torial attempted only to point out that the schedule he proposed for the cuts and his statements about an "orderly transfer" are contradictory. The Senator has created even further concern among scientists with his comment that, if Congress chose to eliminate rather than transfer the funds in question, it "would not be a national calamity."

More on tachyons

In connection with the article "Beyond the Light Barrier" by O-M. Bilaniuk and G. Sudarshan in the May, 1969 issue, readers of PHYSICS TODAY might be interested to know of a further tachyonic domain within relativity theory.

For decades many physicists have believed that a relativistic Newtonian-like particle dynamics, in the old and simple instantaneous-action-at-a-distance sense, is out of the question. It is, however, well within possibility, and it affords scope for many things, superlight velocities included.

A few years ago[1] the conditions were worked out for the "forces" F_i in order that a completely Newtonian-looking particle dynamics $\ddot{x}_i = F_i$ (functions of coordinates and velocities) be Lorentz-covariant. The covariance certainly can not leap to the eye when you have once figured out some suitable F's; but it is equally certainly there, when the forces are tooled up right.

The simplest known example[2] (for two particles in one dimension) is

$$\begin{matrix}\ddot{x}_1 \\ \ddot{x}_2\end{matrix} = \mp \frac{1}{2} \frac{(\dot{x}_1 - \dot{x}_2)^2}{(x_1 - x_2)}$$

Besides the somewhat hidden Lorentz-covariance, this dynamics also has self-evident Galilean covariance. The number c is absent entirely. Relativity's ubiquitous $(1 - v^2/c^2)^{1/2}$ factors are after all nonunique. There is no adjustability whatever of "masses" or "interaction strengths"—the numerical factors $\pm 1/2$ cannot be touched without totally harming Lorentz covariance. Additionally, there is invariance to space and to time reflections and to scale changes in coordinate and in time. There is so much symmetry that the dynamics nearly fails to exist.

It is easy to see that, simply because c is not around for velocities to compare themselves with, not only are superlight velocities possible, but they are mandatory, and the transition from the sub- to superlight velocities and back again is not marvelous. The motion is in fact just (using a principal value integration through $x_1 = x_2$)

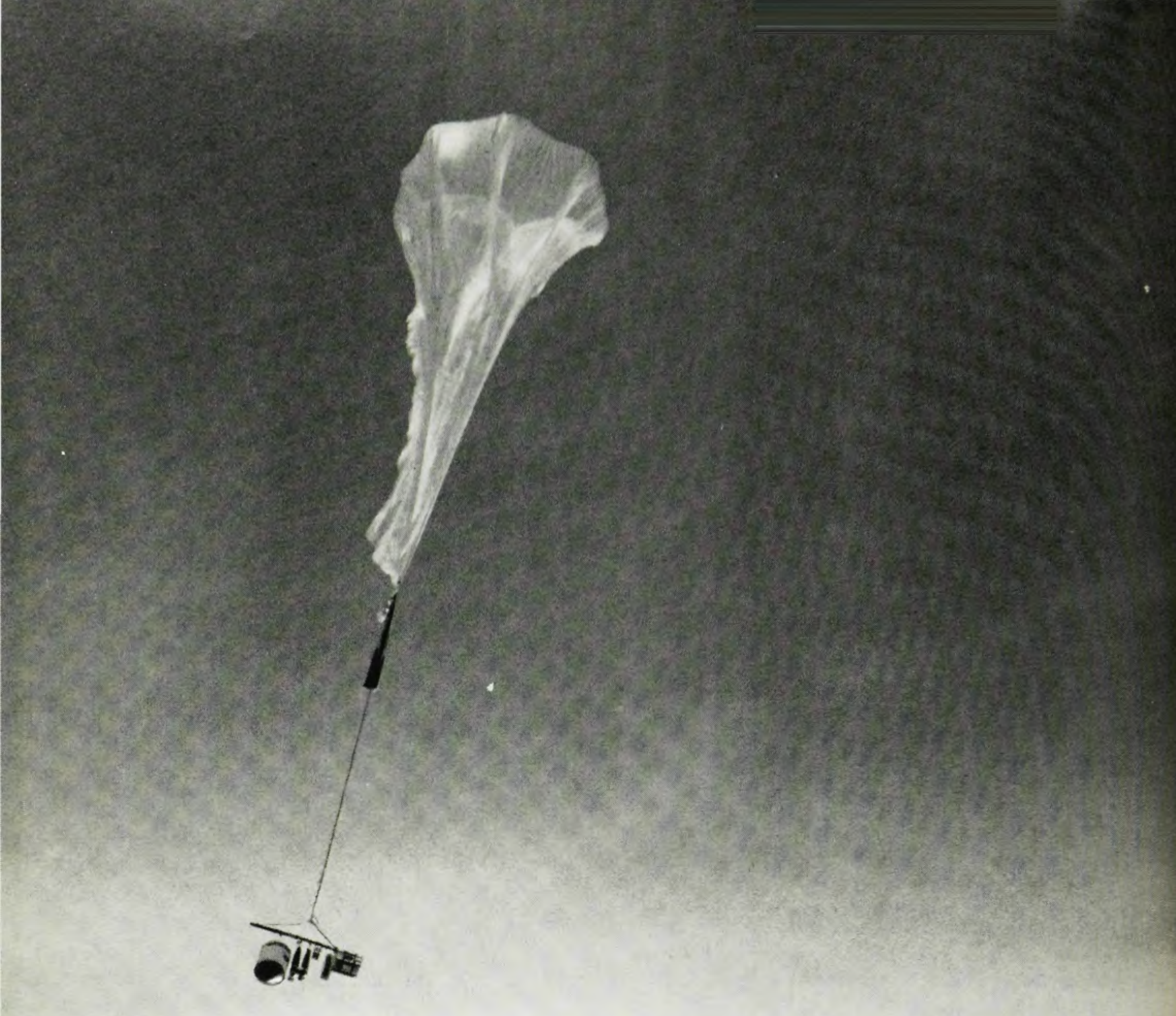
$$\begin{matrix}x_1 \\ x_2\end{matrix} = a + bt \pm |\alpha t + \beta|^{1/2}$$

