

# ELEMENTARY PARTICLES

Two complementary approaches to strong-interaction theory, the multiperipheral bootstrap and the dual-resonance models, appear to be the most promising ways ahead. The idea that emerges is that particles hitherto thought to be "elementary" might instead be composite, all made up of bound states of each other.

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THE THEORY OF STRONG INTERACTIONS has been for many years particularly hard and fruitless. Today, under the momentum of a number of new ideas, theorists in this field appear to have regained confidence and enthusiasm. Both in its mathematical tools and physical ideas, the new trend in strong interactions points to a decisive change from the old one. Is elementary-particle theory undergoing a revolutionary change too?

The merging of quantum mechanics and special relativity has been for

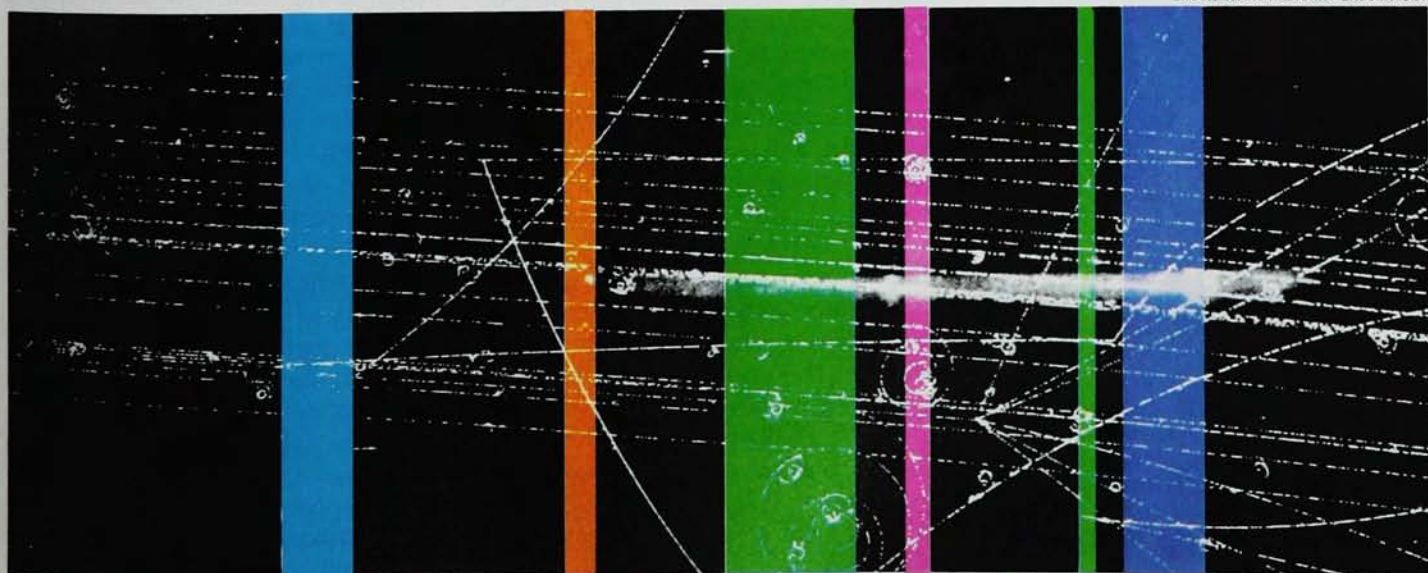
many years, and still is today, one of the big challenges to the theoretical physicist. If successful, such a program would result in a much closer understanding of the properties of what are considered today the ultimate components of matter, and are therefore called "elementary particles." The quantum-mechanical effect should enter into the probabilistic interpretation of the state of the system under observation while relativity would allow us to take correctly into account the fast relative motion of these en-

ties together with the possibility that, at any moment, particles can be created and annihilated from energy and into energy. Thus any relativistic quantum theory is necessarily a many-body problem. This conclusion is not compulsory in the nonrelativistic case (the so-called "potential-scattering" problem) where the number of particles is strictly conserved.

