ISOBARIC ANALOG RESONANCES

These highly excited compound nuclear states, observed in proton scattering from heavy nuclei, have useful applications in nuclear spectroscopy.

W. RORY COKER and C. FRED MOORE

WITHIN THE LAST FIVE YEARS, exploitation of isobaric analog resonances to obtain detailed information concerning the structure of atomic nuclei has become a standard, if somewhat mysterious, technique in the repertoire of experimental nuclear physicists. The precise nature and structure of isobaric analog resonances in nuclei are apparently not well understood by anyone at the present time, because the nuclear configurations involved are exceedingly complicated. Fortunately, however, it is easy to understand many of the properties of analog resonances that make them valuable to experimentalists and a topic of lively interest and sometimes controversy among students of nuclear structure.

The notion of isobaric spin, first clearly set forth in 1937, was soon found to provide a new and useful quantum number for light nuclei. In 1961 the discovery of states of "good" isobaric spin at high excitation energies in heavy nuclei suggested extension of the notion to include all nuclei. In the following year, resonances seen in proton elastic scattering from heavy nuclei were successfully interpreted as being due to isobaric analog states of the compound nucleus, the proton-plus-target system. Such resonances have, in the past few years, proven to be exceedingly useful in nuclear spectroscopy. Studies of inelastic proton scattering through isobaric analog resonances give particularly important information about nuclear states.

Isobaric spin

After the discovery of the neutron in 1932 it became clear that atomic nuclei are built up of protons and neutrons, a nucleus of mass A and charge Z having Z protons and $A - Z$ neutrons. In the same year Werner Heisenberg first introduced the formalism of isobaric spin, to write the Hamiltonian of the nucleus in a manner symmetrical in proton and neutron coördinates.[1] The idea of isobaric spin suggests itself when one notes that neutron and proton have the same intrinsic spin, 1/2, and very nearly the same mass. They differ principally in charge and magnetic moment. It is then convenient to replace proton and neutron by a single entity, the nucleon, which exists in two states, positively charged and uncharged.

Rather than make a direct introduction of an operator for charge, we find it easier to use a formalism analogous to that developed by Wolfgang Pauli for the intrinsic spin 1/2. The nucleon is defined to have isobaric spin $\mathbf{t} = \mathbf{1/2}$, in an abstract three-dimensional space with axes 1, 2 and 3, and there are then two possible projections of this spin vector along the 3-axis, $t_3 = \pm 1/2$. By long-standing convention in nuclear physics, the nucleon state with isobaric spin projection $+1/2$ is the neutron, that with $-1/2$ being the proton. The formalism is summarized in the box on page 56.

It is worth emphasizing in the beginning that the isobaric-spin concept does not embody the laws of physics in any more profound sense than does the "ordinary" nucleon-charge quantum number. The physical properties of a system of nucleons are independent of the method of bookkeeping used to keep track of the charge. However, the

W. Rory Coker took his BS, MS and PhD degrees at the University of Georgia. While a graduate student, he was awarded a fellowship under the National Defense Education Act and a summer fellowship from the National Science Foundation. He has done research at Oak Ridge National Laboratory, and since 1966 he has been at the Center for Nuclear Studies, University of Texas.

C. Fred Moore has been at the Center for Nuclear Studies, University of Texas, since 1965. His BS is from Notre Dame, his MS from the University of Louisville and his PhD from the Florida State University. He has worked at Los Alamos and Argonne National Laboratory, and is currently on leave at the Max-Planck-Institut für Kernphysik in Heidelberg.

isobaric-spin formalism is an extremely powerful and convenient, if only approximate, means of describing certain important characteristics of systems of elementary particles, for reasons that are far from accidental.

If in what follows we seem to concentrate at times on formalism at the expense of physics remember that the power lies with the formalism. Once mastered, the concept of isobaric spin makes a large number of otherwise puzzling facts "line up" in elegant fashion, not merely in nuclear physics, but also, by considerable extension, in particle physics where t_3 is a weight of SU (3).

The isobaric-spin formalism took on a greatly enhanced significance and usefulness with the availability of information concerning the systematics of nuclear stability and the nucleon–nucleon interaction. By 1936 it was evident from analysis of p–p and n–p scattering experiments that the nucleon–nucleon interaction was essentially charge independent. The hypothesis of charge-independence of the nuclear force simultaneously made transparent the variation of nuclear stability and binding energy with mass number A. If we introduce a generalized Pauli principle that requires antisymmetrization of the nuclear wave function with respect to all coördinates (space, spin *and* isobaric spin), we see that each "single-particle" state in a nucleus can hold four nucleons ($s_z = \pm 1/2$, $t_3 = \pm 1/2$), and the relatively high binding energy of those nuclei for which A is a multiple of four becomes readily understandable.

