent of how hard the hadron is hit. Such behavior he calls "limiting fragmentation." GBL

References

1. R. P. Feynman, Phys. Rev. Lett. **23**, 1415 (1969).
2. J. Benecke, T. T. Chou, C. N. Yang, E. Yen, Phys. Rev. **188**, 2159 (1969); T. T. Chou, C. N. Yang, Phys. Rev. Lett. **25**, 1072 (1970).
3. E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan Jr, R. M. Edelstein, N. C. Hien, T. J. McMahon, I. Nadelhaft, Phys. Rev. Lett. **16**, 855 (1966); **19**, 198 (1967).
4. L. G. Ratner, K. W. Edwards, C. W. Akerlof, D. G. Crabb, J. L. Day, A. D. Krisch, M. T. Lin, Phys. Rev. Lett. **18**, 1218 (1967); Phys. Rev. **D3**, 645 (1971).
5. D. Dekkers, J. A. Geibel, R. Mermod, G. Weber, T. R. Willitts, K. Winter, B. Jordan, M. Vivargent, N. M. King, E. J. N. Wilson, Phys. Rev. **137**, B962 (1965).
6. J. V. Allaby, F. Binon, A. N. Diddens, T. Duteil, A. Klovning, R. Meunier, J. P. Peigndux, E. J. Sacharidis, K. Schlupman, M. Spighel, J. P. Stroot, A. M. Thorndike, A. M. Wetherell, in *Proceedings* of the 14th International Conference on High-Energy Physics, Vienna 1968, J. Prentki, J. Steinberger, eds, CERN-Geneva (1968).

Chain of pellets transfers charge in electrostatic accelerator

A new way of transferring charge in an electrostatic accelerator has been developed by the National Electrostatics Corp in Middleton, Wisconsin. Instead of using the traditional Van de Graaff belt-charging system, the "Pelletron" system consists of an endless chain of metal pellets joined by solid links of insulating material such as molded nylon; the pellets carry charge to the high-voltage terminal. Other charge-transfer devices used in electrostatic machines of the past include the rotating glass disc of the Wimshurst machine and the rotating plastic cylinder of the SAMES machines. Pelletron machines offer improved dependability and voltage stability over belt-charged electrostatic machines, according to Raymond G. Herb, president of the corporation.

National Electrostatics is manufacturing Pelletron accelerators. One is a machine with 14 MV on terminal, which will produce 28-MeV protons at the Australian National University at Canberra.

Herb told us that pellet-chain charging evolved from work going on at the University of Wisconsin since 1932 on high-voltage accelerators for nuclear-physics applications. He and Dorr Ralph built a belt whose metal charge carriers consisted of paper staples fastened in an array over the insulating surface of the belt. In the 1950's a wide variety of charge-carrying arrangements were tried by several of Herb's students, with major contributions from Victor Fung. Later James Ferry developed the string-of-beads charge carrier, in which metal pellets were held by a nylon cord consisting of twisted fish line. This technique was used by A. Isoya (Kyusha University, Japan) in a large accelerator he built at Fukuoka, Japan.

The string of beads developed at Wisconsin evolved into the pellet chain (see figure) at National Electrostatics; the chain passes over a 30-cm-diameter pulley. Each metal cylinder is charged by an induction electrode as it leaves the conducting rim of the pulley.

Herb said that voltage fluctuations or ripple are commonly assumed to be introduced by discrete units of charge deposited at the terminal by the car-