Theorists Honor Uhlenbeck with Statistical-Mechanics Meeting

A symposium "Statistical Mechanics at the Turn of the Decade" was held 30–31 Oct. at the physics department, College of Arts and Sciences, Northwestern University, Evanston, Illinois, in honor of George E. Uhlenbeck of The Rockefeller University. Eight invited speakers gave 50-minute lectures that summarized the status of our understanding of selected fundamental problems in statistical mechanics. Emphasis, in the papers that were given at the conference, was on the advances that have been made during the last ten years.

There is of course considerable overlap of statistical mechanics with problems in, for example, solid-state physics, low-temperature physics, plasma-physics and physics of fluids. The intent of the organizers was to avoid competition with meetings in these fields by concentrating primarily on the eternal problems that concern phase transitions and the approach to equilibrium.

E. G. D. Cohen, The Rockefeller University, dealt with the more practical aspect of the approach to equilibrium in his lecture "The Generalization of the Boltzmann Equation to Higher Density." In describing the approach to thermal equilibrium of a moderately dense gas, which at $t = 0$ is not in equilibrium, one wants to know the basic equations describing this approach, and their characteristic features.

Nikolai N. Bogoliubov and many others attempted to answer these questions by generalizing the idea of the virial expansion to nonequilibrium phenomena, that is, by trying to express the properties of the gas as a whole in terms of properties of small isolated groups of molecules. Although this procedure leads in first approximation to the Boltzmann equation with a finite correction for triple collisions, all higher corrections diverge. In the last five years attempts have been made to reformulate Bogoliubov's theory to avoid these divergences. This work has led in particular to an expansion, of the transport coefficients, that contains contributions that depend logarithmically on the gas density. Experimental verification has not yet been conclusive.

Arthur S. Wightman of Princeton University dealt with the fundamental aspect of the approach to equilibrium in a talk entitled "Statistical Mechanics and Ergodic Theory—An Expository Lecture." Wightman explained the significance for classical statistical mechanics of some mathematical developments of the last decade. This lecture traced the development of ergodic theory from Ludwig Boltzmann's ergodic hypothesis to Ya. G. Sinai's proof that hard spheres in a box form a K system (for the definition of these K systems and of the C systems mentioned later, see the book by V. I. Arnold and A. Avez; *Ergodic Problems of Classical Mechanics*, W. A. Benjamin, 1968, page 32, page 55–56).

There have been two main lines of thought; stability theory, in which one finds integrals of motion that make a flow nonergodic, and ergodic theory proper, in which one attempts to understand how flow in phase space can produce ergodicity. The initiator of the first theory was Henri Poincaré, who analyzed the behavior of orbits in a neighborhood of a periodic orbit by expanding the Hamiltonian in a Taylor series around the periodic orbit and keeping only terms of degree less than three. In recent years, Andrei Kolmogorov, Vladimir Arnold and Jürgen Moser have shown that Poincaré's stability conditions imply the existence of invariant tori surrounding the periodic orbit for the exact Hamiltonian.

On the other hand, studies by Eberhard Hopf in the late 1930's disclosed a general mechanism of instability, generalized by D. V. Anosov to yield the definition of a C system. Meanwhile, a quantitative measure of irreversibility, introduced by Kolmogorov and Sinai, the K–S entropy, led to a family of flows called K systems that are very "irreversible." Sinai proved that essentially all C systems are K systems. He also showed, in a paper that is as yet unpublished, that hard spheres in a box yield a K system, and in particular define an ergodic flow in the phase spaces. While this result establishes once and for all that physically interesting ergodic flows exist, the final status of the ergodic hypothesis will have to wait on the study of more general interaction potentials.

C* algebra. On the same level of rigor as Wightman's lecture, David Ruelle of the Institut des Hautes-Études Scientifiques, Bures-Sur-Yvette, France, discussed the C*-algebra approach to statistical mechanics. The main point of this approach is to avoid taking the thermodynamic limit ($N \rightarrow \infty$, $V \rightarrow \infty$, fixed density N/V) by formulating the problem for infinite systems immediately. As examples of the usefulness of this approach Ruelle discussed time evolution, decomposition of invariant equilibrium states into pure thermodynamic phases and the phenomenon of symmetry breakdown. This breakdown is a nontrivial decomposition of invariant equilibrium states into noninvariant equilibrium states. Crystals are nontrivial examples of symmetry breakdown, and some progress has recently been made in this direction.

Continuing the rigorous vein, Freeman Dyson, Institute for Advanced Study, discussed some recent theorems in his lecture, "The Existence of Phase Transitions in One-Dimensional Ferromagnets with Long-Range Interactions." The essential results are that a one-dimensional lattice system, with interaction falling off like $r^{-(1+\sigma)}$ has a phase transition to long-range order if $0 < \sigma < 1$ and has no long-range order at any temperature if $\sigma > 1$. The interesting case $\sigma = 1$ is still unresolved; there are a number of arguments that support alternative views.