A valid quantum number?

Such points were stressed by Eugene Wigner who, in 1937, introduced the idea that the total isobaric spin for a system of nucleons will be a valid quantum number, at least when the energy associated with Coulomb forces is small compared with the energy associated with the nuclear forces.[2]

A system of A nucleons has total isobaric spin $\mathbf{T} = \sum_{i=1}^{A} \mathbf{t}^{(i)}$, and one can construct eigenstates of isobaric spin in exact analogy to the construction of eigenstates of a given intrinsic spin. T^2 operating on such states, $|T, T_3>$, yields eigenvalues $T\ (T+1)$. The eigenvalue of the 3-axis component of $\mathbf{T}$, T_3, ranges from $-T$ to T in steps of unity, and thus it has $(2T+1)$ values. Note the distinction between the operator $\hat{T}_3$ and its eigenvalue T_3. The system with $T_3 = -T$ consists entirely of protons, whereas that with $T_3 = T$ contains only neutrons. In general, if the system of nucleons has N neutrons and Z protons, $T_3 = (N - Z)/2$. One can define isobaric-spin raising and lowering operators

$$T\pm \equiv \hat{T}_1 \pm i\hat{T}_2$$

$$= \sum_{j=1}^{A} (t_1^{(j)} \pm it_2^{(j)}) \equiv \sum_{j=1}^{A} t^{(j)\,\pm}$$

For example,

$$T^- |T, T_3> = \sqrt{(T+T_3)\ (T-T_3+1)}\ |T, T_3 - 1>$$

$\hat{T}_3$ always commutes with the nuclear Hamiltonian, H, by conservation of charge. T^2 commutes with the strong-interaction part of H, if the nuclear force is charge independent. See the box on page 56.

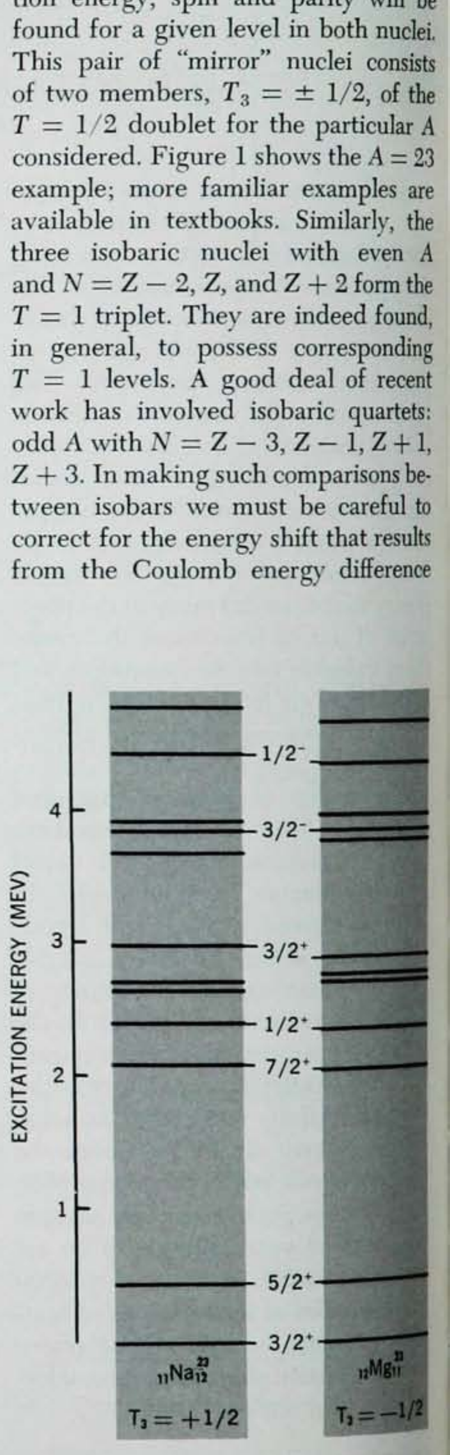
In summary, we have defined a vector operator $\mathbf{T}$, in a purely abstract three-dimensional space. Pictured naively as simply a vector, in analogy with the familiar classical angular-momentum vector, $\mathbf{T}$ has important properties. By construction, for a given physical system in the absence of Coulomb forces, the squared length of the vector is a conserved quantity. Further, the projection of the vector along the abstract-space 3-axis is numerically just half of the difference between the number of neutrons and number of protons in the system. Thus, although a given system might not possess a definite $T(T+1)$ because of a large Coulomb contribution to its energy, it must always possess a definite T_3.

