Quantum nature of superfluid helium

The macroscopic behavior that we describe as "superfluidity" is traceable to the microscopic, quantum-mechanical properties of liquid helium.

Seth J. Putterman and Isadore Rudnick

Low-temperature physics owes its existence to liquid helium, the refrigerant that permits investigation of phenomena occurring at a few degrees Kelvin. But liquid helium is fascinating in its own right, and the goal of understanding its "superfluidity" below 2.17 K has stimulated both experimenters and theorists.

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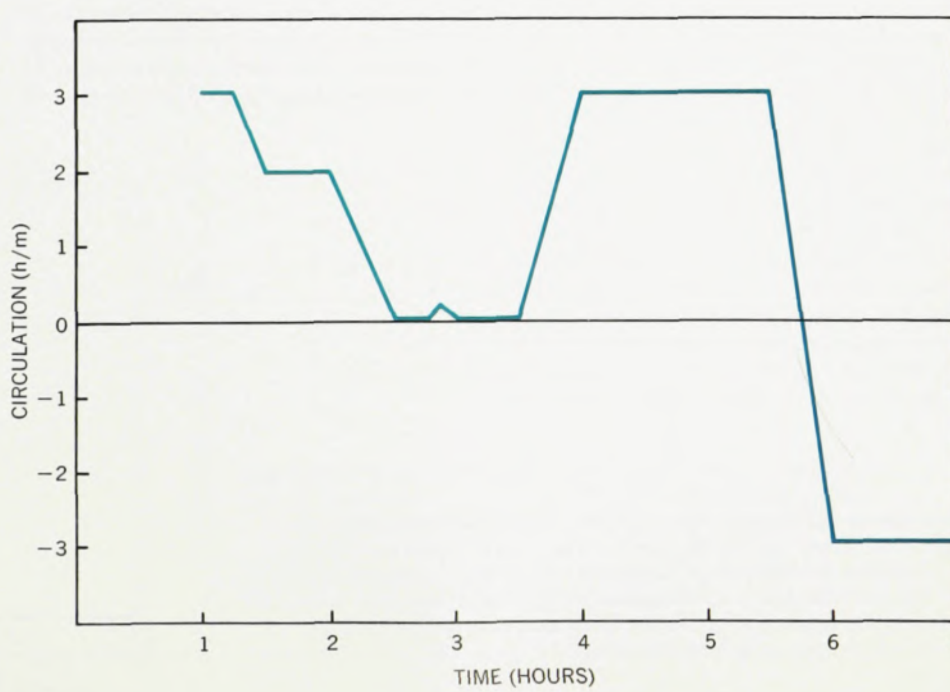
The outstanding fact about superfluid helium, He^4, is that it is a Bose–Einstein quantum liquid, a liquid whose remarkable properties can be directly traced to quantum behavior on a massive scale. Two other systems can be similarly described—the Fermi liquid He^3 and the fermion system of electrons resulting in superconductivity. Superfluid helium deserves to be singled out because of its exotic phenomenology, and because fundamental unanswered questions remain.

Our attention here shall center on the quantum nature of the liquid and the macroscopic properties related to this quantum nature. A group of selected significant experiments will provide the focus for our phenomenological treatment. Our aim is not only to map the past, but also to provide some guide for the future logical development of an understanding of this remarkable liquid.

Early developments

As the temperature of liquid helium is lowered below 2.17 K its dynamical properties change drastically. In 1938 Peter Kapitsa and John Allen and A. D. Misener[1] found that at these low tem-



Quantized circulation. Schematic diagram of data from Stephen Whitmore and William Zimmermann shows that the circulation κ of superfluid helium has only quantized values.
Figure 1

peratures liquid helium could flow through extremely narrow constrictions (less than 10^{-4} cm) with no measurable resistance. The observation led Kapitsa to refer to the liquid as a "superfluid." From this experiment we might guess that liquid helium at temperatures below 2.17 K (commonly referred to as helium II) simply behaved like an inviscid Navier–Stokes fluid. The failure of such reasoning became apparent when W. H. Keesom and George MacWood[2] studied the behavior of an oscillating disc immersed in helium II; the damping of the disc's motion led them to conclude that the viscosity had a finite value.