For the spherical model, which Geoffrey Joyce has solved exactly, the above results are true and the r^{-2} case has no phase transition. The proofs make extensive use of Robert Griffiths's correlation-function inequalities, the most powerful of which states that for ferromagnetic interactions, pair correlation functions increase if the interaction between any pair of spins is increased. Seymour Sherman and C. A. Hurst have shown that this theorem does not hold for Heisenberg

systems, but Dyson conjectures that if "pair correlation functions" is replaced by "average of the total magnetization squared," the theorem may be true. The r^{-2} problem and the generalization of Griffiths's inequalities to Heisenberg models will be challenging and, we hope, solved, problems in the next decade.

Still in the realm of Ising models and phase transitions was Cyril Domb's (King's College, London) talk on the Curie point. This was a historical survey and a summary, but stressed plausibility arguments more than strict mathematical proofs. Searching through the literature, Domb found apparently the first experimental observation of the Curie point in a book by William Gilbert in 1600: "Magnets lose their strength when put in a furnace." Experimental progress since this time has been considerable, but theoretical progress since Lars Onsager's exact solution of the two-dimensional Ising model, which showed clearly the inadequacy of the classical Curie–Weiss and van der Waals theories, has been more erosive than exact.

The most systematic and successful approach to the three-dimensional problem to date has been via generation of series expansions for thermodynamic quantities. Graph classification and counting and high-speed computers are the essential tools in this approach, and by now we know a considerable number of series coefficients for a variety of three-dimensional lattices. These series have been analyzed by several methods, and it is felt that critical behavior (critical points and critical exponents characterizing singularities) is now known fairly well.

An interesting development in the past decade has come from the phenomenological arguments, known as scaling and homogeneity arguments, that suggest relations between various exponents. Inequalities between exponents follow from thermodynamics (or convexity and monotoneity), and the essential consequence of scaling and homogeneity arguments is that these inequalities are actually equalities. Some exactly soluble models (notably the spherical model) have exponents that satisfy these relations, and numerical results from series expansions also seem to satisfy the relations, but the evidence, particularly for some relations involving dimensionality, is not yet conclusive. Recent attempts have been directed towards finding the graphs that dominate the series coefficients in the critical region.

The van der Waals approximation for the isotherm of a fluid with Maxwell (equal-area) construction has undergone several changes of status since its inception, and with it the Weiss theory of ferromagnetism. Twenty years ago many workers considered these mean-field theories to be of historical interest only. In the 1960's, however, a revival of interest in these old approximations led Joel Lebowitz and Oliver Penrose to the rigorous proof of a limit theorem, which states in essence that the van der Waals approximation with Maxwell construction becomes exact when the attractive part of the intermolecular potential is made infinitely weak and of infinite range with its space integral held fixed. The rough approximation for the pressure of the hard-sphere gas in the original van der Waals equation must of course be replaced by the exact pressure resulting from short-range repulsive interaction.

Arnold J. F. Siegert of Northwestern University in his lecture "From the Mean Field Approximation to the Method of Random Fields" outlined these developments. The method of random fields replaces all or part of the interaction between the molecules of a fluid or the spins of an Ising model by impressed fields; one then averages over the fields with a Gaussian probability distribution specified by the interaction. The resulting representation of the partition function was first suggested by Mark Kac. The representation is exact for a class of interactions sufficiently wide to include many cases of physical interest, and it contains the mean-field theories as lowest-order approximation in the limit defined above. Applications of the formalism to obtain corrections near this limit were reviewed in the lecture.

Critical-point dynamics. Pierre C. Hohenberg of Bell Labs reviewed some experimental and theoretical studies of the dynamical properties of systems near critical-point phase transitions. He described a phenomenological theory that generalizes the van Hove theory of critical slowing down to include the "scaling" properties of static and dynamic correlation functions. Applications were made to thermal transport near the λ-point of liquid helium and to critical inelastic neutron scattering near the Néel point of $RbMnF_3$.

Paul C. Martin of Harvard University spoke on superfluidity and superconductivity, which are both the result of long-range phase coherence. In three-dimensional samples, this coherence extends to infinite distance below a transition temperature at which a Bose condensation occurs, and results in striking macroscopic quantum effects. Thermodynamics and hydrodynamics of the superfluid state are relatively well understood.

Even in one-dimensional samples where there is no transition, the long-range phase coherence produces qualitative changes in resistance. The nature of this resistance, the resistance of metastable current-carrying states and the resistive fluctuations near the transition temperature are only beginning to be understood. It is in these areas that we need important new insights.

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ARNOLD J. F. SIEGERT
COLIN J. THOMPSON
Northwestern University

Delaware Luminescence Meeting Covers Organics and Inorganics

Luminescence embraces a variety of disciplines, so that a conference on luminescence attracts a heterogeneous group including organic chemists, solid-state experimentalists and theorists, spectroscopists, magnetic-resonance specialists and radiation chemists. About 400 of these specialists assembled for the 1969 International Conference on Luminescence held 25–29 Aug. at the University of Delaware. This conference, chaired by Ferd Williams of Delaware, was one of a series that have been held every three or four years; the most recent was the 1966 conference in Budapest. Delegates from 15 countries heard reports on luminescence research on both organic and inorganic materials in a program of 16 invited and 71 contributed papers. To permit inorganic specialists to hear reports on lumines-