being parabolas in the x,t plane. The figure shows a plot of $t \pm |t|^{1/2}$. Around the "collision point" $t = 0$ the motion is arbitrarily fast. The Lorentz covariance stands out sharply now: Parabolas go into parabolas under any nonsingular linear transformation in x and t: the general linear group, including the Lorentz group merely as a special case, is the invariance group of the dynamics.

Given a basically Newtonian structure of dynamics, be it Lorentz-covariant or otherwise, the question of causality comes out to be an empty one. Simply, the motion unfolds, either forwards or backwards in time, from initial data on positions and velocities. The old Newtonian world view does have relativistic survival. The sending and receiving of "signals" is not in the picture. They are not needed. Observability is something else. In the relativistic Newtonian dynamics, unlike the Galilean, forces are not superposable (the sum of two relativistic forces is not a relativistic force). So when observation apparatus (this could be one particle) is brought in, the total dynamics of particle world-lines and apparatus world-lines (the latter not necessarily simple or gently coupled to the former), has to be surveyed somehow all the way down to what is recorded in the apparatus.

Well, concerning the example, isn't that just a toy? No doubt. But the machinery that could produce it would seem to bear study.[3] Perhaps, in the machinery's close concentration directly on ensembles of world-lines and their rules of construction, some wider view of relativistic nature is possible. On this view, for instance, preoccupation with momentum-energy four-vectors of the usual type is fruitless. The generalization to *system* momentum-energy, which has the interaction wound up in it, becomes more central.

There is of course a specially pointed reason for attempting the Newtonian view besides mind-broad-



ening. What quantum theory we think we understand places the time in a Newtonian role. To begin with, dynamical considerations on the same basis—while still holding to Lorentz-covariance—may show a path to putting h and c together consistently.

There is a final puzzle inside the toy. If we make Hamiltonian clothing for the dynamics with a view to quantization, it can be arranged for either Lorentz or Galilean transformations to be canonical transformations, so that in equivalent Lorentz frames, or else in equivalent Galilean frames, calculations are carried out equivalently. Classical calculations in the end will be indifferent to whether Lorentzian or Galilean viewpoints are taken—the motion is the motion is the motion. But quantal calculations will differ, owing to quite different meanings of canonical variables in the two cases, and to structurally different Hamiltonians. The same physical problem looks really very different through Lorentzian as compared to Galilean lenses. Both being allowed equally, which is "correct," and why?

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It appears that if particles of real mass and separate particles of imaginary mass are postulated, the formalism should be completed by positioning a general complex mass particle of which tachyons and tardyons are but two special cases. Thus, if $m = m_r + im_i$, the requirement that the quantity $E/|m|c^2$ be real necessitates the assumption of a complex velocity, so that $v/c = v_r/c + i\ v_i/c = \beta_r + i\beta_i$. Some algebraic manipulation shows that

$$\frac{E}{|m|c^2} = \rho^{-1/2} e^{j(\varphi - \theta/2)}, \qquad (1)$$

where

$$\rho = [(1 - \beta_r^2 + \beta_i^2)^2 + 4\beta_i^2\beta_r^2]^{1/2}$$

$$\varphi = \tan^{-1}\Gamma$$

$$\theta = \tan^{-1}\left(-\frac{2\beta_r\beta_i}{1 - \beta_r^2 + \beta_i^2}\right)$$

$$\Gamma = \frac{m_i}{m_r}.$$

Thus, equation (1) is real only if

$$\varphi = \frac{\theta}{2} \pm m\pi,\ m = 1, 2, 3 \ldots$$

Neglecting the $m\pi$ term, this gives

$$\frac{\Gamma}{\Gamma^2 - 1} = \frac{\beta_i\beta_r}{1 - \beta_r^2 + \beta_i^2}$$

as the necessary condition for real-particle energy. One may now make a substitution in equation (1) to derive the form of E and to obtain the special cases of superluminal and subluminal particles.