Wigner applied the conservation of total isobaric spin to the problem of beta decay and obtained selection rules obeyed by T for the two types of nonrelativistic nuclear matrix elements involved.[3] This pioneering work was not at once followed up. Experiments taking full advantage of conservation of isobaric spin were not done until 1953, when Denys Wilkinson[4] and his collaborators were able to perform spectroscopic studies of light nuclei using recently published derivations of the T selection rules for electric-dipole transitions and for various nuclear reactions, particularly those involving $T = 0$ projectiles such as deuterons and alpha particles.

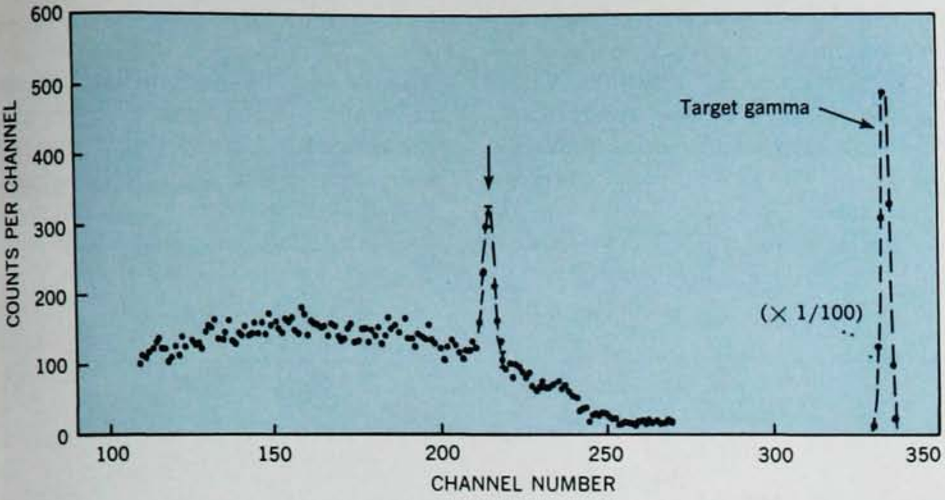
Light nuclei ($A \leq 25$) were the focus of both experimental and theoretical work in the next decade; experimentally, because light nuclei were the easiest to explore with the bombarding energies then available; theoretically, because the Coulomb energy in medium and heavy nuclei is comparable to the nuclear energy, and it seemed improbable that isobaric spin could be a useful quantum number.

Isobaric multiplets

Perhaps the most impressive result of the work up to 1961 was the exhibition of the beautiful isobaric multiplets formed by the light nuclei. An odd-A nucleus with $N = Z + 1$ should have a structure identical to that of the isobar with $N = Z - 1$; the same excitation energy, spin and parity will be found for a given level in both nuclei. This pair of "mirror" nuclei consists of two members, $T_3 = \pm 1/2$, of the $T = 1/2$ doublet for the particular A considered. Figure 1 shows the $A = 23$ example; more familiar examples are available in textbooks. Similarly, the three isobaric nuclei with even A and $N = Z - 2$, Z, and $Z + 2$ form the $T = 1$ triplet. They are indeed found, in general, to possess corresponding $T = 1$ levels. A good deal of recent work has involved isobaric quartets: odd A with $N = Z - 3$, $Z - 1$, $Z + 1$, $Z + 3$. In making such comparisons between isobars we must be careful to correct for the energy shift that results from the Coulomb energy difference



STATES OF MIRROR NUCLEI Na^{23} and Mg^{23}. The spins and energies are taken from nuclear data tables. –FIG. 1



TIME-OF-FLIGHT NEUTRON SPECTRUM from proton bombardment of V^{51}. Time calibration of the system is 1.8 nsec/channel, and increasing time of flight is towards the left. The flight path was 8.7 m, and the incident proton energy was 14.8 MeV. The principal background obscuring neutrons corresponding to the isobaric state is "boil-off" neutrons from compound-nucleus formation. The vertical arrow shows the neutron group that corresponds to the analog state. —FIG. 2

between the various nuclei, as well as the small shift due to the neutron–proton mass difference. The excellent agreement of the various isobaric states, when such a correction is made, is an independent confirmation of the charge independence of nuclear forces.

