

Meson factories and the two-nucleon problem

The advent of ultrahigh-intensity accelerators should encourage interest in an old unsolved problem in physics: What is the interaction between two nucleons?

Michael J. Moravcsik

The interaction between two nucleons was first investigated almost 40 years ago and still occupies a central place in contemporary research. Efforts to understand this complex phenomenon have been hampered by the lack of precise data for all the pertinent observables. The experimental situation will be remedied in 1972 when accelerators producing beams 10 000 times more intense than presently attainable begin operation. The new breed of accelerator, informally called "meson factories," will initially operate in the energy range below 1 GeV but may someday reach 20 GeV.

Neither the problem nor the technique should be unfamiliar to readers of *PHYSICS TODAY*. The two-nucleon interaction was reviewed recently by M. H. MacGregor¹, while meson factories were described a few years ago by L. Rosen.² I will refer to these articles for background and concentrate on the connection between the two.

The two-nucleon problem has been studied for a variety of reasons. First, it is a challenging and interesting problem in itself. In this context, the aim is to describe the interaction of the two-nucleon system as completely, as simply, and as accurately as possible. A second reason for our interest is that the two-nucleon interaction is, at least in principle, the basis of all of nuclear physics. From this point of view, the goal is to determine accurately those features of the interaction that play a role in calculations of nuclear structure.

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A third source of interest is the relevance of the two-nucleon problem, as the most studied and best known strong interaction, to elementary-particle physics. Our relatively precise knowledge of the two-nucleon interaction at low energy allows us to test new ideas and models in strong-interaction physics. In this sense, the two-nucleon interaction serves as a *tool* for testing general principles. It can also serve as a successful *example* on which we can pattern models and from which we can bolster our intuition.

Nuclear structure

To calculate nuclear structure, we in principle need to know the matrix elements of the interaction between all possible pairs of initial and final two-nucleon states, including those pairs that do not conserve energy and momentum. Before we enter a detailed discussion, it should be mentioned that this information may not be sufficient for such structure calculations. It is possible that there are also genuine three-body forces among nucleons, as distinguished from three nucleons interacting simultaneously through three two-body interactions. There is no evidence against this possibility from nuclear-structure experiments themselves.¹ Presumably a careful analysis of very precise meson-factory experiments on three-nucleon states, such as the scattering of protons by deuterons, could shed light on this point.

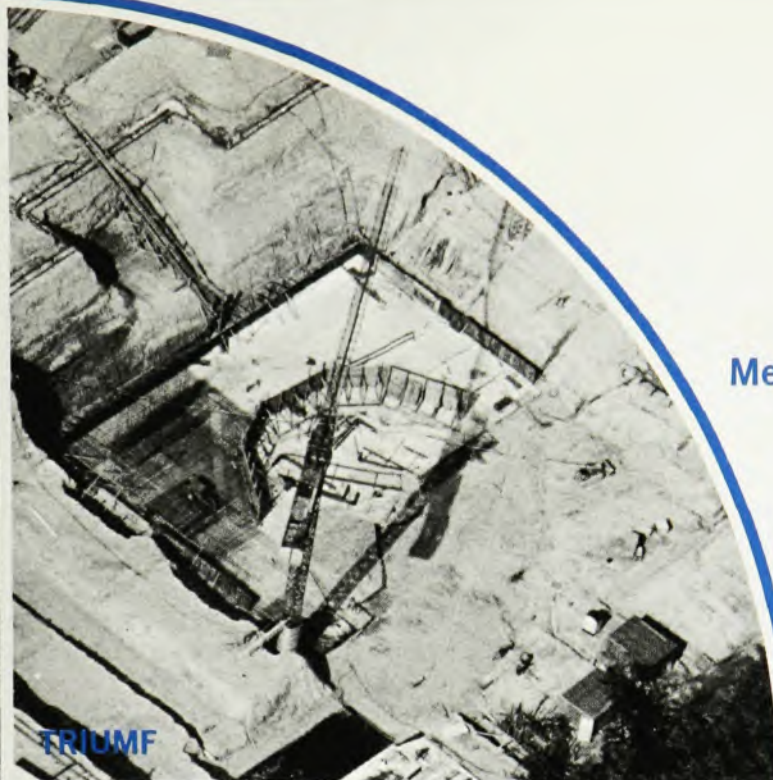
For two-nucleon elastic scattering, only final states that conserve the initial energy-momentum can be reached; we speak of being restricted to the energy shell. However, for inelastic

events, one or more new particles are created in the final state, and hence the four particles comprising the initial and final nucleon pairs need not (in fact can not) satisfy energy-momentum conservation. The interaction that takes place between nucleons under these conditions is described as being off the energy shell. These nonconservative interactions contribute to nuclear structure through virtual transitions inside the nucleus.

