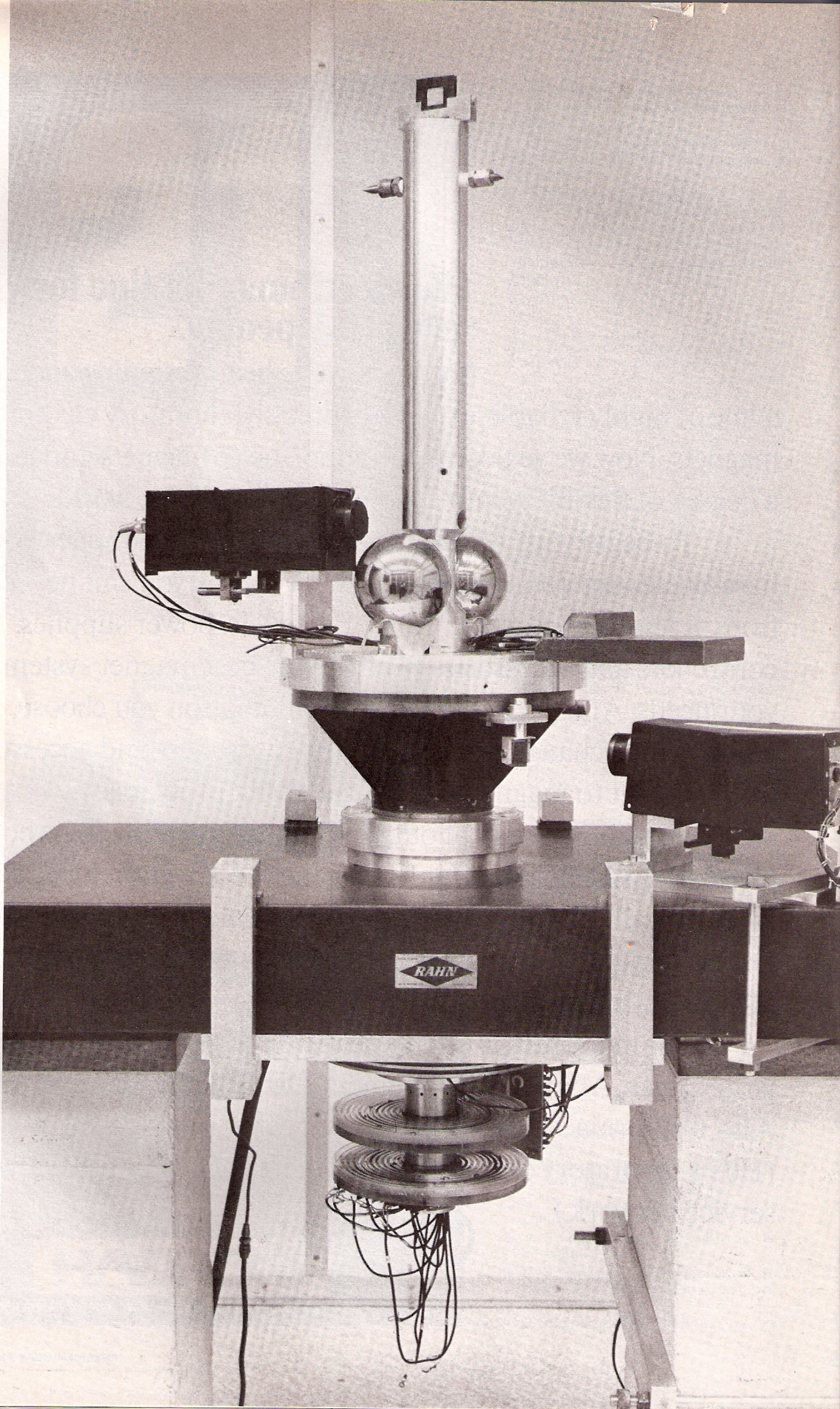
RAHN

G-measuring device designed by Beams and his coworkers includes a circular rotating table and two mass systems. The tungsten spheres visible in the photograph form the "large mass" system; the "small mass" system is concealed in the vertical cylinder. Figure 1

Finding a better value for G

Surprisingly, we know the gravitational constant only to within about half a percent. With this elegant method, we hope for a precision of at least one part in ten thousand.