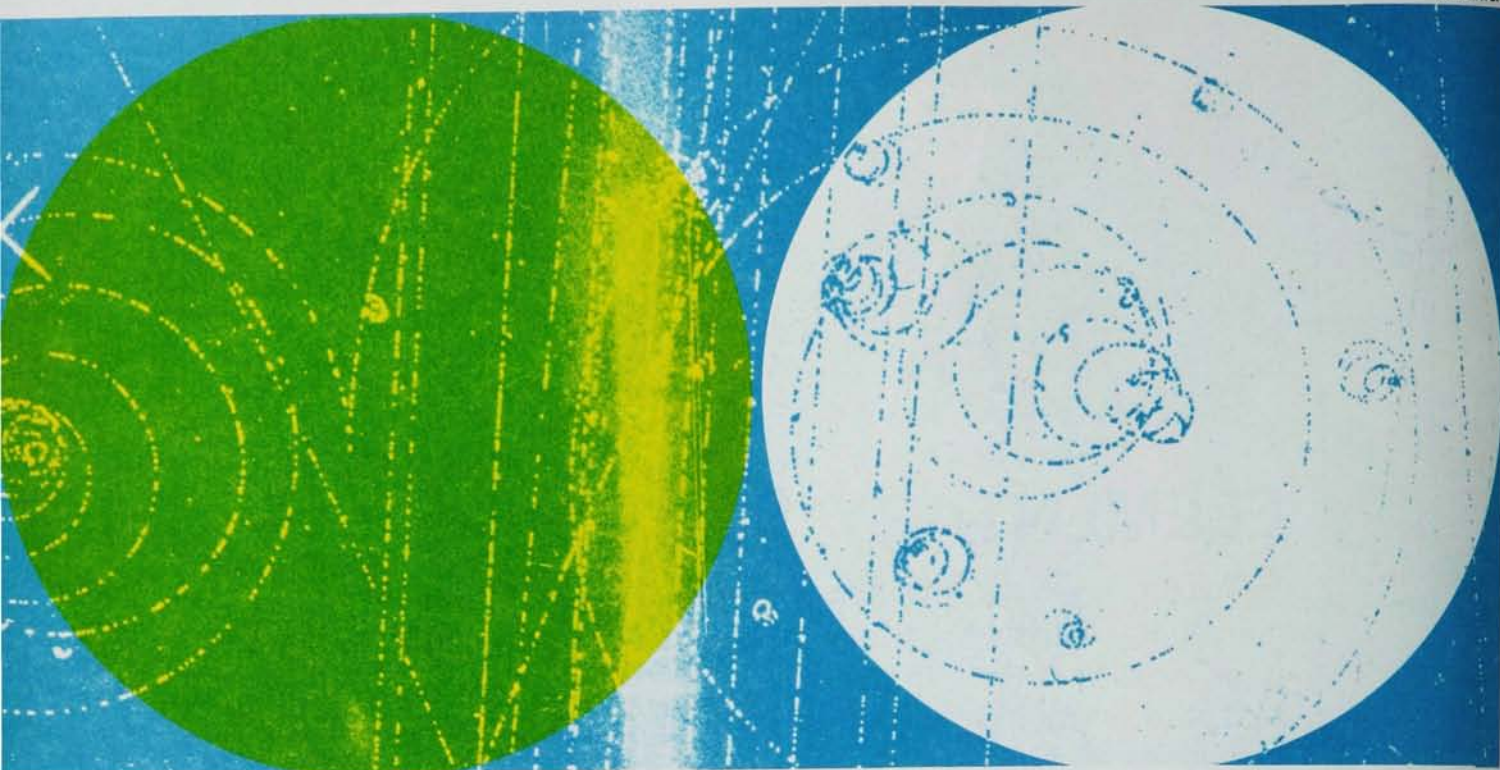
## *Strong and weak interactions*

For still unexplained reasons the fundamental interactions among elemen-

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tary particles fall into a few categories according to their strength. In order of increasing strength (or coupling constant) we find gravitational, weak, electromagnetic and, finally, strong interactions. The last in this list, strong interactions, are responsible for most of the properties of nuclei and of the elementary particles, and here we shall mainly restrict ourselves to them. For a wider outlook on elementary-particle theory see Abraham Pais's *PHYSICS TODAY* article.<sup>1</sup>

A good treatment of strong interactions appears to be the necessary first

step towards a complete understanding of elementary particles. The other, weaker types of interactions, we hope, can be taken into account in a second stage to give corrections.

Unfortunately the theory of strong interactions is the most difficult to deal with, because of the impossibility of escaping many-body complications. In fact in cases where the strength of the interaction is "small," like electrodynamics and weak interactions, one gets around that difficulty by means of perturbation theory. Creating more particles means going to a higher order in the coupling constant  $g$  and, if (in the appropriate units)  $g$  is much less than one, we can consider these effects negligible corrections to the lowest-order calculation, or alternatively we will be able to evaluate approximately the most important part of them.