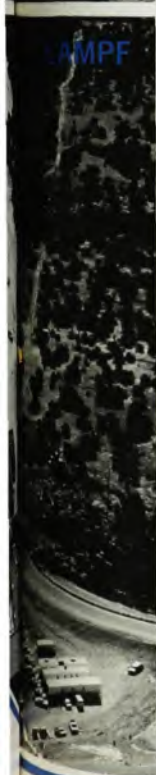




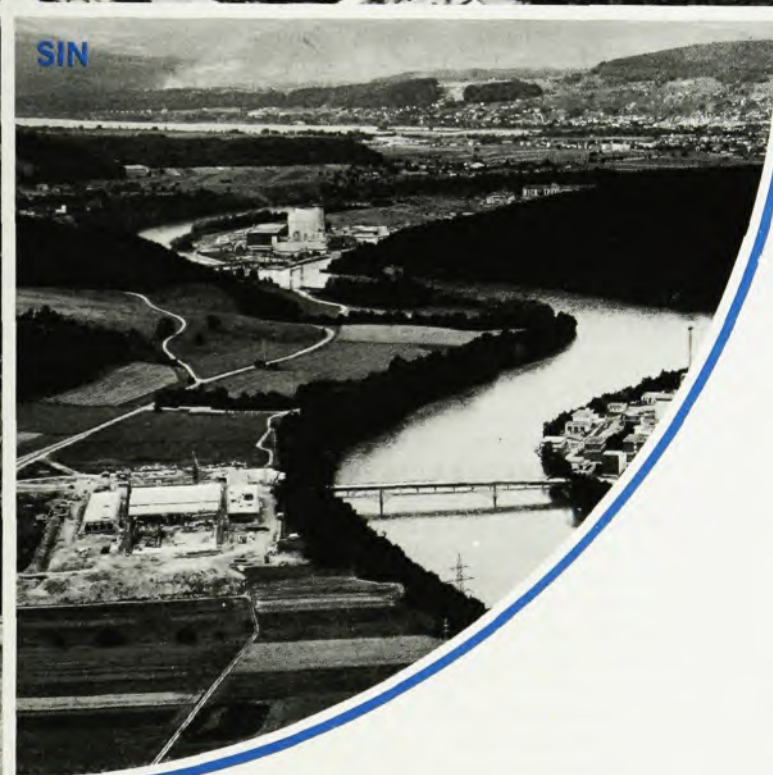
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Meson factories around the world

A spirit of healthy competition exists as construction on four ultra-high-intensity accelerators proceeds on schedule with full operation expected between 1972 and 1974. The emphasis at all the meson factories will be on accommodating many users simultaneously with a minimum of interference. Elaborate beam switchyards will route the multiple secondary beams to the various experimental areas. Some beams are used more than once and the nuclear by-products are carefully husbanded; the meson factories are "economic" enterprises aiming at maximum utilization of resources.

The Los Alamos Meson Physics Facility (LAMPF) of the University of California will produce the most intense and most energetic beam: one milliamper of protons at a variable energy of 100-800 MeV. LAMPF, the only linac among the four, incorporates a newly-developed $\pi/2$ -mode waveguide to achieve the final energy.

The Tri-University Meson Facility at Vancouver (TRIUMF) has now added a fourth participant to its consortium. The original three, the Universities of British Columbia and Victoria and Simon Fraser University, have been joined by the University of Alberta at Edmonton. TRIUMF is built around a negative-ion cyclotron, which will produce 100 microamp of H^- at 500 MeV. Beam extraction, by stripping conversion to positive ions, is 100% efficient.

The Swiss Institute for Nuclear Research (SIN) will accelerate protons by a two-stage process. A 70-MeV cyclotron will inject into an isochronous (constant-frequency) ring cyclotron, which raises the energy to 580 MeV.

The Joint Institute for Nuclear Research at Dubna, USSR, is converting its 310 cm synchronocyclotron to isochronous operation by spiral ridging the electromagnet. Protons with energies of 700 MeV are expected.

Among the inelastic processes, nucleon-nucleon bremsstrahlung is one of the best tools to investigate the region off the energy shell. This process, in which a nucleon scatters from a nucleon and a photon is also emitted, is advantageous because the photon is coupled to the nucleons only by the well understood electromagnetic interaction. Detailed work, both experimental and theoretical, has been confined to a domain only slightly off the

energy shell, and even there, uncertainties remain substantial.