To Laszlo Tisza and Lev Landau[3] these experiments brought out the need for a fundamentally new "hydrodynamic" description of He II. The "two-fluid" theory was first presented by Tisza (1938) and later (1941) extensively developed by Landau. This phenomenological theory pictures He II as consisting of two interpenetrating fluids, the superfluid and normal fluid, that move with independent velocities and without mutual interactions. The superfluid flows without friction and carries no entropy; the normal fluid carries all the entropy, experiences viscous stresses and for the most part behaves like an ordinary Navier–Stokes fluid. It is now clear that when we study superflow we are measuring the frictionless flow of the superfluid (the normal fluid will be held back by its viscous interaction with the narrow walls), and that Keesom and MacWood's oscillating disc could not avoid contact with the normal fluid, which then sticks to the disc and damps out its motion.

Superfluid helium is called a quantum fluid because attempts to understand its properties from the microscopic or atomic point of view must start with Schrödinger's equation and not Newton's laws. This necessity becomes clear if we realize that the thermal de Broglie wavelength $h(mkT)^{-1/2}$ of the individual helium atoms is comparable at these low temperatures with the interatomic spacing. It is crucially important, for example, that helium atoms obey Bose statistics. However, the Landau two-fluid theory, which has been extremely successful in describing the macroscopic behavior of He II, does not contain Planck's constant, so that from the strictly phenomenological viewpoint we might argue that the theory is just as classical as, say, the Navier–Stokes equations.

Onsager-Feynman condition

This situation changed when Lars Onsager (1948) and Richard Feynman (1955)[4,5] proposed a macroscopic wave function ψ, interpreted analogously to the ordinary quantum mechanics, to describe the superfluid component of He II. Thus for instance $\psi^*\psi$ is taken to be the local superfluid density ρ_s, or, including a possible phase factor ϕ, we have

$$\psi = \rho_s^{1/2} e^{i\phi}$$

where ϕ is real. The macroscopic velocity of the supercomponent $\mathbf{v}_s$ is the same function of ψ as is the probability current in ordinary quantum theory

$$\mathbf{v}_s = \hbar/m \nabla\phi \qquad (1)$$

where m is the mass of a helium atom. From equation 1 we obtain immediately the restriction

$$\nabla \times \mathbf{v}_s = 0 \qquad (2)$$

with which Landau always supplemented his theory. Because ψ is single valued we find, in multiply connected geometries, the quantization of circulation (line integral of $\mathbf{v}_s$ around a closed contour)

$$\oint \mathbf{v}_s \cdot dl = nh/m \qquad (3)$$

where n is an integer, which is clearly analogous with the angular-momentum quantization of the usual quantum theory. Finally, again in analogy with the ordinary quantum theory, the wave function vanishes at a boundary

$$\psi \to 0 \quad \text{at a boundary} \qquad (4)$$

Conditions 3 and 4 go beyond the basic Landau theory and are of special interest, because through them (particularly through equation 3) Planck's constant enters the dynamics of the fluid flow. The important theoretical problem suggested by the Onsager–Feynman idea is that of determining the single unified theory (that is, the *macroscopic Schrödinger theory*) that contains the Landau two-fluid theory as well as the quantum ideas in equations 2, 3 and 4. There appears to be little theoretical research in this direction; so we will here concentrate on experimental developments that in recent years have strongly verified the macroscopic quantum restrictions of equations 2, 3 and 4.

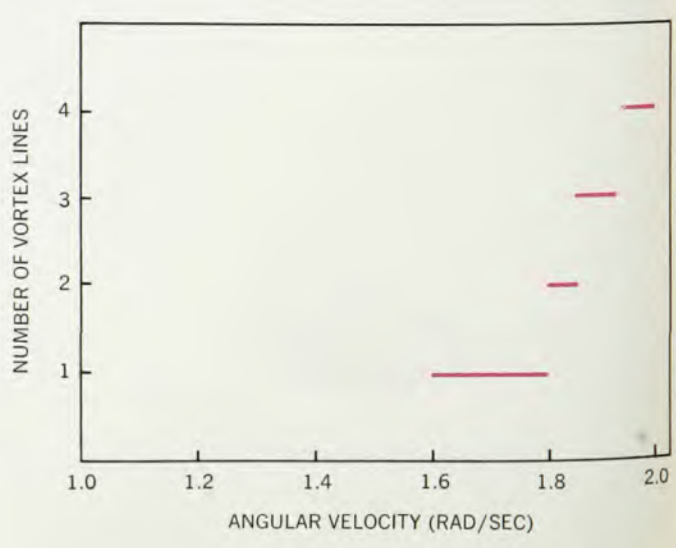
Quantization of circulation

The quantization of circulation (equation 3) is an extraordinary condition: It implies that a careful macroscopic measurement (note that h/m equals 10^{-3} cm^2/sec) of the superfluid velocity can be used to determine Planck's constant. To appreciate the restrictions imposed by the quantization of circulation consider the case where there is a rotation of a fluid about a straight nonsuperfluid core, that is, a linear vortex in the superfluid. The velocity field of a vortex line is, in cylindrical coordinates,