The existence of a meaningful perturbation approach together with the knowledge of the basic interaction explains the well known success of quantum electrodynamics (where  $g^2 \approx 1/137$ ).

Weak-interaction theory (for example, beta decay) has not gone as far, mainly because of our ignorance of the exact form of the interaction. On the other hand the possibility of neglecting higher orders (here  $g^2 \approx 10^{-13}$ ) has allowed us to determine quite a number of details about such interactions. One result has been to connect some weak interactions to the electromagnetic ones through a rota-

tion in the space of isobaric spin and, more recently, the so-called "current algebra" has been able to make the connection much closer.

### *Symmetries and invariance*

Because of their strength, nothing of that sort can be used in strong interactions (here  $g^2 \approx 1-10$ ). The only results that can be drawn independently of the perturbation expansion in  $g$  are those coming from the invariance of the theory with respect to some transformations (so-called "symmetries"). An example is relativistic and translational invariance, which leads to conservation of linear momentum, energy and angular momentum. Another very fruitful application of this method, which uses mainly group theory, is the invariance of strong interactions under isobaric-spin rotations that transform, for instance, the proton into the neutron or the positively charged pion into the neutral one. The scheme was further extended to include the so-called "strange particles" (K mesons,  $\Lambda$  and  $\Sigma$  baryons, for example) in the successful scheme of a broken  $SU(3)$  symmetry of strong interactions.

These methods, however, can not say much about the detailed dynamics of strong interactions. Because the possibility of a perturbative field-theoretical approach to strong interactions appears to be hopeless, for the reasons I have given, several theorists have tried to overcome the difficulty



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by other methods. They impose directly the fundamental properties that the scattering probability amplitude (here called the S matrix) should have for a reasonable quantum-relativistic theory and then require some further particular properties that characterize the differences that strong interactions should show when compared with the other interactions.

### A new approach

This approach to the study of elementary particles (S-matrix theory) was first proposed by Werner Heisenberg.<sup>2</sup> The guiding idea underlying it is that one should deal only with quantities that, as well as describing our microscopic system, are also directly measurable. Scattering probability amplitudes are such quantities. More recently this line was reconsidered and emphasized by several theorists and in particular by Geoffrey F. Chew in the so called "bootstrap" program.<sup>3</sup>

We will find it useful to review briefly the fundamental general assumptions of the present S-matrix theory:

- **Unitarity** is the statement of conservation of probability. As probability is the square modulus of the amplitude unitarity turns out to be a nonlinear condition.

- **Analyticity** is the mathematical expression for causality. It is mainly used in the form of dispersion relations, which hold in the classical theory of light dispersion as a consequence of a precise time relation between cause and effect.

- **Crossing** is a purely relativistic property that connects two different processes obtained, one from the other, by interchange of one initial and one final particle of the process. Figure 1 indicates two processes connected through crossing.

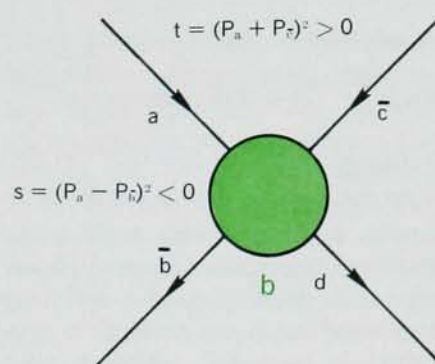
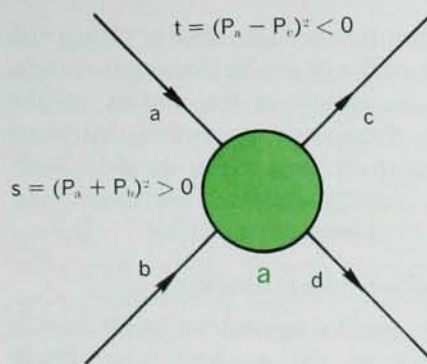
Before going into the details of the most recent developments, we have to introduce the concept of "resonance" and that of "Regge pole."