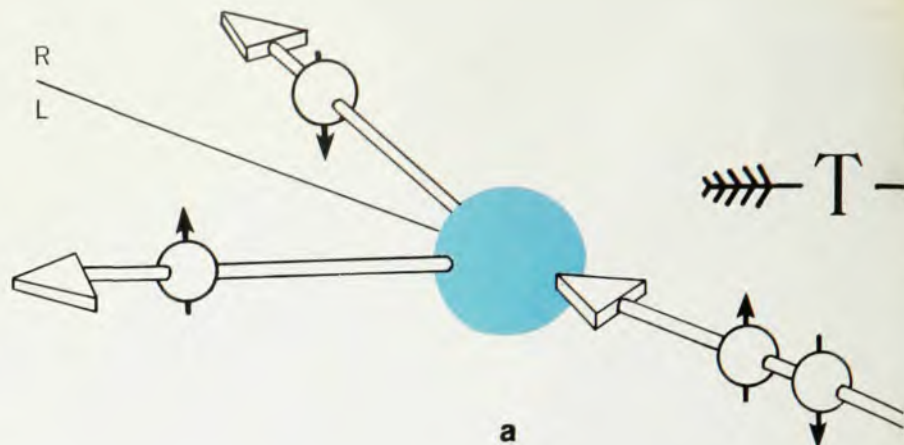
The production of pions in inelastic nucleon-nucleon collisions can carry the investigation further off the two-nucleon energy shell, because of the non-zero mass of the pion. Scattering experiments with more than two particles in the final state can be most successfully performed in meson factories where the high-intensity beam can compensate for low cross sections. Many

experiments that were previously impractical because of low anticipated counting rate must be reconsidered in terms of meson-factory capabilities.

The question arises how far off the energy shell one needs precise information to calculate a given nuclear property accurately. Although it is not possible to give a general answer, it would be very helpful for the design of new experiments to run some parameter studies in the nuclear-structure

The time-reversal operator T reverses the spins and momenta of all particles. In event A, both incident beam and target are unpolarized; spin-down particles are scattered mostly to the right, and spin-up, mostly to the left. Time-reversal invariance requires that when a beam polarized spin-down is scattered by the same unpolarized target, more particles go to the right than to the left as shown in event B.

Figure 1



calculations to explore this dependence.

Even for interactions on the energy shell, it is not clear what degree of accuracy is required for nuclear calculations. For example, if we want to calculate the energy of the first excited state of Ca^{40} with an accuracy of 5 per cent how well must we know the $^1\text{D}_2$ nucleon-nucleon phase shift between 100 and 200 MeV?

The general state of our knowledge of the two-nucleon interaction on the energy shell can, however, be summarized. This knowledge can be expressed in terms of several, presumably equivalent, formalisms, notably phase shifts, potentials, and boundary-condition models. These three semiphenomenological approaches have different assets and handicaps but are interconvertible. Each includes beside free parameters some well established theoretical elements usually pertaining to the form of pion-exchange processes. Which of them can express the experimental information with the least number of parameters and the lowest chi-square is a question to which the answer is constantly changing and which also involves some personal judgement concerning the credibility of small differences in chi-square, the trade-off of some parameters against a change of chi-square, a selection of data, and so on. In any case, it is both educational and good insurance to continue work along all three of these lines.

Of the three formalisms, the phase-shift treatment has received the most attention but it is not clear that it will remain the best approach in the range of 400–1000 MeV. The phase-shift description is useful either at low energy where only a few partial waves contribute significantly or at resonances that occur in single angular-momentum states. According to present knowledge the two-nucleon interaction is distinguished among elementary particles by the complete absence of resonances, except perhaps the S-states at the very lowest energies. Above 400 MeV the number of angular-momentum states that must be included is large,

and the phases are complex because inelastic channels are open. The numerous two-nucleon phase shifts are slowly varying and generally quite small. Furthermore, none of the current theories appears naturally in a partial-wave form. The two-nucleon interaction might therefore be better studied in this energy range by giving up the usual practice of measuring over a fine mesh of angles and concentrating instead on more elaborate and accurate measurements at a few well chosen angles.

It should be added that the absence of resonances in the two-nucleon system between 400 and 1000 MeV is not firmly established experimentally. A narrow resonance could well have been overlooked. It would be very worthwhile to measure nucleon-nucleon elastic and inelastic total cross sections at a fine mesh of energies to test this point.