$$\mathbf{v}_s = \kappa/2\pi r\, \mathbf{e}_\theta$$

where the circulation κ is a measure of vortex strength and $\mathbf{e}_\theta$ is a unit axial vector. The vortex line is the singular region at $r = 0$ that extends along the z-axis. In a classical liquid κ can assume any value, but in He II the quantum restrictions imply that the circulation of any vortex line must be given by nh/m. The number of macroscopic states that can be realized is limited by the macroscopic quantum restrictions!

William F. Vinen was first to observe this effect experimentally in 1961; he measured the force on a wire around which there was a superfluid circulation.[6] This work has been recently repeated in a more elaborate form by S. C. Whitmore and William Zimmermann.[7] According to hydrodynamics, a force is exerted on a moving object around which there is a circulation of fluid. The force is perpendicular to the plane of the circulation vector (determined by the right-hand rule) and to the velocity of the object relative to the



Number of vortex lines in a sample of He II is determined in an experiment by Richard Packard and Michael Sanders. Their results are obtained by collecting the electrons captured by the vortices. The vortex lines appear (and vanish) approximately at the angular speeds required by the quantum conditions, but there are strong metastability effects.
Figure 2

fluid far away from it; this is the Magnus effect. Thus, if a wire with circulation around it is made to vibrate, the Magnus force will act to change the plane of vibration of the wire. The vibration plane is experimentally observed by measuring the emf induced in the wire as it cuts across the lines of force of an externally imposed magnetic field. The value of the circulation that is found by this method obeys the quantum condition of equation 3 most of the time. Certainly only the quantized values were found to be stable. Observed values of n ranged from -3 to $+3$; figure 1 is a schematic diagram of their results for the time variation of the circulation.

We have the clearest evidence for the quantization of circulation in the fundamental experiments of George Rayfield and Frederick Reif,[8] who measured the energy and velocity of free charged ions in He II. At low temperature they found the unexpected result that the greater the energy of the ion, the smaller its velocity. This unusual behavior is just what one expects from a vortex ring of a given circulation. (A vortex ring is a vortex line that closes on itself.) If Rayfield and Reif assumed that the ions became trapped on vortex rings, then their data indicated that the circulation of the rings was h/m, within experimental accuracy.

When a vessel containing He II is brought into rotation, the motion of the superfluid component will have to be quite different from that of an ordinary fluid because of the quantum restrictions. The superfluid, like an ordinary fluid, will tend toward a state that maximizes the angular momentum for a given kinetic energy. In other words the free energy

$$\int \rho_s \mathbf{v}_s^2/2 - \boldsymbol{\omega}\cdot(\mathbf{r} \times \rho_s \mathbf{v}_s)\, dV = \text{minimum} \qquad (5)$$

where ω is the angular velocity of the vessel containing the He II. The true minimum of equation 5 is achieved by solid-body rotation ($\mathbf{v}_s = \boldsymbol{\omega} \times \mathbf{r}$), but $\mathbf{v}_s = \boldsymbol{\omega} \times \mathbf{r}$ violates the macroscopic quantum restrictions. The solution[9] of equation 5 subject to the quantum restrictions reveals that for each ω there are a finite number of quantized vortex lines in the superfluid, and that for ω less than a critical angular velocity

$$\omega_{cr} = \hbar/mR^2 \log R/a$$

where R is the radius of the cylindrical vessel and a is the radius of the nonsuperfluid core of the vortex line, there are no vortices in the superfluid ($\mathbf{v}_s = 0$). As ω is increased beyond ω_{cr}, first one quantized vortex line will appear (it is on the axis $r = 0$ and extends from the bottom to the top of the vessel) and then for still higher ω, two and so on. In this way the number of vortex lines versus ω displays a quantum step structure that we might call "quantum phase transitions." There are of course other ways that the superfluid could rotate and still obey the quantum restrictions; an example is the quantized vortex sheet model proposed by Fritz London.[10] Experiments suggest, but do not conclusively favor, the vortex line model for superfluid rotation.