### Resonance

The first concept, resonance, is a classical one. Looking again at the classical theory of dispersion we know that, when a resonating mode is excited around some frequency  $\omega_r$ , the complex amplitude behaves near  $\omega_r$  as

$$A(\omega) = \text{const.} \frac{1}{\omega_r - \omega - i\Gamma} \quad (1)$$

$\Gamma$  is a measure of the friction (absorp-



**CROSSING.** The reactions  $a + b \rightarrow c + d$  (in part a of the figure) and  $a + \bar{c} \rightarrow b + d$  (in part b) are related by crossing. They are described by the same scattering amplitude  $A(s, t)$  in different regions of the  $s, t$  plane. —FIG. 1

tion), and  $1/\Gamma$  is then proportional to the lifetime of the oscillation. From equation 1 we see that  $\Gamma$  is also the width of the resonance. Equation 1 is an analytic expression for  $A$  that obeys a dispersion relation

$$A(\omega) = \frac{1}{\pi} \int \frac{\text{Im } A(\omega')}{\omega' - \omega - i\epsilon} d\omega'$$

$$\text{Im } A(\omega') = \frac{\text{const. } \Gamma}{(\omega' - \omega_r)^2 + \Gamma^2}$$

$\Gamma = 0$  corresponds to a stable resonance with an infinite lifetime; we shall call it a "narrow" resonance. It corresponds to a pole of  $A(\omega)$  on the real axis at  $\omega = \omega_r$ . In this limit

$$A(\omega) \approx \frac{1}{\omega_r - \omega - i\epsilon} = P \frac{1}{\omega_r - \omega} + i\pi\delta(\omega_r - \omega)$$

The same phenomenon occurs in strong-interaction physics where, around some energy  $E_r$  of the system of incident particles, a state can be excited and then decays with a lifetime  $1/\Gamma$ . Here the inverse relation of the width and the lifetime is a consequence of the uncertainty principle. A resonance has a definite and quantized value of total angular momentum, its spin, which is given by the orbital angular momentum  $\hbar l_0$  of the incident particles if they are spinless. In other words, a resonance of spin  $l_0$  is a pole in the partial-wave amplitude  $A_{l_0}(E)$

$$A_{l_0}(E) \approx \text{const.} \frac{1}{E_r - E - i\Gamma} \quad \text{when } E \approx E_r$$

### Regge poles

Tullio Regge had the idea of investigating  $A_l(E)$  as a function  $A(l, E)$  in the complex  $l$  plane. His studies in

potential scattering (Schrödinger equations) revealed the existence of poles in the  $l$  plane (Regge poles) at a position  $l_{\text{pole}} = \alpha(E)$ , which was moving with the energy (the Regge trajectory) to give

$$A(l, E) \approx \frac{\beta(E)}{l - \alpha(E)}$$

where  $l \approx \alpha(E)$  (2)  $\beta(E)$  is the residue at the pole (the Regge residue function). For  $l$  at an integer  $l_0$  and  $\text{Re}\alpha(E_r) = l_0$  we find from equation 2

$$A_{l_0}(E) \approx \text{const.} \frac{1}{l_0 - \alpha(E)} \quad (3)$$

$$\approx \text{const.} \times \frac{1}{l_0 - [l_0 + \alpha'(E - E_r) + i \text{Im}\alpha(E_r)]} = \frac{\text{const.}}{\alpha'(E_r - E - i\Gamma)}$$

(valid for  $E \approx E_r$ , with  $\alpha' = d\text{Re}\alpha(E_r)/dE$  and  $\Gamma = \text{Im}\alpha(E_r)/\alpha'$ )

We see then that the pole in  $l$  (the Regge pole) and the pole in  $E$  are manifestations of the same entity: a resonance. However, the Regge approach provided new links among resonances of different spin, by explaining them as manifestations of a single Regge trajectory passing through successive position integral values (the Regge recurrences). The other very interesting feature of Regge poles is that they control the behavior of scattering amplitudes at high energy (Regge behavior). To see this better let us introduce the variables  $s$  and  $t$  as shown in Figure 1. They are related to the incident energy  $E$  and the scattering angle  $\theta$  by simple algebraic relations. Then, roughly speaking, a Regge pole at



$l = \alpha(s)$  means that

$$A(s, t) \sum_{l=0}^{\infty} (2l+1) A_l(s) P_l(\cos \theta_s) \\ \approx P_{\alpha(s)}(\cos \theta_s) \approx (\cos \theta_s)^{\alpha(s)} \approx t^{\alpha(s)} \\ (\text{when } \cos \theta_s \rightarrow \infty \text{ and } \alpha(s) \text{ is fixed}).$$

Here  $\theta_s$  is the scattering angle in the center of mass of the  $s$  channel (figure 1a). This limit of fixed  $s$  and very large  $\cos \theta_s \approx t$  is not physical in non-relativistic potential scattering ( $|\cos \theta_s| < 1$ ). It can, however, be used instead in the relativistic situation where, through crossing,  $t$  is identified as the squared energy and  $s \approx \cos \theta_s$  as the cosine of the scattering angle of the process obtained from the original one by crossing (see figure 1). We see that Regge poles in the  $s$  channel (that is, in the complex plane of the angular momentum in the process shown in figure 1a) give the resonances that dominate low-energy scattering, whereas Regge poles in the  $t$  channel control the high- $s$  behavior.