Meson factories will play a crucial role in the 400–800 MeV range in resolving a question related to nuclear structure that is much discussed these days, namely the detailed properties of the repulsive core of the nucleon. A number of studies³ have been made recently on the compatibility of present information on the two-nucleon interaction with cores of various sorts differing in radius, height, and hardness. The conclusion appears to be that with the present data there is a very considerable leeway in these parameters. On the other hand, at least some nuclear-structure calculations depend quite strongly on these details, both physically, particularly on the radius, and in terms of computational ease, particularly on the hardness and height. There is little doubt that the greater penetration of 400–800 MeV scattering would considerably clarify the problem of the repulsive nucleon core.

Even below 400 MeV, meson factories will help to resolve ambiguities in nucleon-nucleon scattering. It would be of interest to start a comprehensive experimental program to remeasure the two-nucleon interaction up to 400 MeV, with an order of magnitude better accu-

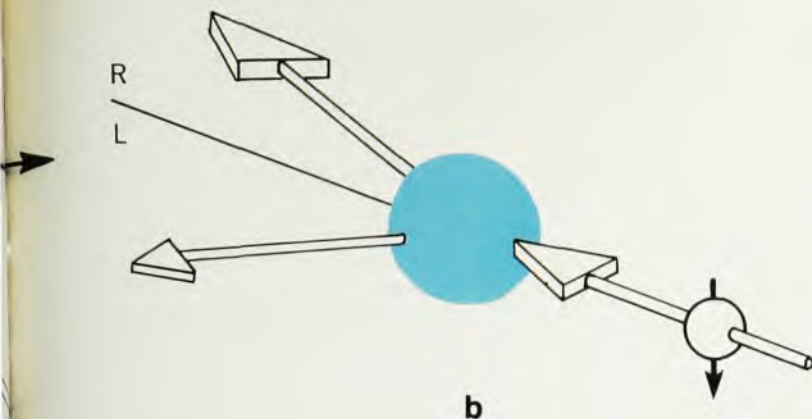
racy. Such a set of measurements would not simply be chasing the next decimal place but would almost certainly yield qualitatively new results. It might be appropriate to make an estimate of the time and money required for this task.

Charge independence

An important example of an aspect of the two-nucleon interaction that can be studied below 400 MeV is the validity of the charge-independence hypothesis. Mathematically expressed as a symmetry in isotopic spin space, this general principle has consequence in elementary-particle physics as well as nuclear structure. There are a number of ways one can check charge independence. The most general method would be to carry out an analysis of the neutron-proton data alone, determining from it both the isovector and the isoscalar phases, and then to compare the isovector phases with those obtained from the proton-proton data. The deviation from exact charge independence might well be a function of energy, and such an analysis would give detailed evidence in this problem. However, to sort out and separately analyse the isovector and isoscalar parts of neutron-proton scattering would require a larger set of data involving many more types of observables.

A more specialized test of charge independence is the comparison of the values of the pion-nucleon coupling constants obtained from the one-pion-exchange contribution in proton-proton and neutron-proton scattering analyses. This method has been used⁴, but an approximate error of ± 1 on the coupling constants establishes only a weak upper limit on violations of the symmetry. If the precision on the coupling constants could be improved by an order of magnitude, the conclusion could be much firmer.

Charge symmetry, a weaker assumption than charge independence, merely states that the nuclear force between a pair of neutrons is the same as that between a pair of protons. Although this



hypothesis is presumably well established from the study of nuclear structure it would be interesting to test it in scattering processes, particularly at fairly high energy where the nuclear structure evidence does not apply. Either neutron-deuteron scattering or a colliding neutron-beam experiment could be performed.

Another exciting possibility opened by meson-factory capabilities is the detailed study of spin dependence of the forces in nuclear physics. Polarized initial states can be prepared and the polarization of final states can be analysed without producing unacceptably low counting rates, because of the very high intensity of primary beam available in the meson factory. Multiple scattering experiments of all kinds will become more practical. When polarized beams are combined with polarized targets, nuclear studies will have reached a new level of sophistication.

Relevance to particle physics

The strong interaction has the highest degree of symmetry of the four basic interactions of physics. Nucleon-nucleon scattering experiments can be devised to test the discrete symmetries of the interaction. Charge independence has been already discussed in the context of nuclear structure. Other symmetries that can be checked are parity conservation *P*, time-reversal invariance *T*, and the product *PT*, which is equivalent to a test of charge conjugation, if the *PTC* theorem holds.

The conservation laws predict certain relations between the observables for a given experiment and a twin experiment obtained by the transformation appropriate to the symmetry operator. Many of these relations are independent of our knowledge of the dynamics of the nucleon-nucleon interaction and they hold at any angle and energy. Furthermore, as they are linear, they also hold when integrated over a range of energies and angles.