Jesse W. Beams

The gravitational proportionality constant G, although one of the first constants measured and perhaps the most fundamental and universal constant in Nature, is now the least accurately known. G, which relates the force F between two particles to their masses m_1 and m_2 and the distance d between them

$$F = -Gm_1m_2/d^2$$

is now known only with an accuracy of about one half of one percent. Because many contemporary techniques, such as in geophysics, require a good knowledge of G, as well as for purely theoretical interests, a group of us at the University of Virginia, Charlottesville, have been developing the apparatus shown in figure 1 to measure the absolute value of G as well as its possible variation with time or other factors.

Significance of G

Our own interest in measuring G is motivated by its very fundamentalness; the gravitational constant should really be known as accurately as possible. But there are more "practical" reasons as well. Finding values for the densities and density distributions in the interiors of the earth, moon, outer planets and stars, for example, have recently become important, and these determinations require precise knowledge of G. Precise measurements of the orbits of celestial bodies give us values of GM, where M is the mass of the body, rather than values of G.

Seismological exploration, along with other precise geophysical measurements, may soon be able to give us equations of state for the interior of the earth that are accurate to about 0.1 to 1.0 percent. Comparison of these results with laboratory experiments on the equation of state should greatly increase our knowledge of the earth's interior, as well as provide a sensitive check on the extrapolation of the lab studies. But to get at these values, we shall probably need to know G to at least one part in 10^4, about 100 times more precisely than any of the values so far determined.

Cosmological theory, according to Robert Dicke,[1] predicts a possible annual change in G of one part in 10^8–10^{10}. Testing these predictions experimentally would of course be very exciting.

Past attempts to measure G

The gravitational constant is difficult to measure precisely because of the extreme weakness and great universality of gravitational interactions. The gravitational attraction between a proton and an electron, for example, is roughly 10^{-40} times the electrostatic attraction, and careful studies show no variation of G with the magnitude of the masses, the distance between them, their temperature and so on.[1,2] Consequently it is necessary to measure very small quantities under conditions such that one can not isolate the desired variable; it is not possible to shield the test masses from the gravitational fields and field gradients produced by all the other matter in the universe.

Both large-scale and laboratory-scale experiments have been used to measure G. In one large-scale experiment in Peru, Pierre Bouguer[3] estimated the relative masses of an isolated mountain and the earth from their volumes and densities, and then found a value for G from the deflection of a plumb line near the mountain. George Airy and his collaborators,[3] in another experiment, estimated the relative masses of a spherical shell of the earth and of the entire earth. By recording the oscillation periods of a pendulum at ground level and one at the bottom of a mine shaft, they effectively measured the acceleration of gravity at the earth's surface and at the bottom of the shaft, and so determined a value for G. The result is valid only if the earth is spherical and homogeneous throughout, or if its density distribution is known.

Henry Cavendish did the first (1798)[4]