The "acteon" suggested here seems to be only mathematically acceptable unless some plausible interpretation of complex velocity is offered.

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In connection with the recent discussion of causality objections against tachyons, the following remarks may clarify matters.

Any physical theory has two ingredients: differential equations of motion and prescriptions of how to impose boundary conditions. Any acceptable theory must be self-consistent. To achieve self-consistency, it is not sufficient to require that the equations of motion have solutions that are single valued. In addition, one also has to require that the boundary conditions be compatible with each other and with the equations of motion.

Our conventional physical theories are self-consistent (as far as we know). The equations of motion do have single-valued solutions, and the compatibility of the boundary conditions is ensured by requiring that they satisfy the principle of "retarded causality."

According to this principle, no effect may precede its cause. In addition to this principle, other causality

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1330 E. GOLF ROAD, PALATINE, ILLINOIS 60067 PHONE 312/529-4600 NUCLEAR DATA GMBH, FRANKFURT, GERMANY principles can also be defined. For example [*Phys. Rev.* **180**, 1266 (1969)], one may define the principle of "advanced causality," according to which no effect can follow its cause, or "full causality," according to which (roughly speaking) an effect may both precede and follow its cause with the same probability.

The important point to notice now is that, while self-consistency is a requirement that all acceptable theories must satisfy, retarded casuality is not. In fact, contrary to an often held belief, the more general causality principles do not necessarily lead to self-contradictory theories.

Applying this result to the discussion concerning tachyons, we may conclude: Even if it were true that some interactions involving tachyons could not be explained by any theory satisfying *retarded* causality, that would not necessarily imply that tachyons cannot exist.

Certain causality principles (but not all of them) are physically equivalent. For example, let us assume that time-reversal invariance holds. Then, quite obviously, whenever a series of events can be explained by a theory satisfying retarded causality, the same events can also be explained by a theory that satisfies advanced causality and vice versa. Schematically:

Retarded causality $<\!\!==\!\!>$ Advanced causality
(If time-reversal invariance holds)

Let us now assume that tachyons do exist, and have the properties ascribed to them in the literature. Then there exists a coordinate frame in which the event of a tachyon emission is described by a theory satisfying retarded causality (for example, in the rest frame of the tachyon emitter). In any other frame, then, this event can be described by a theory satisfying either retarded or advanced causality, depending on whether the Lorentz transformation switches the time direction or not. (Switching can happen if velocities $> c$ can exist.) With time-reversal invariance, it follows from equation 1 that, viewed from any frame, the event can be explained by a theory satisfying retarded causality. (However, two observers in two different frames, both able to explain a sequence of two events, and both using the language of retarded causality, may then disagree about which of the two events caused the other.) The final conclusion does not depend on the assumption of time-reversal invariance.

In usual physics we easily avoid incompatible boundary conditions, with our eyes closed, so to say, because we have had a great deal of experience with retarded causality.

For example, let us describe a series of events in the coordinate frame K. Let us specify the boundary conditions B_0 (for example, emission of particles) at time $(t_0)_{\rm K}$ as measured in K. Let the system develop according to certain known equations of motion between $(t_0)_{\rm K}$ and $(t_1)_{\rm K}$ (for example, emitted particles travel in vacuum). Let us specify certain additional boundary conditions B_1 at $(t_1)_{\rm K}$ [for example, particles absorbed by an instrument at $(t_1)_{\rm K}$] as well as boundary conditions B_2 at $(t_2)_{\rm K}$ [perhaps, emission of some other particles at $(t_2)_{\rm K}$]. We may give B_2 in such a manner that they depend on the state of the system at $(t_1)_{\rm K}$ [for example, if particles are absorbed at $(t_1)_{\rm K}$, then some other particles are emitted at $(t_2)_{\rm K}$, but not otherwise]. At times between $(t_2)_{\rm K}$ and $(t_3)_{\rm K}$, let the system evolve according to certain known equations of motion, then let us specify boundary conditions B_3 at $(t_3)_{\rm K}$, and so on.