With the advent of meson factories many more of these tests could be carried out with much greater precision.

At the present, analyses of this type can confirm *P* and *T* only to an accuracy of about 5 per cent and because these tests presently average several amplitudes, they do not allow exploration of the possibility that particular amplitudes in *S*-matrix show relatively large symmetry violations, while others hardly any at all. A complete list of such tests is easy to construct from the general prescriptions.⁵

As a simple example of time-reversal invariance, consider an unpolarized beam scattered by an unpolarized target. There can be no left-right asymmetry in the differential cross section averaged over polarizations, assuming parity conservation. However because of spin-dependent forces, spin-down particles may be scattered preferentially to the right and spin-up to the left. Now consider a beam polarized spin-down scattered by the same unpolarized target. Time-reversal invariance requires that more particles be scattered to the right than to the left (see figure 1). Such a test could undoubtedly be carried out in meson factories to an accuracy of much better than 1 percent.

In special situations, additional tests of symmetries might become available. For example, at energies where phase shift analysis is feasible, relations between phases and mixing parameters can test symmetries. In particular, the fact that the partial-wave parameterization of two coupled angular momentum states in nucleon-nucleon scattering can be done in terms of two phases and one mixing parameter is a consequence of time-reversal invariance.

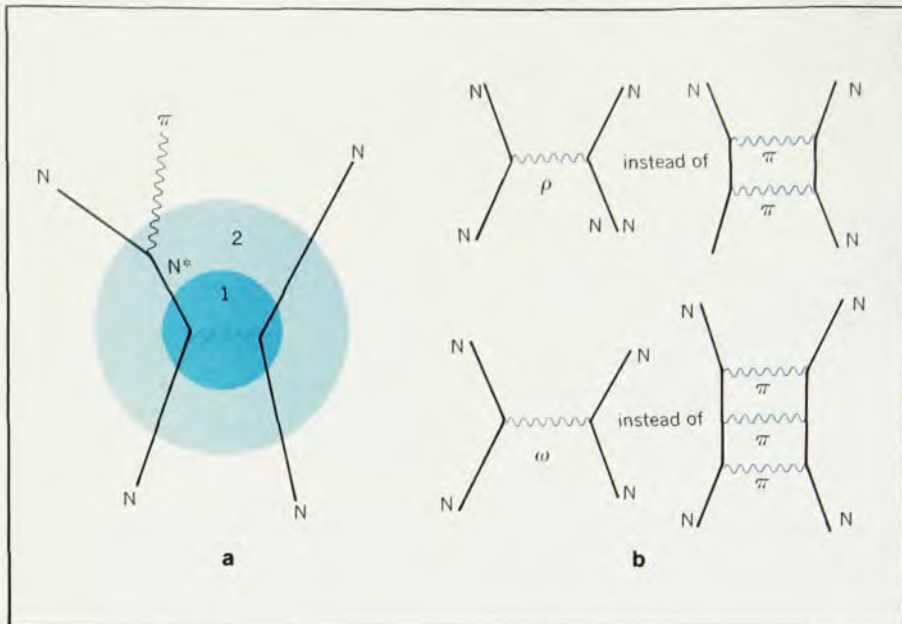
Another problem in particle theory that can be explored with the two-nucleon interaction is the analyticity of the *S*-matrix. The *S*-matrix is essentially the scattering amplitude in terms of phase shifts as a function of an energy variable (for instance, the kinetic energy in the center-of-mass system) that takes on real, positive values. In *S*-matrix theory, one assumes that this function may be extended into the entire complex plane by analytic continuation.

As is known from the theory of a

complex variable, the behavior of an analytic function is completely governed by the location and strength of its singularities. That is, if we know the poles and their residues as well as the branch cuts and the associated discontinuities, we can calculate the value of the function at any point. The *S*-matrix, considered as an analytic function of energy, is analogous to the electrostatic potential produced by line and point charges according to Coulomb's law. In this analogy, the residues are the magnitudes of the point charges while the discontinuities at branch cuts become the line charge densities.

In practice, we try to deduce the singularity structure of the *S*-matrix from experimental data, which give the behavior of the *S*-matrix in a physical region removed from the singularities. A successful example of such an analysis is the establishment of the one-pion exchange pole in the two-nucleon scattering amplitude. By "establishment" we simply mean that the inclusion of a pole of the correct location and strength yields a more economical representation of the data than is obtained without it. Because the pion is the lightest particle that can be exchanged by two nucleons in a strong interaction, it has the longest range, given roughly by $r = \hbar/mc$, where m is the pion mass. The location of the pion pole determines the functional form of the high angular-momentum phase shifts to within a multiplicative constant. In earlier work, these phases for higher angular-momentum states were arbitrarily set equal to zero. The multiplicative constant is given by the residue of the pole and is proportional to the pion-nucleon coupling constant. Our confidence in the meaningfulness of the pole contribution is bolstered by the agreement between the value of the coupling constant so obtained and that deduced directly from pion-nucleon scattering experiments.