Isobaric analog states

In 1961 John Anderson, Calvin Wong and John McClure performed a series of experiments that led to the unexpected conclusion that isobaric spin might well be a valid quantum number for nearly all nuclei.[5] Anderson's group studied the p,n reaction at 15 MeV on a number of nuclei with masses ranging from 48 to 93, using time-of-flight techniques to obtain the neutron energy spectrum. They observed a very strong neutron group in each spectrum, as shown for the V^{51} case in figure 2. The energy at which this neutron group occurred, in each instance, was such that $E_p = E_n + \Delta_c$, where E_p is the kinetic energy of the incident proton in the center-of-mass system, E_n similarly is the center-of-mass kinetic energy of the outgoing neutron, and Δ_c is the Coulomb energy of a single proton in the particular residual nucleus involved. In short, in each case examined the reaction Q value, Q^{a}_{pn}, the smallest center-of-mass energy at which the reaction could take place, was Δ_c. Thus the residual nucleus of the reaction has one less neutron and one more proton in a particular state than the target, and it has absorbed just the energy Δ_c from the system to achieve such a transition. The reaction can therefore be pictured as the exchange of the charge of the incident proton with that of a neutron in the target: $p + N_c \rightarrow n + N_a + Q^{a}_{pn}$. The residual state of N_a, highly excited, has the same quantum numbers as the ground state of N_c, except that $T^{a}_{3} = T^{c}_{3} - 1$. N_a is usually called the isobaric analog of N_c. N_c itself is sometimes referred to as the "parent" nucleus.

Of course, the "ordinary" p,n reaction on a nucleus ${}^{N}_{Z}X^{A}$, leaving behind the nucleus ${}^{N-1}_{Z+1}X^{A}$ in its ground or low lying states that have no particular relation to those of the target, can occur at a much lower energy. It provides the continuous background seen underlying the analog peak in figure 2. The distinction between these two p,n processes should be kept in mind during discussions to follow.

The experimental existence of states Ψ_a suggests strongly that Ψ_a and Ψ_c are related by $\Psi_a = T^{-}\ \Psi_c$. Such an equation requires great care in its interpretation. It would seem to be implied that the state functions Ψ_a and Ψ_c are the same except for their isobaric-spin part, but this clearly cannot be the case in general. The energy difference between the two states is given by $E_a - E_c = Q^{a}_{pn} - \delta_{np}$, where δ_{np} is the n − p mass difference. Experimentally, the Coulomb displacement energy of a proton is $Q^{a}_{pn} = \Delta_c \approx 1.33\ (Z/A^{1/3})$ MeV. As one looks at heavier and heavier nuclei, Δ_c becomes larger and larger while the binding energy of the last neutron tends to decrease. Thus a point is reached at which the proton analog of a bound single-neutron state becomes unbound, and the space parts of the state functions must be very different.

The Lane equations

In 1962 Anthony Lane pointed out that the p,n process would be a direct consequence of a nucleon–nucleus potential of the form $V(r) = V_0(r) + V_1(r)\ (\mathbf{t} \cdot \mathbf{T})$, where $\mathbf{t}$ is the isobaric spin of the nucleon, $\mathbf{T}$ the isobaric spin of the nucleus.[6] Comparison of the optical-model potentials for elastic scattering of protons and neutrons from nuclei shows that the optical potential contains a "symmetry term," usually written $\pm U_1\ (N - Z)/A$, which can be recognized as just the term $V_1 t_3 T_3$ in disguise. We see $V_1 = (4\ U_1)/A$, in fact. As well as this diagonal term, the Lane potential also provides the off-diagonal pieces

$$V_1 \frac{(t^{+}T^{-} + t^{-}T^{+})}{2}$$

It is the term $V_1 t^{+}T^{-}/2$ that accounts for the quasi-elastic p,n process observed by Anderson, Wong and McClure. Picture $t^{+}T^{-}$ operating on the system of proton plus nucleus N_c. The operator t^{+} operates only the proton, changing it into a neutron. Similarly, the operator T^{-} operates only on the nucleus N_c, changing it into its isobaric analog nucleus N_a. Clearly, in order for such a process to occur, the incident proton energy has to be at least as great as the difference in internal energy of the initial nucleus N_c and final nucleus N_a, which is just the additional Coulomb energy the nucleus gains by converting one of its neutrons into a proton. Both off-diagonal terms together, under certain conditions, can account for a resonating p,p process at proton energies less than Δ_c, as we shall soon see. Writing the total nucleon–nucleus wave function as