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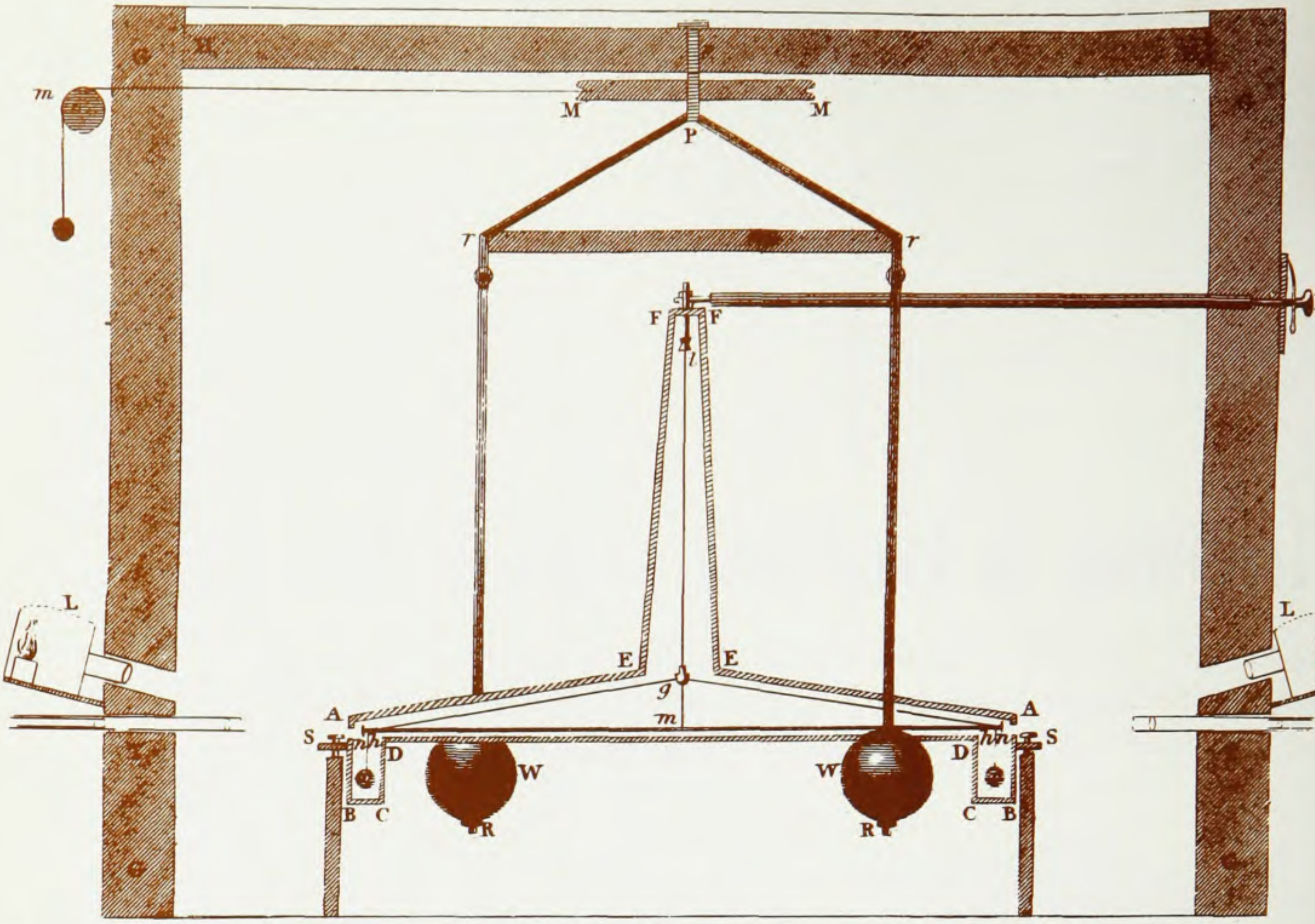


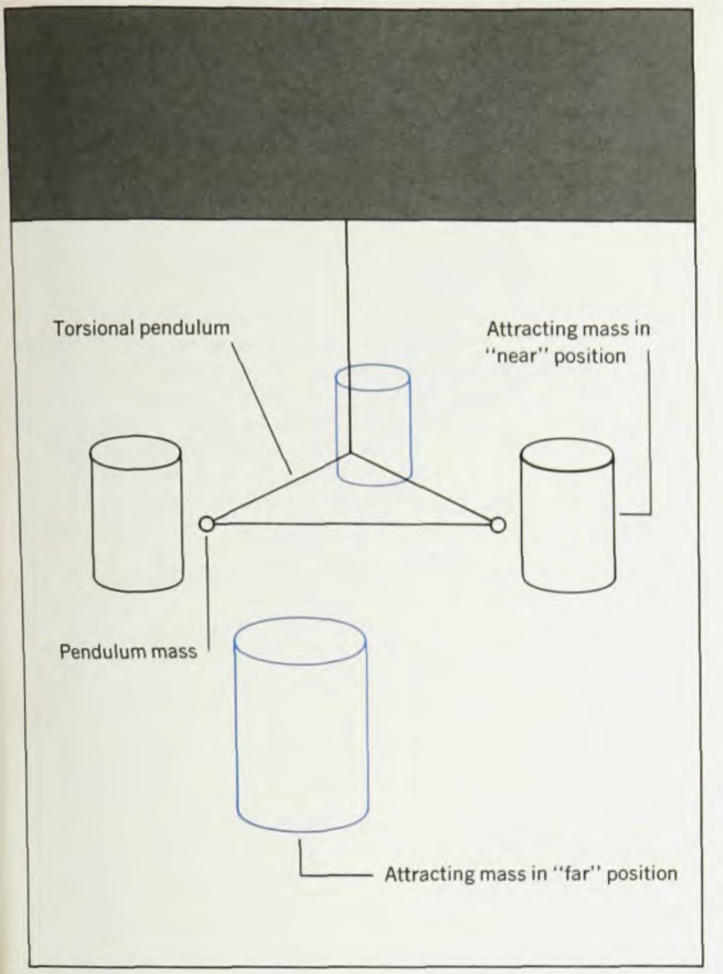
laboratory-scale determination of G with apparatus designed several years earlier by John Michell, who invented the torsion balance for this experiment. Cavendish's experiment appears to have been the first measurement of any of the so-called "fundamental constants." He mounted two solid homogeneous spheres (for which the total mass may be considered as at the center), each with mass m on the ends of a light stiff rod that is suspended in the horizontal plane by a thin fiber. If two masses m' are now brought near the spheres (see figure 2), the gravitational attraction between the masses m and m' produces a torque that twists the suspension fiber. When the masses m' are moved into symmetrical positions, the deflection is in the opposite direction. If the restoring constant of the fiber is known, the value of G can be determined.