The controversial and unexplored part of the *S*-matrix lies beyond this one-pion pole, and is dominated by the heavier mesons. We have some ideas



Resonances, a pervasive feature of hadronic physics, are extremely short-lived "particles" that manifest themselves indirectly. (a) In pion production from nucleon-nucleon collision, the N^* formed in region 1 is not directly observed but instead decays in region 2, resulting in a correlation between the momenta of the pion and nucleon. (b) The shorter-range force between two nucleons is mediated by the exchange of single, heavy bosons instead of by the uncorrelated exchange of their multipion decay products. **Figure 2**

about that region based on the analytic structure of the one-boson exchange models, which are quite successful in explaining short-range, low angular-momentum scattering. Therefore, the assumption of the analytic behavior arising from one-boson-exchange leads to good agreement with experimental data. However, the necessity and uniqueness of this structure has not been established. In fact, the postulated analyticity of the exact S-matrix has neither been deduced from any set of basic axioms nor used significantly in calculating theoretical predictions for comparison with experiment. Attempts to establish uniqueness have not produced definitive results so far.⁶ More accurate data would give impetus to this kind of analysis, and some thought should be given to the type of experiment and the kinematic range that would best serve the purpose.

One of the most interesting questions about the one-boson-exchange model is when it will break down. According to present ideas, these schemes cannot be an expression of a rigorous theory, and in fact there are tentative experimental indications that processes other than one-particle exchange must be included. Furthermore, because only the first term of the Born expansion is included, the one-boson-exchange model of the nuclear force is only approximately unitary and there are an infinite number of ways to make it satisfy the unitarity principle.⁷ It would be of interest to determine from experimental results which of these methods is the most satisfactory.

More basically, one should even consider the unlikely possibility that the apparent dominance of poles has no physical significance and is nothing more than the consequence of mathematically fitting an inaccurate set of

data. In other words, it is conceivable that any set of data, if sufficiently inaccurate, can be represented by a series of pole contributions. There has been some work along the lines of replacing the pole contributions of the heavier bosons by an uncorrelated two-pion contribution that would give a branch cut instead of a pole.⁸

However, most successful theories to date consider only the exchange of single bosons in evaluating the internuclear force. Instead of including the exchange of two uncorrelated pions, the nucleons are assumed to exchange a rho meson, which is a composite of two pions and can, in fact, decay into two pions. Similarly, the exchange of an omega meson replaces the exchange of three pions (see figure 2).

Resonances

This tendency of two elementary particles to combine and form a temporary "particle" or dynamical resonance is a very general aspect of hadron physics. Another example of this mechanism is the interpretation of inelastic channels in terms of resonances and quasi-two-particle intermediate states. Thus pion production in nucleon-nucleon scattering is often mediated by the formation of an N^* resonance, the most important being the lowest at 1236 Mev. The N^* is a very short-lived association of a pion and a nucleon, and the N^* -nucleon intermediate state is not directly observed. Instead the N^* manifests itself as a correlation between the energy-momentum of the pion and one of the nucleons in the final state. In pion-nucleon elastic scattering, the N^* appears as a strong peak in the $J = 3/2$ cross section.