The quantum phase transitions were observed by Richard Packard and Michael Sanders.[11] While rotating a vessel containing He II they send in electrons, which are captured by vortex lines. Electrons in helium are in bubbles because the strength of the Pauli exclusion principle does not allow them to occupy the already filled shell. The pressure at the core of a vortex is reduced by the Bernoulli force, and the reduced pressure accounts for the capture of the electron bubbles. Packard and Sanders count the captured electrons by applying a field parallel to the vortex line and measuring the charge accumulated on an electrode. If we assume that each vortex captures the same number of electrons, their method could be used to determine the number of lines present. The results (see figure 2) agree qualitatively with the prediction of equation 5 and the quantum restrictions, but indicate a strong metastability, which may be the quantum analog of the supercooling of a saturated vapor.

Persistent currents

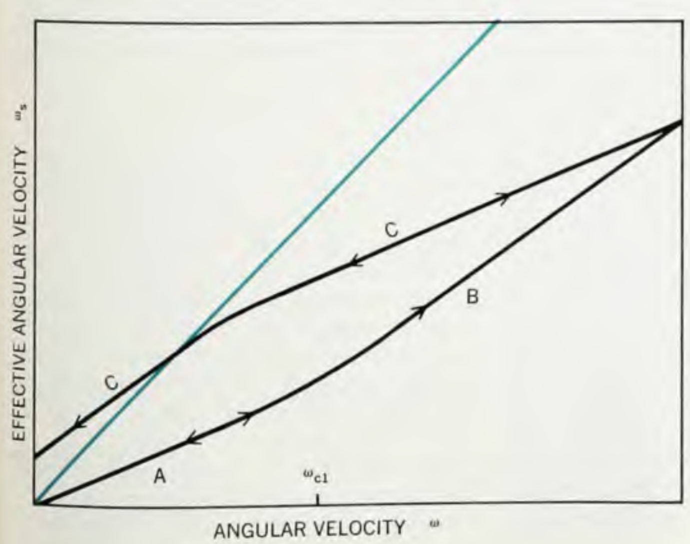
The quantum condition $\nabla \times \mathbf{v}_s = 0$ implies that superflow is something more than frictionless flow. Suppose that liquid helium in the normal state is set into rotation at an angular velocity less than ω_{cr}. Clearly the liquid, like any ordinary viscous liquid, will be in solid-body rotation. If the helium is now cooled below T_λ the superfluid will, according to the quantum restrictions, come to rest ($\mathbf{v}_s = 0$). Were it simply characterized by frictionless flow it would continue in solid-body rotation on cooling through T_λ.

George Hess and William Fairbank[12] have shown that, in the above situation, the superfluid does come to rest; they observed the effect by measuring the angular momentum transferred to the walls of the vessel as the temperature was lowered.

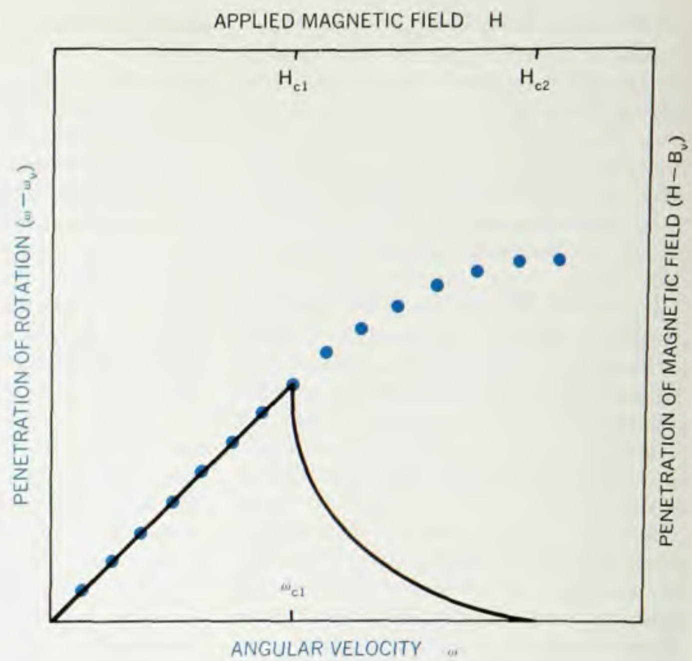
A more subtle effect has not yet been observed. When $\mathbf{v}_s$ equals zero, it is zero only for an observer in an inertial system! Thus He II contained in a vessel at rest in the laboratory will, to a laboratory observer, appear to be in motion. [Note that (24 hours)$^{-1}$ is less than ω_{cr} for reasonable geometries.] This relative motion of container and He II will persist indefinitely, and no external forces are needed to maintain it. We can call this a persistent current without circulation, because no vortex lines are present.