Other experimenters, including Charles Vernon Boys and Roland von Eötvös, have repeated Cavendish's experiments with some variations.[5] In many of these experiments the two masses m were placed at different heights on the torsion balance instead of in the same horizontal plane; this arrangement allows m and m' to be brought closer together, increasing the gravitational torque between them and simplifying the calculations. It has the disadvantage, however, of sensitivity to gravitational gradients and is in fact a method of measuring these gradients. Still other experimenters, such as Phillip Johann Gustav von Jolly and John Henry Poynting, used a chemical beam balance to measure G.[6] With this kind of balance, equal masses are placed on each of the balance pans, and the pointer deflection is noted when a large mass is placed first under one pan and then under the other.

The most reliable laboratory experiments until now were probably the work of Paul R. Heyl and his coworkers[7] at the US National Bureau of Standards. In their apparatus (see figure 2) two pendulum masses are mounted on a torsion balance, the attracting masses are placed on the imaginary line that passes through them, and the period of the pendulum is measured. The two at-

Table 1. Past Results

Experimenter	Year	Method	G (Nm²/kg²)
Cavendish	1798	Torsion-balance deflection	6.754
Wilsing	1889	Metronome balance	6.596
von Jolly	1881	Common balance	6.465
Poynting	1894	Common balance	6.698
Boys	1895	Torsion-balance deflection	6.658
Braun	1896	Torsion-balance deflection	6.658
von Eötvös	1896	Torsion-balance deflection	6.65
Crémieu	1909	Torsion-balance deflection	6.67
Heyl	1930	Torsion-balance period	6.670
Heyl and Chrzanowski	1942	Torsion-balance period	6.673



Torsion balances as used by Henry Cavendish and Paul Heyl in their measurements of G. Cavendish (1798) measured the force exerted by two 12-inch diameter lead spheres (W) on two 2-inch lead spheres (x), as shown in the engraving at left. The deflection was observed through telescopes (T) outside the room. In Heyl's experiment (1930's), the period of a torsional pendulum is measured when the attracting masses are in the "near" and "far" positions. Figure 2

tracting masses are next placed in a line that is perpendicular to the line joining the masses and passes through its center, and the period is measured again. The advantage of this method is that periods of the pendulum can be measured with greater accuracy than the small deflections in the previous methods.

A. H. Cook of the University of Edinburgh[8] and Antonio Marussi of the University of Trieste are now planning an experiment that is essentially based on Heyl's. The study, to be carried out at the Grotta Gigante in Italy, will have a torsion-wire suspension of about 90 meters with mass m equal to 10 kg and m' equal to 500 kg.

The accuracies of reported values for G are, except for Heyl's work, difficult to judge. Table 1 is a partial list of reported values, and, although an analysis[9] of Heyl's work gives $(6.670 \pm 0.015) \times 10^{-11}$ Nm^{2}kg^{-2} for G, it is doubtful that any of the data in the table are reliable to better than half a percent. A recent analysis of Heyl's data by L. M. Stephenson[10] shows an annual variation of G that is larger than the statistical error, with a maximum at the vernal equinox and a minimum at the autumnal equinox.