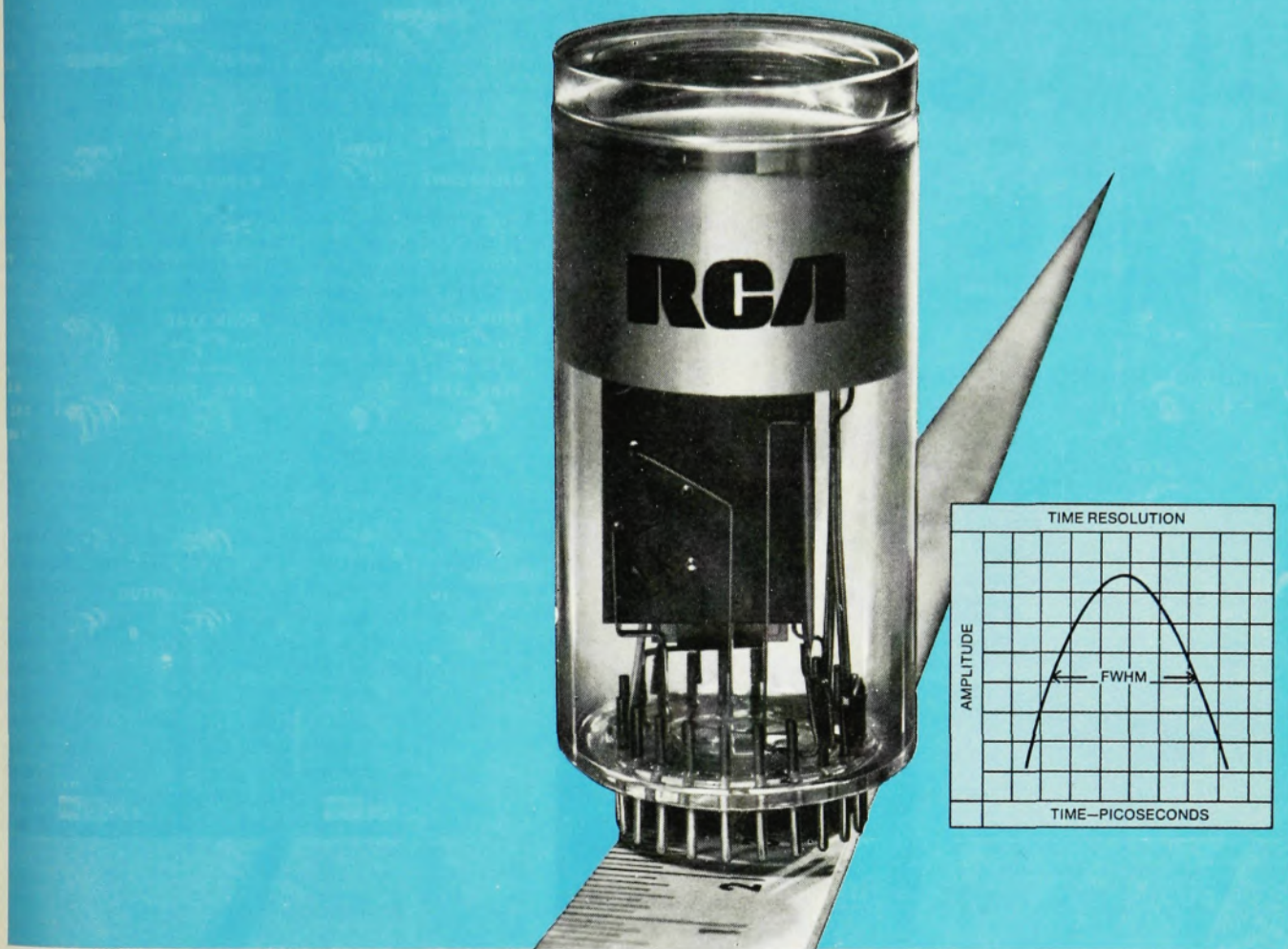
The widespread success of the resonance models for hadronic phenomena suggests the notion that the appearance

of a group of uncorrelated particles (that is, not in a resonance) is strongly suppressed in nature. The further study of resonance models in meson factories would help explore quantitatively the extent to which this fascinating conjecture is valid.

Another interesting topic to be considered in connection with inelastic processes is the role of final-state interactions. Scattering events are usually described by Feynman diagrams in which the number of vertices equals the number of times the particles interact. As is often stated, the large size of the strong coupling constant means that the series does not converge rapidly and that adding new vertices to a diagram need not greatly decrease its contribution to the total scattering amplitude.

Therefore, after the creation of a new particle in an inelastic collision, it is necessary to include interactions among the three outgoing particles in addition to the basic production mechanism. This so-called "final-state interaction" is especially large when two particles lie close together in energy-momentum because, loosely speaking, they have more time to interact. By imposing the appropriate kinematical restraints, which would be feasible with meson-factory intensities, one can separate out those cases where the interaction is virtually confined to two of the three final-state particles. By this method, the final-state interaction can be used to obtain information about systems not directly available for experimentation, such as the neutron-neutron and the pion-pion systems (see figure 3).