The major experimental difficulty here is posed by the extremely small size of the angular velocities; ω_{cr} is about 2×10^{-3} sec^{-1} for R equal to 1 cm. Very recent experiments have indicated that if He II is contained in the multiply-connected geometry created by packed fine powder (powder grains 10^{-4} to 10^{-6} cm in diameter have been used) the critical angular velocity for the entrance of the first vortex line is increased. Before discussing these experiments, we should consider persistent currents *with* circulation (persistent currents due to vortex lines), which are much easier to observe.

John Reppy and J. Mehl and Zimmermann in 1965 observed that when liquid helium contained in a rotating packed-powder geometry (called a "superleak")



Effective angular velocity of superfluid (ω_s) in a "superleak" shows hysteresis for $\omega \geq \omega_{c1}$. Results are interpreted to indicate that below ω_{c1} no vortex lines are present, that the vortex lines entering the superfluid at ω_{c1} are not destroyed by small counterrotation, and that a residual persistent current, with circulation, remains when the superleak is brought to rest. Line at 45 deg ($\omega = \omega_s$) represents what would happen if there were no relative velocity, as in an ordinary fluid. Figure 3



Superfluidity and superconductivity. Penetration of rotation (colored dots) in a superleak, by formation of quantized vortices above ω_{c1}, can be compared with penetration of an external magnetic field H in a type-II superconductor (solid line) through formation of quantized flux lines above H_{c1}. Below ω_{c1} the Landau state in helium II is analogous to the Meissner state in a superconductor, but there is no evidence that ω_{c2} is finite—it may be effectively infinite. Magnetization curve shown is for a typical reversible type-II superconductor. Many irreversible type-II superconductors exist whose magnetization curves resemble the He-II curve much more closely. The resemblance extends to the hysteresis effects.
Figure 4

above T_λ was cooled and then brought to rest, a substantial residual angular momentum remained in the system,[13] as verified by gyroscopic techniques. These persistent currents are generally accepted as metastable equilibrium states, with cosmic lifetimes, that are caused by the quantized vortices that thread through the powder.

Comparison with superconductors

Superfluids and superconductors, we have long known, possess many similar properties; the persistent mass flows we have been discussing are analogous to the persistent electric currents seen in superconductors. Only through very recent experiments in our lab at UCLA has it become apparent that a much more precise comparison can be made between type-II superconductors and superfluid helium contained in a superleak. Our experiments measure the relative velocity of the container and the superfluid for any angular velocity ω of the container and not only for $\omega = 0$, as in the previously described work. Our studies use the dependence of the speed of the wave mode (called "fourth sound"), that propagates in helium contained in a superleak, on the relative velocity of the container and superfluid; by observing fourth sound with respect to a laboratory rotating with the container we determine the relative velocity of container and superfluid for any ω.

Typical plots of the effective angular velocity ω_s of the superfluid versus ω for He II formed by cooling stationary He I are shown in figure 3. For any history of rotation the state of the system will be uniquely determined by ω so long as $|\omega|$ is less than a critical angular velocity ω_{c1} for the entire history; this region, A in the figure, is the reversible region. As ω increases past ω_{c1} (region B) the slope of the graph changes, and the behavior is no longer reversible; hysteresis sets in. If the container, for example, is now slowed to rest (region C) a residual angular momentum is observed.

How do we interpret these results? Below $\omega_{c1} \simeq 4$ Hz (for R about 2 cm) we claim that no vortex lines are present and that one has a persistent current without circulation. (The work of Mehl and Zimmermann had already suggested that there exists an ω below which we have irrotational and circulation-free flow.) Classical hydrodynamics tells us that a sphere brought into motion in an irrotational inviscid fluid imparts to the fluid half the momentum needed to bring the fluid occupying the same volume as the sphere into the same motion. In these experiments fluid and packed powder occupy roughly the same volume, and, indeed, the line in region A has a slope of about one half! We also expect such an effect to be reversible, as it is. Note that here the system has angular momentum and no circulation. Above ω_{c1} vortex lines enter the superfluid and the slope changes (region B). As we decrease ω we do not retrace curve B but follow curve C. We can take this hysteresis to show that the vortices created by a certain rotation in B are not destroyed by the same counterrotation; in this sense it is easier to create than to destroy the vortices in a superleak. The path in region C is at first parallel to A because as we slow down, the circulation-free component of the motion is changing. As the system is slowed still further, behavior like that in region B may be realized, and, in general, when the superleak is brought to rest ($\omega = 0$) a residual persistent current (with circulation) is observable. This residual persistent current is precisely what Mehl, Zimmermann and Reppy observe.