### Duality

What is the connection between these two properties of Regge poles? We can see today, with hindsight, that it was mainly the lack of an answer to this question that prevented a fast development of strong-interaction theory in the 1960's. The solution of this problem came in a quite indirect way: Sergio Fubini and his collaborators, starting from studies of Murray Gell-Mann's current algebra (that is, of nonstrong interactions) discovered the so-called "superconvergence relations" for strong processes. Several groups extended these equations to more general cases, and after that it was relatively easy for Richard Dolen, David Horn and Christopher Schmid<sup>4</sup> in 1967 to see what solution of the above problem was suggested by these equations. Their proposal, known today as Dolen-Horn-Schmid duality, was that the high- $s$  behavior predicted from Regge poles in the  $t$  channel and the low-energy resonances also predicted by Regge poles (but in the  $s$  channel) were two self-complete descriptions of the same phenomenon. If one extrapolates the high-energy prediction to lower energies, the result is an average description of the resonating phenomena occurring there.

Consequently one should not add together the two contributions at all energies (double counting). The idea of duality summarized above has been crucial in the latest developments of

strong interactions. It has played the role of the dynamical assumption characterizing strong interactions in the two dynamical approaches that seem most promising today in this field: the "multiperipheral bootstrap" and the "dual-resonance models."

### Multiperipheral bootstrap

The multiperipheral bootstrap has its origin in the application of Regge theory to particle production, namely, processes in which the two initial particles produce a large number of final particles. In this case we believe that the amplitude is described, for high values of the fixed subenergies, by exchanges of Regge poles in the channels indicated in figure 2. Because a property of a Regge pole is that its contribution decreases rapidly with increase of the momentum transfer  $t_i$ , we conclude that, at high subenergies, the amplitude is large only in the so-called "multiperipheral" kinematically allowed region. This is the region where all the  $t_i$  are small. At this point one uses unitarity in the usual way. Defining  $S \equiv 1 + iT$  we have for the  $T$  matrix (from  $S+S=1$ )

$$-i(T^+ - T) = T^+T \quad (4)$$

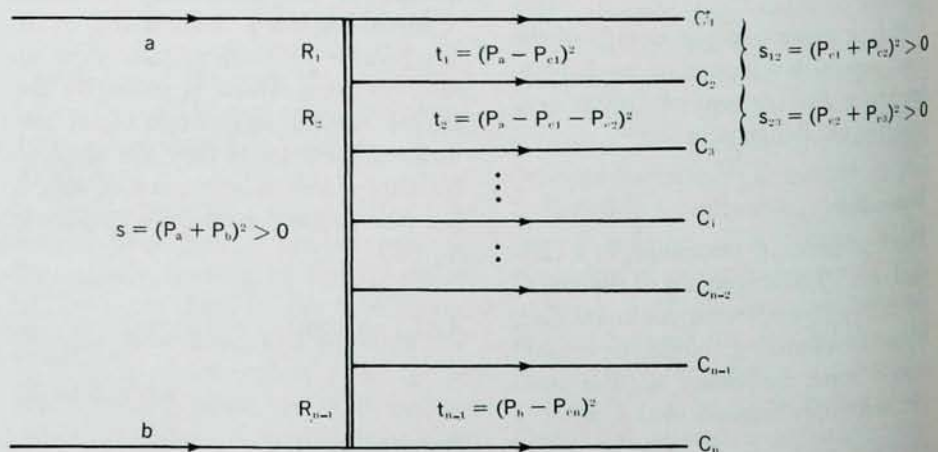
Equation 4 is an operator equation, and its expectation value is taken between the initial and final states. Inserting as usual a complete set of states in the right-hand side of equation 4 one gets

$$\langle \text{final} | i(T - T^+) | \text{initial} \rangle \\ = \sum \langle \text{final} | T^+ | \text{inter} \rangle \\ \langle \text{inter} | T | \text{initial} \rangle \quad (5)$$

If the incident energy is large, a

great variety of intermediate states contributes to the sum in equation 5, and, because the number of particles is not conserved in the relativistic theory, the difficulty of many-body states pointed out earlier is again showing up. Before the proposal of the multiperipheral bootstrap it was generally supposed that we could neglect many-particle production and restrict ourselves to the contribution of the intermediate states of lowest mass, but the results were not quite satisfactory. Having now a model (the multi-Regge model) to calculate the production of many-particle states, we can overcome this difficulty in the multiperipheral bootstrap. Duality plays a crucial role in that it allows us to replace the multi-Regge expression even when some of the subenergies in the intermediate state are not very large. Therefore we can be sure that, on the average, we shall not introduce a large error. We then see that, in this new approach, Regge theory with duality replaces the role of dominance of low-mass states in equation 5 as the special assumption to make for strong interactions. In fact the result of the calculations indicates that such dominance was not justified.