How can one insure the compatibility of all these boundary conditions B_i $(i = 0, 1, 2, \ldots)$? One easy way is to proceed as follows. First, make sure that B_1 is compatible with B_0 and the equations of motion that govern the system between $(t_0)_{\rm K}$ and $(t_1)_{\rm K}$. Second, make sure that B_2 is compatible with B_1 and whatever is known about the system between $(t_1)_{\rm K}$ and $(t_2)_{\rm K}$ (for example, the workings of the instrument), and so forth. In other words, make sure that B_{i+1} is compatible with B_i and the behavior of the system between $(t_i)_{\rm K}$ and $(t_{i+1})_{\rm K}$, for all i. Now it is well known that in this case any B_i will be compatible with any B_j (not just for $t_i = i + 1$), *provided* that the times t_i are chosen so that $(t_i)_{\rm K} \leq (t_{i+1})_{\rm K}$ for any i, because if the B_i are compatible over all successive time intervals, then they are compatible over the whole history.

$$\left.\begin{array}{l}\text{Making sure that any } B_i\\ \text{is compatible with } B_{i+1}\end{array}\right\} <\!\!==\!\!>$$

$$\left\{\begin{array}{l}\text{Making sure that any } B_i\\ \text{is compatible with any } B_j\end{array}\right. \qquad (2a)$$

whenever $(t_i)_{K_i} \leq (t_{i+1})_{K_i}$ for any i (2b)

and all K_i are the same (2c)

As an illustration, consider the following. The B_0 means the appearance of monoenergetic protons at the point **x** at $(t_0)_K$, all of the protons travel in vacuum until they hit a counter at point **y** at $(t_1)_K$, at $(t_2)_K$ protons with the same energy are emitted at point **y** whenever protons hit the counter at $(t_1)_K$, but not otherwise, and, finally the protons travel in vacuum until at $(t_3)_K$ they reach point **x**. This is a perfectly consistent way of specifying boundary conditions in usual physics, provided that all $(t_i)_K \leq (t_{i+1})$. If, however, this condition is violated, for example, if $(t_2)_K > (t_3)_K$, then the above set of boundary conditions are no longer necessarily all compatible with each other. In particular, if $t_3 = t_0$, then they are certainly incompatible, because B_0 states that at **x** all protons travel towards **y**, at t_0, while B_3 states that they travel in the opposite direction at $t_3 = t_0$.

Equation 2 can immediately be generalized, remembering that whenever all signals and coordinate frames travel with velocities less than c with respect to each other, the time order of any causally connected two events is not interchanged by Lorentz transformations: if $(t_i)_K \leq (t_{i+1})_K$ then $(t_i)_{K'} \leq t_{i+1})_{K'}$ where K and K′ are any two rest frames of observers or particles. In other words, if none of the velocities is superluminal, then condition 2c can be dropped, and the frames K_i may be all different; 2b alone will insure that 2a holds.

It is important to keep in mind that 2a can not be dropped in general if some particles have superluminal velocities. This is so, because then it may happen that $(t_i)_K \leq (t_{i+1})_K$ but $(t_i)_{K'} > (t_{i+1})_{K'}$ if K and K′ are two suitably chosen frames, even if the events at t_i and t_{i+1} are causally connected. Therefore, the usual procedures of specifying boundary conditions do not guarantee that these conditions are all compatible with each other, unless condition 2c is valid. An analysis of the unusual paradoxes quoted in connection with tachyons shows that all of them arise because the boundary conditions were given in various frames in a manner that satisfies condition 2b, but in violation of 2c. That such a procedure does not lead to a self-consistent solution, that is, that condition 2a is not valid, is, in view of what was said above, not surprising. However, while these paradoxes do not at all demonstrate the logical inconsistency of a theory that permits the existence of tachyons, they do clearly illustrate that boundary conditions have to be given with care.

Independently of whether tachyons exist, the principle of retarded causality may prove to be too narrow to describe nature on the deepest level. The possibility in a more general theory (based on full causality) of defining and discussing the "velocity of time," the possibility of closing causal chains, and of changing the singularity structure of scattering amplitudes, to mention just a few, appear to indicate this. During the last century many geometers believed that Euclidean geometry was the only logically possible one. Today more general geometries are widely accepted. I believe that, quite similarly, retarded causality, which is almost universally believed today, will prove to have been merely a hangup with which many of us were inflicted, and that the time is not far away when a more general causality will be accepted as a matter of course. Then many will no longer know that the more general causality principles were not always around, and few will remember where they came from. People will probably say: "Full causality? Why not? Violation retarded causality? Who cares?"—And they will be right.

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In their rebuttal (December, 1969) of several letters concerning their article on tachyons, Bilaniuk and Sudarshan represent the point of my paper (Phys. Rev. **163**, 1274, 1967) as "So what?" in reaction to the causality objections. That entirely misses its intention. Rather than trying to make the argument here in a few lines, I refer the interested reader to my forthcoming article in *Science*.

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