Two presently popular approaches to elementary-particle dynamics are Regge poles and current algebras. The low energy two-nucleon interaction has little bearing on Regge-pole theory, which describes events at much higher ener-



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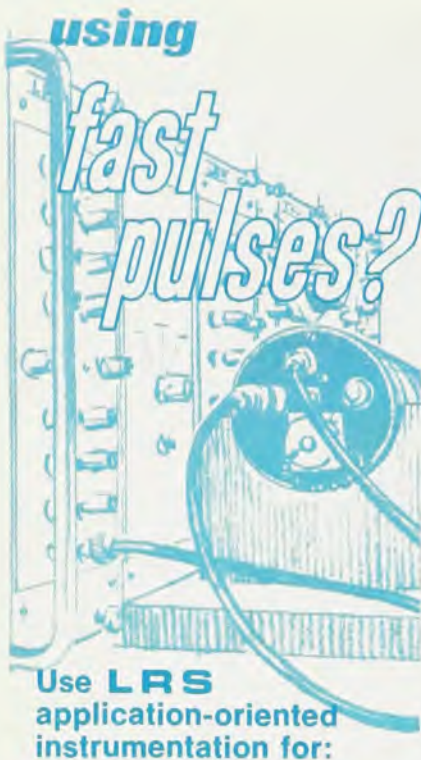
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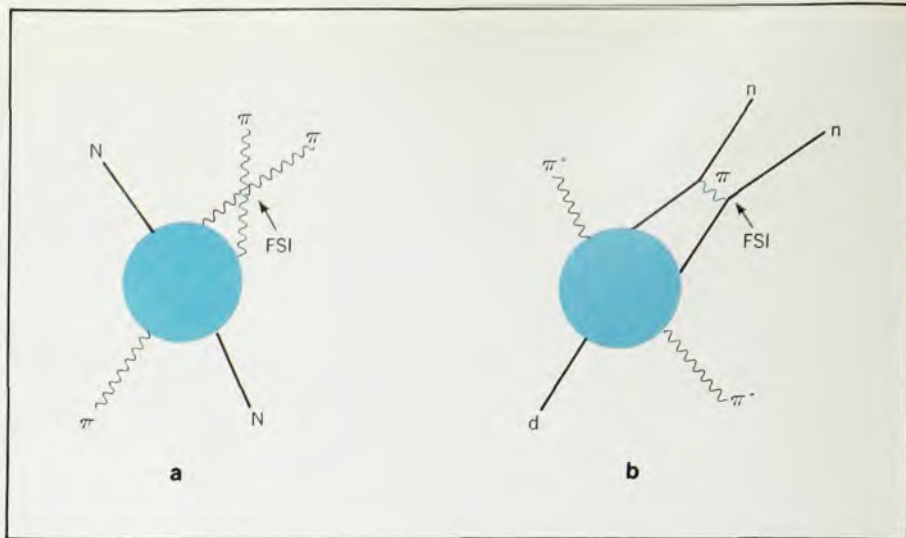
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Final-state interactions modify the distribution of the outgoing particles from what would be theoretically expected if the particles were free after the primary interaction. This extent of this redistribution is a measure of the cross section for events that are difficult to study directly. (a) The pion-pion cross section can be deduced from the final-state interaction in inelastic pion-nucleon scattering. (b) In the charge-exchange reaction of a negative pion with a deuteron, the two neutrons in the final state can interact if the kinematics are favorable.

Figure 3

gies. The situation is more favorable with respect to current algebra. This theory is designed, among other things, to establish a connection between a process and the same process with an additional low-energy pion emitted. A detailed study of pion production in nucleon-nucleon collision near threshold would be valuable in checking current-algebra predictions.

The ability of meson factories to provide experimental data for specific initial and final-state polarizations has great relevance for particle physics. Particle physics in general has suffered much in the past from the availability of only total or differential cross sections averaged over polarizations. The confrontation of theoretical ideas with experimental results only through polarization-averaged observations is insufficiently rigorous. This is especially true for particles of high spin, which have been discovered recently. But it is also valid for the low spins that occur in nucleon-nucleon scattering.

As an example consider the havoc wreaked among Reggeists not very long ago by a few pieces of polarization data after a long period of rosy optimism based on differential cross sections alone. A great contribution of the increased beam intensity of meson factories will be a detailed exploration of the spin structure of the S-matrix and a corresponding establishment of a much stricter standard of satisfactory agreement between theory and experiment.

There are those who believe that the breakthrough in particle physics will come from superhigh energy accelerators. Some others, including myself, are inclined to guess that instead of such an

all-illuminating Deus ex machina at 938.256 GeV, eventual progress will come from careful, detailed experiments at already available energies utilizing intensities that will become available only with the advent of meson factories. If this view is correct, the real impact of meson factories on particle physics will come with the second generation of meson factories, which are expected to operate in the 5–20 GeV energy range. But even with the machines presently under construction, some central problems of particle physics can be attacked with considerably increased power and efficiency.

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