Our method

The method that we are developing at the University of Virginia[11] has two novel features that give it a potential for improved accuracy. A rotational acceleration, rather than a deflection, indicates the interaction force of the masses, so that the effect of the interaction is cumulative and integrable over a long period of time. And the mass systems rotate about an axis many times during a measurement, cancelling all but the higher-order effects of gravitational fields or field gradients from extraneous masses.

In our arrangement (see figure 3), two large tungsten spheres (the "large mass system") are mounted on a table that can be rotated. Mounted from the same rotary table is a gas-tight chamber in which a horizontal cylinder (the "small mass system") is suspended from a quartz fiber. The gravitational interaction between the two mass systems tends to deflect the small mass system in the direction that brings its axial center line in alignment with the line connecting the centers of the two large spheres, changing the angle θ between the two lines. A beam from a light source mounted on the rotary table is reflected from a mirror mounted on the small mass system, near the axis of the quartz fiber, and falls on a photodiode that is also mounted on the rotary table, so that there is an angle β between the incident and the reflected light beams.

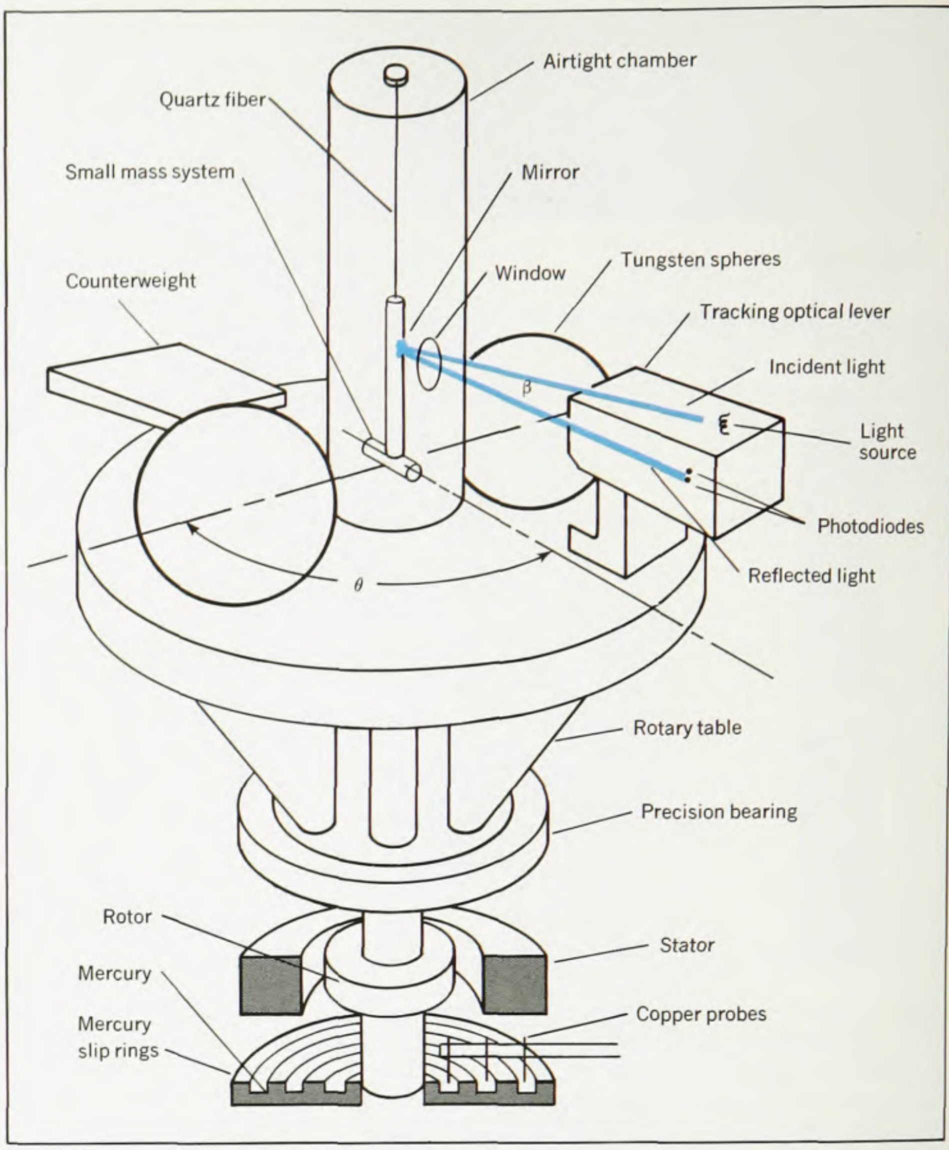
When the gravitational interaction between the large and small systems begins to change, β also begins to change. Even very minute changes in β are sensed by the photodiode, which sends an "error" signal to the motor that drives the table. The table then rotates to maintain β, and therefore θ, constant. Because θ remains constant, the small mass system experiences a constant torque, which causes a constant angular acceleration of the rotary table. By measuring the period of the table, we can determine the acceleration very accurately, and this acceleration is a direct measure of G.