The Landau state

The comparison with superconductors is clear in figure 4. Superfluid helium tries to exclude the externally imposed rotation, just as a superconductor tries to exclude the externally imposed magnetic field H. Below a certain critical field H_{c1}, surface currents do keep out the external field (Meissner state), but above H_{c1} the external field penetrates the superconductor in the form of quantized flux lines. The magnetic field B_v of the quantized flux lines is clearly equal to zero for H less than H_{c1}. In superfluid helium there are similarly no quantized vortices below ω_{c1}. Thus the effective angular velocity due to quantized vortices (ω_v) is zero in this region. We call this the Landau state in superfluid helium. It is analogous to the Meissner state.

In superconductors, for H less than H_{c1} the external field is excluded by circulation-free surface currents (actually they are currents that run around within a penetration depth λ of about 10^{-4} cm of the surface). In He II, however, the circulation-free currents characteristic of the Landau state ($\omega < \omega_{c1}$) penetrate throughout the volume. This behavior is consistent with the idea that the penetration depth in He II is infinite. Above ω_{c1} quantized vortices penetrate the He II, but instead of $\omega - \omega_v$ dropping to zero (as does $H - B_v$ in superconductors), it appears to saturate at a nonzero value. In a superconductor $H - B_v$ equals zero at a large enough value H_{c2} of the external field; we might have to say that ω_{c2} in He II is effectively infinite.

The presence of quantized flux lines in superconductors has been unquestionably established by experiments that have actually photographed their locations.[14] Several attempts are now underway to photograph the quantized vortices in He II. Packard, Reif and Sanders are trying to accelerate electrons that have been captured by a vortex line into a phosphor screen, so that they can photograph the image. At UCLA we are trying to see directly the accumulation of small solid particle grains that should be captured on the cores of the vortices, like tea leaves in a stirred cup of tea.

Helium vapor forms a film on any surface that comes in contact with it, as do ordinary vapors. However, unlike ordinary films the helium films have superfluid properties, and various attempts are being made to use the films to observe persistent currents. Reppy and his group at Cornell,[15] using gyroscopic techniques, have been able to observe persistent currents for films that coat a superleak. Our group has tried to observe persistent currents in films adsorbed on the outside of a cylindrical block. The experiment consists of cooling the rotating block through T_λ, bringing it to rest and then looking for a relative motion of film and block by investigating the propagation of third sound (the wave mode that propagates in a helium film). Results so far indicate that there is no persistent current. Also when the cylinder is brought from rest into rotation one finds no relative velocity between cylinder and film. Note that a measurable persistent current in this geometry would be one with an enormous quantum number n (see equation 3).

Quantum boundary effects

According to the macroscopic wave-function description of the superfluid, the superfluid density must vanish at the boundary of the helium. One expects this drop in ψ to occur over very short distances, so that boundary effects become important only in very narrow geometries, such as are experimentally achieved in a thin helium film or a superleak with very narrow channels.

In figure 5 we see the behavior of the macroscopic wave function when boundaries must be taken into account. Far from the boundary, ρ_s assumes its "bulk" value, determined by the pressure and temperature through an equation of state. It drops from this bulk value to zero over a distance a called the "healing length."

Experimentally this quantum-mechanical boundary effect is investigated by measuring quantities that are functions of the average amount of superfluid component in the liquid. The square of the speed of the wave-mode fourth sound that propagates in a He-II filled superleak is, for example, proportional to the average amount of superfluid in the superleak: the narrower the channels, the greater the contribution of the boundary effect and, for a given temperature and pressure, the greater the reduction in the average value of ρ_s and hence the speed of fourth sound. The narrowest channels are obtained by packing carbon particles less than 100 Å in diameter into the sound resonator under high pressure (up to 40 000 psi). From the results[16] we can determine a, but rather than describe details of the analysis, which is complicated by the uncertain geometry, we shall turn to another method for which the geometry is well defined.