Equation 5 will act as a self-consistency equation, because we shall use as input a multi-Regge formula with a definite form of trajectory function  $\alpha(t)$  and residue function  $\beta(t)$ , and we demand that the output be again a Regge formula with the same functions  $\alpha(t)$  and  $\beta(t)$ . This kind of self-consistency method is what has been called "bootstrap". It looks as if only now, through an extensive use of dual Regge poles, it may lead to a successful theory of strong interactions.



PRODUCTION PROCESS  $a + b \rightarrow c_1 + c_2 + \dots + c_{n-1} + c_n$  and its kinematics.  $s$  is the total incident squared energy in the center of mass.  $s_{i,2}, s_{i,3}$  are final squared subenergies,  $t_i$  squared momentum transfers and  $R_i$  Regge particles.

—FIG. 2



The results obtained first by Chew and A. Pignotti<sup>5</sup> along these lines were quite encouraging, although their derivation was not formally very neat. Later Chew, Marvin Goldberger and Francis E. Low gave<sup>6</sup> a more elegant formulation of the multiperipheral bootstrap in the form of an integral equation expressing the self-consistency condition between input and output Regge parameters in a convenient mathematical form.

**Comparison of characteristics**

Summarizing the ideas that underlie the multiperipheral-bootstrap approach to strong interaction we could say that it is characterized by

- applying Regge theory in its extended form to production processes, together with the multiperipheral nature of high-energy scattering (dominance of small values of the momentum transfers)
- using Dolen-Horn-Schmid duality to extend meaningfully the multi-Regge formula to low subenergies, where presumably resonance production dominates over Regge exchange

- imposing unitarity as the fundamental dynamical condition.

It is worth noticing that, because of the bounds imposed by unitarity on the scattering amplitudes, one of the predictions of the multiperipheral approach is the relative smallness of the resonance widths. This fact turns out to be important for the dual-resonance models that we shall discuss now.

Dual-resonance models are characterized by

- a complete description of scattering processes (including production) in terms of narrow resonances only. These resonances should be infinite in number and unbounded in their angular momentum. They will then be classified in the Regge language by infinitely rising trajectories  $J = \alpha(s) \rightarrow \infty$  as  $s \rightarrow \infty$
- use of the Dolen-Horn-Schmid duality in order to use the resonance model in the high-energy region, where Regge exchange is supposed to be a better description than resonance formation
- imposing analyticity and crossing as fundamental dynamical conditions.

We see immediately by comparison of the three characteristics of the two approaches that they are somehow complementary to each other. Concerning the second characteristic of each method, Dolen-Horn-Schmid duality is used in the two extreme ways in representing the amplitudes by Regge poles only or by resonances only. The first characteristic of each method is also complementary. We have said that the multiperipheral bootstrap explains the narrowness of the resonances, which appears in the first characteristic of dual-resonance models. Conversely a consequence of dual-resonance models is the justification of the multiperipheral assumption for production processes, which appears in the first multiperipheral-bootstrap characteristic. Even Regge asymptotic behavior comes out naturally from the dual-resonance model.

Finally, in the third characteristic of each method, the general principles of a quantum relativistic theory are shared between the multiperipheral bootstrap and the dual-resonance model, with the first taking the more difficult part (because of the nonlinear nature of unitarity).