Theoretical analysis of the gravitational torque system gives

$$G = \alpha \pm \dot{\omega}_0 A\,(1 + B + C \ldots)$$

where $\alpha = \dot{\omega} \pm \dot{\omega}_0$ is the measured angular acceleration, $\dot{\omega}$ is the angular acceleration of the torsion pendulum *with* the large spherical weights on the table and $\dot{\omega}_0$ is the angular acceleration with the weights *removed*. A, B and C and functions of the length L and the radius a of the suspended cylinder, the distance $2R$ between the centers of the large spheres, the moment of inertia I_s of the system suspended from the quartz fiber, the masses m and M of the small and large mass systems and the angle θ.

The experimental problem then is to measure α, $\dot{\omega}_0$, R, M, a, L, I_s, m and θ as accurately as possible. Five of these parameters (a, L, I_s, M, m) are constants of the apparatus and so measur-



In the present experiments a helium-filled chamber, supported on a rotary table and precision air bearing, encloses a torsional pendulum. Two tungsten spheres are placed on the table. Any small change in the angles β (and therefore θ) is sensed by the photodiodes and rectified by rotation of the table. The acceleration of the table is a measure of the gravitational interaction. Figure 3

able independently of the actual experiment; the other four are determined in the course of the experiment.

Our procedure is first to level the table and check the vertical alignment of the cylindrical chamber. We adjust the quartz fiber to coincide with the axis of rotation, fill the chamber with helium, and assure ourselves that the effective twist in the fiber is as close to zero as possible. The tungsten spheres, already at room temperature, are put in place and their heights adjusted so that their centers of mass are in the same horizontal plane as the axis of the cylindrical rod. The spheres are placed so that their centers of mass are on an imaginary line through the perpendicular to the axis of rotation of the table, with the radial distances of the two spheres from this axis equal to within $\pm 10^{-5}$ cm, and R is measured.

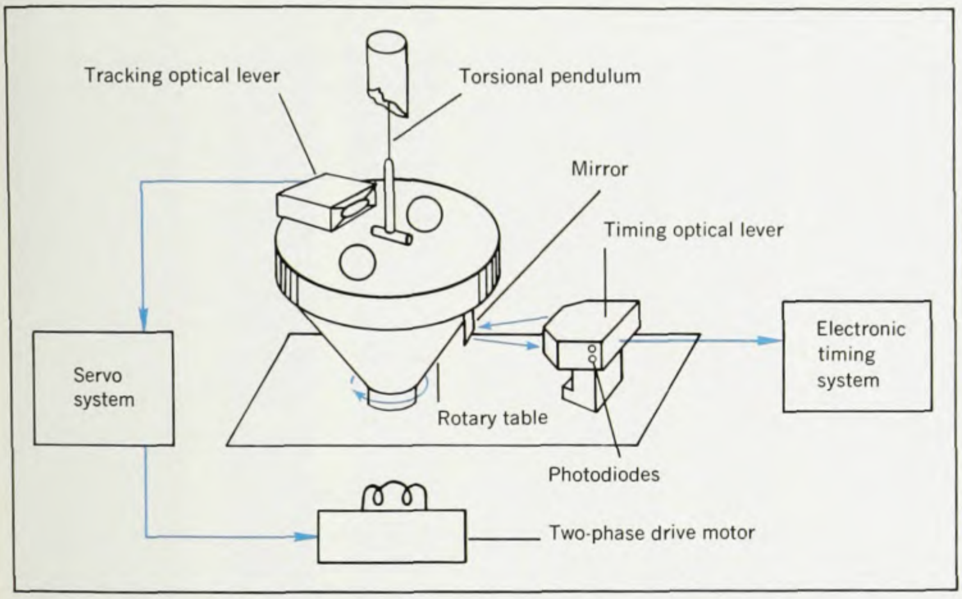
The dimensions of the small mass system are a compromise between keeping the distance between the spheres as small as possible (to maximize $\dot{\omega}$) and using feasible-size spheres. (The precise dimensions of the system are in table 2). Because θ must be not only measured but also held constant, it should be about 45 deg, the angle for which the torque on the small mass system, and therefore the angular acceleration $\dot{\omega}$ of the table, is a maximum. The error-angle β is held constant by means of a servo system (see figure 4).