A surface in contact with the helium vapor at the equilibrium vapor pressure has a film about 100 atomic layers thick form on it. In such a film the depletion in ρ_s due to boundary effects is too small to measure (except near T_λ). Arbitrarily thin films (as thin as an atomic layer), however, can be obtained on surfaces brought into contact with helium vapor *below* the saturated vapor pressure. A surface wave ("third sound") propagates on the helium films. It is quite analogous to the shallow-water gravity waves that travel with the speed

$$v = (gd)^{1/2}$$

where d is the depth of liquid and g the gravitational acceleration. In a film, the van der Waals attraction f between the film and the surface, which is proportional to d^{-4}, plays the role of restoring force and takes the place of g in the above expression. Note that f is many orders of magnitude greater than g. Moreover, because the normal fluid is locked by its viscous interaction with the boundary, only the superfluid participates in the wave motion, accounting for an additional factor of the average value of ρ_s/ρ (denoted by $\langle\rho_s/\rho\rangle$) in the formula for the speed C_3 of third sound

$$C_3 = (\langle\rho_s/\rho\rangle fd)^{1/2}$$

Here ρ is the total fluid density.

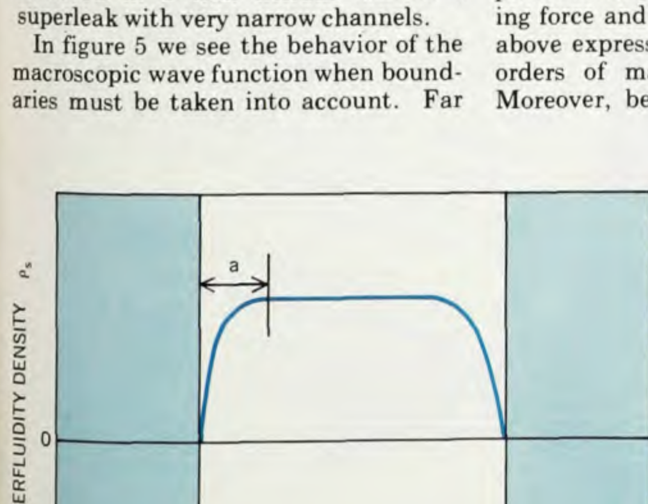
From thermodynamics and the known pressure and van der Waals force, we get d, and hence from measuring the speed of the third sound we determine $\langle\rho_s/\rho\rangle$. This has been done for films as thin as four atomic layers. The results[17] indicate a healing length of about one atomic layer at low temperatures. At higher temperatures the healing length increases, and is about three atomic layers at 2 K. Experiments suggest the empirical relation

$$a \approx 1.5T/\langle\rho_s/\rho\rangle T_\lambda$$

atomic layers for T between 1.0 K and T_λ.

The theory that has been most successful in describing the quantum-mechanical boundary effects is that of Vitali Ginzburg and L. P. Pitaevskii.[18] It is an adaptation of Landau's theory of second-order phase transitions to the He I–He II transition. Their approach is quite controversial, and oddly enough one of the major uncertainties in assigning an exact value to the healing length is lack of agreement on the correct theory to which the experimental results should be matched. Another difficulty in interpreting the experiments is inexact knowledge about the nature of the van der Waals attraction between film and substrate.

Macroscopic measurement on the microscopically thin films (less than 15Å) are surprisingly accurate. One of the intriguing aspects of superfluid physics is the success of macroscopic techniques and ideas in dealing with microscopically small systems, and third sound is one such instance. So far as can be determined, hydrodynamics continues to work Another particularly striking example is provided by Charles Anderson and Edward Sabisky's[19] observation of ordinary



Near a boundary the superfluid density $\rho_s = |\psi|^2$ drops to zero over a distance a called the "healing length." Figure 5

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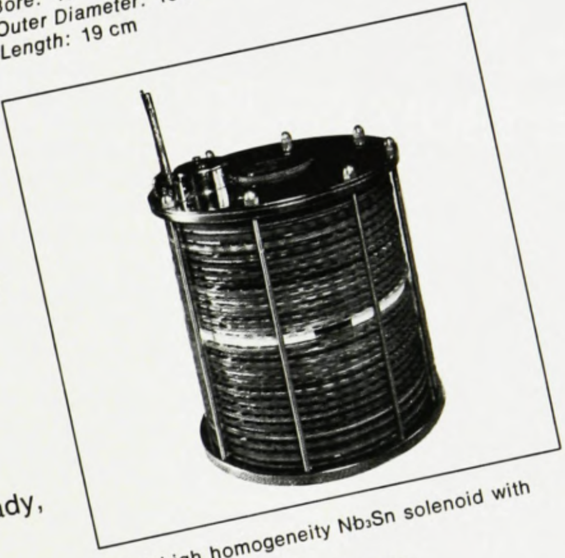

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Independent determinations of the depleted ρ_s and the healing length are available from another direction. We have already mentioned gyroscopic methods for the detection of persistent currents in helium films. These methods can also yield a quantitative measurement of the angular momentum of a persistent current. From measurements of angular momentum versus thickness of film for a given persistent current *for a given* $\mathbf{v}_s$ *field*, we can extrapolate to the thickness at which the mass participating in the persistent current, and therefore the angular momentum, vanishes. This thickness is roughly twice a.