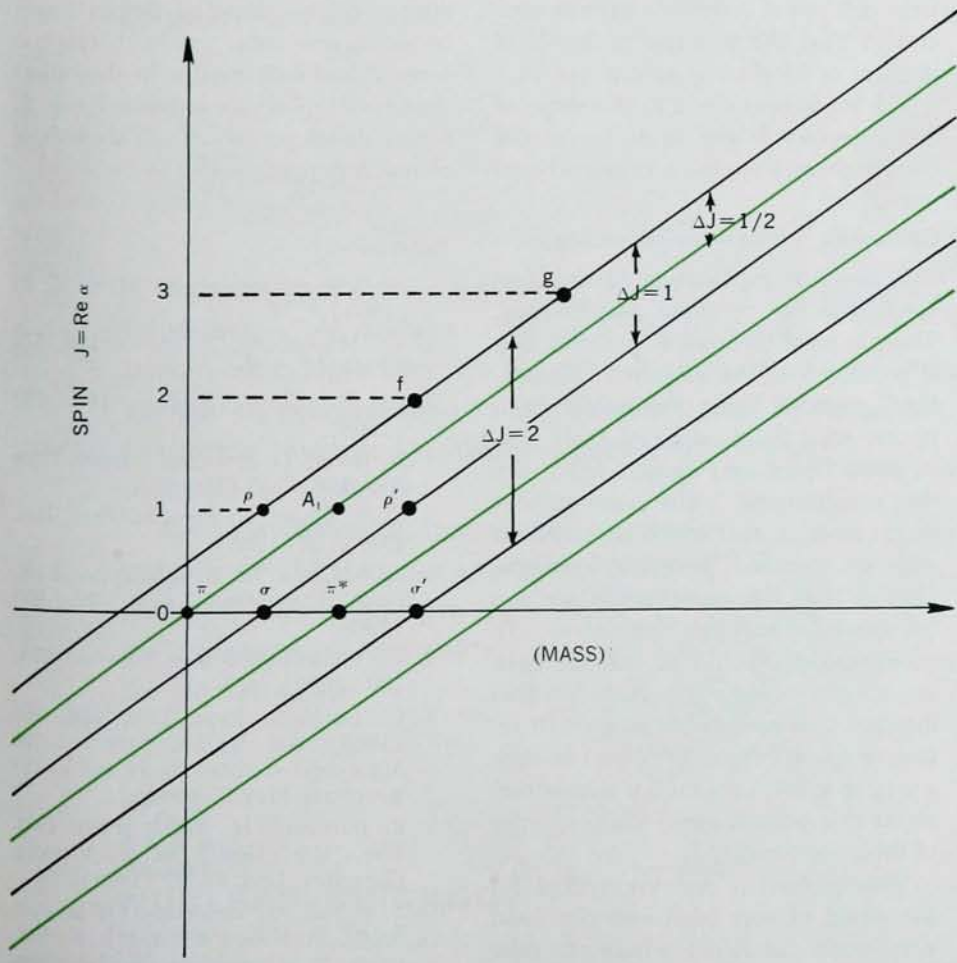
**Results of dual resonance**

Instead of going into the technical details of the dual-resonance model, I will just mention the main results of these models. As anticipated, one needs an infinite number of linear and parallel Regge trajectories; these trajectories will produce, for physical values of the angular momentum, a great variety of resonances (see figure 3). Nevertheless it is surprising that, with a scattering amplitude consisting only of simple poles, we can fix the constraints of crossing symmetry, Regge behavior and duality.

Not only does a solution exist; it is very simple. For a scattering of two identical spinless particles the scattering amplitude is given by<sup>7</sup>

$$A(s,t) = B(-\alpha_s, -\alpha_t) = \int_0^1 x^{-\alpha_s-1} (1-x)^{-\alpha_t-1} dx \quad (6)$$

where  $\alpha_s = a + bs$  and  $\alpha_t = a + bt$  are the Regge trajectories in the  $s$  and  $t$  channels respectively.  $B$  is the Euler beta-function. It is well known that  $B(x,y)$  is symmetric in  $x$  and  $y$  (crossing) and that it has simple poles at the negative integral values of  $x$  and  $y$  (analyticity). Moreover, the asymptotic limit ( $x \rightarrow \infty$  and  $y$  fixed)



DOUBLE-RESONANCE MODEL prediction for the spectrum of the lowest-mass non-strange mesons. The parallel black and colored lines, separated by units of  $\Delta J = 1/2$ , correspond to particles of opposite polarity. —FIG. 3



is  $x^y$  (Regge behavior) and we can write

$$B(x, y) = \sum_{m=0}^{\infty} \frac{1}{x+m} C_m(y) \\ = \sum_{m=0}^{\infty} \frac{1}{y+m} C_m(x) \quad (7)$$

where  $C_m(t)$  is a polynomial of degree  $m$  in  $t$ . Equation 7 states that the amplitude can be expressed *either* as a sum of the poles in the  $s$  variable *or* of those in the  $t$  variable (Dolen-Horn-Schmid duality) and that summing together both of them gives twice  $A$  (double counting of the interference model). Apparently equation 6 satisfies all the requirements we wanted to impose except unitarity. This violation is necessary for zero-width resonances because the scattering amplitude becomes arbitrarily large near a pole and thus gives eventually a probability greater than one.