Determining the angular accelerations $\dot{\omega}$ and $\dot{\omega}_0$ requires determining the period of the rotary table. The optical arrangement is such that the rotation period is equal to the time between two voltage pulses, and is measured in microseconds with a precision of about one part in 10^7. After

Table 2. Mass Systems

Large masses (high-density tungsten spheres)		
	Sphere 1	Sphere 2
Mass (kg)	10.489980 ± 0.00007	10.490250 ± 0.00007
Diameter (cm)	10.165072	10.165108
Distance between center of mass and geometrical center (cm)	4.610×10^{-4}	7.569×10^{-4}
Sphericity (cm)	12×10^{-6}	12×10^{-6}
Small mass system (copper cylinder)		
Length L (cm)	3.9649 ± 0.0004	
Diameter a (cm)	0.19824 ± 0.0004	
Mass m (gm)	4.0512 ± 0.0001	
Moment of inertia of aluminum-alloy stem (gm cm^2)	0.0408 ± 0.0001	
Angle between the axis of copper cylinder and the perpendicular to tracking mirror	44 deg 43 min $\pm$ 0.5 min	

Servo and timing systems. Servo motor rotates table to ensure the constancy of the angle between the small and large mass systems. Timing arrangement measures the period of rotation by counting pulses from the photodiode and is accurate to at least 1 in 10^7 Figure 4



successive rotation periods are measured, the constant acceleration of the rotary table can be computed.

Once our apparatus is going properly, the angular acceleration of the table is constant; the constancy of this acceleration is in fact a good measure of system noise. After the acceleration goes on for several hours (between 4 and 5×10^{-6} rad/sec^2), and the table is rotating at about one or two revolutions per minute, we carefully remove the tungsten spheres and continue tracking for several hours longer. This second acceleration $\dot{\omega}_0$ is a measure of the gravitational interaction of small mass system with the table and with other fixed masses turning with it, as well as a measure of fiber twist and other perturbations.