Crucial to the interpretation of these results is that, as we change the thickness of a film undergoing a persistent current, only the mass partaking in the current (the mass of supercomponent, which equals $\int \rho_s dV$), and not the velocity field $\mathbf{v}_s$ changes. That this is in fact true is not obvious, because the vapor that condenses into the film to increase its thickness is stationary, whereas the persistent current has a velocity, and we might wonder whether or not momentum is transferred. The answer is that momentum transfer between He II and its vapor affects only the normal-fluid velocity, and furthermore the liquid and vapor are in equilibrium with respect to net momentum transfer when the normal-fluid velocity and vapor velocity are equal,[20] as is true here. (See reference 20 for a description of the experimental verification and for a discussion of the thermodynamic theory.) Finally, the vapor condenses into the film (as the pressure is increased) without *net* angular-momentum transfer: Part of the increase in the mass of the film goes into the supercomponent and part into the normal component; the increase in angular momentum of the supercomponent is balanced by the change in angular momentum of normal fluid plus walls (superleak), and the torque needed to bring the normal fluid plus walls back to rest equals the net increase in angular momentum of the He II film for this process. The results for the healing length from the measurements[15] agree fairly well with the third- and fourth-sound measurements.

Thickness of a moving film

Although the techniques and interpretations involved in experiments relating to the quantum-mechanical boundary effects are *macroscopic*, the effects become more and more important only as the geometry approaches *microscopic* dimensions (except near T_λ), and we might call them "semimacroscopic" quantum effects. On the other hand the quantized vortex lines and $\nabla \times \mathbf{v}_s = 0$ are fully macroscopic quantum effects because they can be observed in bulk systems. We are led to wonder whether situations might arise in which the behavior of the macroscopic wave function near the boundary could also be important in bulk systems. In this light we shall discuss a recent experiment of William Keller's.[21]

Keller's aim was to measure the thickness of a moving He II film. The film is set in motion by increasing the level of the He II, contained in a beaker, relative to the surrounding bath. Because the film wets all the walls it forms a continuous path from the He II in the beaker to the bath, and the resulting flow is like that in a siphon. According to a simple application of the thermodynamics of He II,[22] the thicknesses d *and* d_0 of the moving and stationary films should be related by an equation that expresses the equilibrium balance at the free surface between the Bernoulli pressure and the gravitational and van der Waals forces

$$(\rho_s/\rho)(v_s^2/2) = gz\left(d_0^3/d^3 - 1\right) \quad (6)$$

Here v_s is the speed of the film and z is the height of the point in question (on the film) above the level of the bulk He II in the beaker. According to the thermodynamics, then, a moving film should become thinner. Keller measured the thickness of a moving film with capacitance techniques and found with quite good accuracy that $d = d_0$. A moving film did *not* become thinner, apparently contradicting the basic thermodynamics of He II.

The van der Waals attraction between moving film and substrate is not fully understood, and we can not rule out the possibility that work in this area might shed light on Keller's results. However, we speculate that this experiment might someday be understood in terms of a thermodynamics generalized to include the quantum-mechanical boundary effects. As with all thermodynamic relations yielding the shape of a free surface, the quantities in equation 6 are to be evaluated at the free surface. If we now put in the boundary condition $\rho_s = 0$ at the free surface, we find $d = d_0$ for all v_s. If there is any truth in this idea, then Keller's experiment points the way toward possible observations of quantum boundary effects in bulk systems. Consider the example of He II contained in a rotating vessel with the superfluid at rest, $v_s = 0$ (such states have been obtained in metastable situations by Reppy and Cecil Lane[23] for rather high ω). According to the thermodynamics of He II the shape of the free surface in this case should be parabolic, and the height z of the free surface above the minimum is

$$z = \rho_n/\rho\, \omega^2 r^2/2g \quad (7)$$

where r is the radial distance from the axis of rotation. Equation 7 differs from the corresponding formula for a classical liquid by a factor of ρ_n/ρ, which is the fraction of normal fluid. If we now impose on equation 7 the condition $\rho_s \to 0$, and hence $\rho_n = \rho$ at the free surface, then we would find that the shape of the surface is always classical and not shrunken ($\rho_n/\rho < 1$) as indicated by equation 7. In such a way we might investigate the effects of the macroscopic quantum boundary condition in a "bulk" system.

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