### Unitarity

Attempts to find unitarity corrections to equation 6 have been, and are being, tried by several groups. I will mention only the work of C. Lovelace<sup>8</sup> which shows that, with a proper introduction of finite resonance widths, dual-resonance models are in very good agreement with many experimental data and are therefore more than pure mathematical models. Another good feature of equation 6 is that, in spite of having poles in  $s$  and  $t$ , it never gives a "double" pole of the form

$$\frac{1}{s-s_0} \cdot \frac{1}{t-t_0}$$

This form is in fact forbidden. Mathematically the reason is that  $s$  poles appear in equation 6 from  $x \approx 0$  and poles in  $t$  from  $x \approx 1$ , and they are therefore mutually incompatible.

This fact can be generalized to the case of production processes<sup>9</sup> where, say, two initial particles produce  $N$  final particles (where  $N$  is greater than two). This success has led to a generalization of equation 6 to an arbitrary process, and it is amusing that the formulas one obtains satisfy the requirements that were used in the multiperipheral bootstrap (multi-Regge behavior, multiperipheralism).

### Probabilistic interpretation

The mathematics underlying these formulas is both new and elegant. It consists of a set of nonlinear equations that relate the variables  $x_i$  appearing

in the generalization of the integral of equation 6 as raised to the power  $-\alpha_i - 1$ . (For a production process we have many energy variables like  $s$  and  $t$ ). The equations appear to connect different simple and composite probabilities  $P_i$  through a normalization condition  $\sum P_i = 1$ . The system of equations turns out to have a solution precisely of the type needed and with the right number of free parameters. I should underline at this point that this probabilistic interpretation is still far from clear.

It looks, however, as if statistical elements come out in other contexts of these models as well. More precisely, one can ask whether the energy levels of these dual-resonance models are degenerate or not. It is possible to investigate this problem (which is necessary also for the unitarization of the dual-resonance models), and the surprising result<sup>10</sup> is that the number of degenerate states present in the  $n$ th energy level increases essentially as the number of partitions  $P(n)$  of  $n$  in nonnegative integers, which for large  $n$  is proportional to  $\exp(\text{const} \cdot n^{1/2})$ . As the energy (or mass) of the  $n$ th level is also proportional to  $n^{1/2}$  (this comes from  $\alpha(s) = a + bs$ ) we have in conclusion that the number of levels of mass  $m$  is increasing as  $\exp(m/m_0)$  where  $m_0$  is connected to the slope of the trajectory  $b$  and must be, in the dual-resonance model, a universal constant.

### Composite "elementary" particles

The physical explanation of this fact can have far reaching implications. There is good theoretical evidence that it is related to the so-called "elementary" particles being themselves composite, with the number of their constituents being very large. What are the constituents? The answer that might emerge, and which is consistent with the so-called "bootstrap" philosophy, is that the constituents are the "elementary" particles themselves. In other words, it could be that we have an infinite variety of particles that interact with each other in a small region of space (about  $10^{-13}$  cm) in such a way as to form bound (or resonating) states that possess again the properties of their constituents.

This picture is not unphysical for the world of very high energies (and very small distances) where the relativistic regime of motion is responsible for the creation and annihilation of particles and where the counting of

the constituents might lose a precise meaning. In this case the methods of statistical mechanics could be the most appropriate to describe these systems with so many degrees of freedom.

The analogy with statistical mechanics is made even more striking if one compares the dual-resonance model with the results obtained by R. Hagedorn<sup>11</sup> in his studies of hadronic collisions at very high energies. He used a thermodynamic relativistic model, and found, as a consequence of a thermodynamic "bootstrap" condition, that the mass spectrum of hadrons has to increase as  $\exp(m/kT_0)$  ( $k$  = Boltzmann's constant) where  $T_0$  is a universal temperature ( $T_0 \approx 160$  MeV =  $1.3 \times 10^{12}$  K) determined from experiments at very high energy.

It is perhaps too early to claim that the analogy is more than a formal one. Nevertheless, it does appear that the physical assumption of a self-consistent description of elementary particles as bound states of themselves, which is common to the two approaches, has led, through different methods of investigation, to very similar results. Furthermore, it has naturally introduced into the game a new universal constant (the slope of Regge trajectories in one case, maximal temperature of hadronic matter in the other) that could play an important role in future developments of our theories of elementary particles.

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