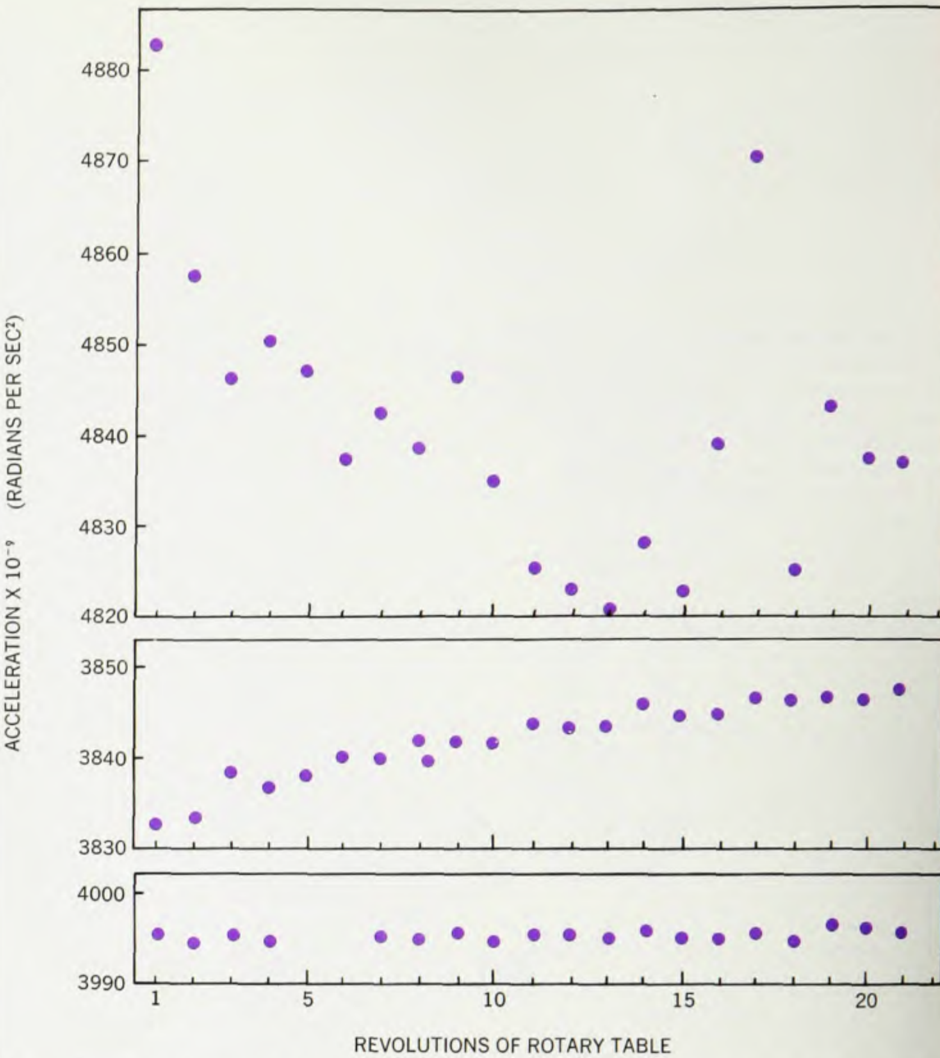
We have done experiments with the table decelerating as well as accelerating, but in all experiments we let the table rotate a whole number of turns, so that the effect of ever-present mass asymmetry surrounding the apparatus is negligible.

Our preliminary values for G have revealed several sources of uncertainties that limit the precision of our measurements, and consequently we are trying to improve the various parts of the apparatus and to develop better ways to do the delicate measurements. In our earlier experiments the major error was in measuring ω and resulted from variations in spindle friction, instabilities in the electronic circuits and temperature gradients produced by the motor and bearing. We substituted a gas bearing for the precision ball bearing, improved and stabilized the electronic circuitry and thermally isolated the motor. Our results improved (see figure 5); the acceleration now remains constant to the order of one part in 10^4, and we believe this precision can soon be improved by at least a factor of ten. Before the recent improvements, our best value was $G = (6.674 \pm 0.012) \times 10^{-11}$ Nm^2/kg^2, where 0.012 is three standard deviations. This result agrees very well with Heyl's value of $(6.670 \pm 0.015) \times 10^{-11}$ Nm^2/kg^2.

The precision with which the center of mass of the two tungsten spheres can be determined is also a limiting factor, and we are encouraged to believe[12] that by substituting for the tungsten spheres we can measure G much more precisely. Certain metals with intermediate to high densities have density variations of less than one part in 10^7 per centimeter. These metals can not be formed into precise spheres, because they will distort under their own weight, but they are usable as rectangular blocks. The use of blocks as the large mass system complicates the theory but not beyond the capabilities of computer analysis.

In our original conception of this experiment, magnetic rather than quartz-fiber suspension of the small mass system was planned. We still believe this method will ultimately give greater accuracy, primarily because the restoring torsion of the small-mass suspension system would be very significantly reduced.

Although I hesitate to estimate the ultimate precision with which this method can measure G, our experience has shown that many important improvements can be made. The possibilities include a completely new design of the rotary table and chamber that gives special attention to thermal and vibrational isolation; certain changes in the servo-motor drive system; improved metrology; substitution



Constancy of measured acceleration results from refining experimental methods. Data with ball bearing and no temperature control (a) form erratic curve. Better results were found (b) when the room temperature was controlled, the photodiodes and other electronics were improved, and a gas bearing replaced the ball bearing. When the motor temperature was also controlled (c), Beams found more improvement in the data; changing the rotation direction caused the two points to be missed. **Figure 5**

of the metal blocks for the tungsten spheres, and an accurately ground quartz cylinder in the small mass system. Measuring G to one part in 10^6 is not beyond conception, once these improvements are made, nor is detecting variations in G of one part in 10^6.

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