$$\Psi^{+} = \alpha_p \phi_p^{+}\ (\mathbf{k}_p, \mathbf{r}_p)\ \Psi_c + \alpha_n \phi_n^{+}\ (\mathbf{k}_n, \mathbf{r}_n)\ \Psi_a$$

and using $\Psi_a = T^{-}\Psi_c$, Lane obtained two coupled Schrödinger equations for the wave functions ϕ_p^{+} and ϕ_n^{+} of the outgoing proton and neutron. These coupled equations are usually referred to as the Lane equations, and they have become a basic tool in the theoretical study of isobaric analog states.

Influence of Coulomb forces

Why do isobaric analog states exist? Lane and John Soper examined this problem in 1962.[7] One can shift the emphasis of the initial question to ask if the existence of isobaric analog states in all nuclei implies that isobaric spin is always a good quantum number. Lane and Soper found that, for nuclei near the stability line, T is relatively pure; indeed it even increases in purity with A, because of the pure isobaric spin of the excess neutrons. In a heavy nucleus, to be specific, there is a "core" of Z protons and Z neutrons, which ideally has pure isobaric spin $T = T_3 = 0 = T_c$ and is surrounded by $N - Z$ neutrons. The $N - Z$ neutrons have pure isobaric spin

$$T = T_3 = (N - Z)/2 \equiv T_0$$

so that "ordinary" low lying states of heavy nuclei have

$$T = T_3 = (N - Z)/2$$

There is not very much mixing of states of differing isobaric spin, as we can understand in part by considering the transition, produced by the Coulomb potential, between two such states. If the initial state has core isobaric spin $T_c = 0$, and total isobaric spin $T =$

ISOBARIC-SPIN FORMALISM

For the sake of completeness and consistency, we present here the isobaric-spin formalism, as customarily applied in nuclear physics, in some detail. The two isobaric basis states of the nucleon, in column matrix form, are

$$\alpha_n = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \alpha_p = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In exact analogy with the Pauli spin operator σ, we define the isobaric spin operator $\mathbf{t}$, with components

$$t_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, t_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } t_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

These components satisfy the commutation relations $[t_j, t_k] = i\, t_l$ (j,k,l is a cyclic permutation of 1,2,3) as is easily verified directly. Note that $t_3\alpha_n = a_n/2$ and $t_3\alpha_p = -a_p/2$. Thus t_3 keeps track of the *charge* q ($= 1$ or 0) in the sense that $q = (1/2) - t_3$. One can construct charge-raising and charge-lowering operators $t^{\pm} = t_1 \pm i\, t_2$, with the properties that $t^{+}\alpha_n = 0$, $t^{+}\alpha_p = \alpha_n$, $t^{-}\alpha_n = \alpha_p$, $t^{-}\alpha_p = 0$, and satisfying $[t_3, t^{\pm}] = t^{\pm}$.

The total isobaric spin of a system of A nucleons (Z protons, N neutrons) is

$$T = \sum_{i=1}^{A} t^{(i)}$$

Similarly, $$\hat{T}_3 = \sum_{i=1}^{A} t_3^{(i)} \rightarrow (N - Z)/2$$

and $$T^{\pm} = \sum_{i=1}^{A} t^{(i)\pm}$$

By combining basis states in the appropriate way, one can construct isobaric spin states $|T, T_3>$ of appropriate symmetry, for nuclear systems. The simplest example is the two-nucleon system with its familiar antisymmetric singlet state

$$|\,0, 0> = \frac{[\alpha_n(1)\,\alpha_p(2) - \alpha_n(2)\,\alpha_p(1)]}{\sqrt{2}}$$

and symmetric triplet

$$|\,1, 1> = \alpha_n(1)\,\alpha_n(2)$$

$$|\,1, 0> = \frac{\alpha_n(1)\,\alpha_p(2) + \alpha_p(1)\,\alpha_n(2)}{\sqrt{2}}$$

$$|\,1, -1> = \alpha_p(1)\,\alpha_p(2)$$

Defining

$$T^2 = \hat{T}_1^2 + \hat{T}_2^2 + \hat{T}_3^2 = \frac{T^{+}T^{-} + T^{-}T^{+}}{2} + \hat{T}_3^2$$

We have as usual

$$T^2\,|\,T, T_3> = T(T+1)\,|\,T, T_3>$$

$$\hat{T}_3\,|\,T, T_3> = T_3\,|\,T, T_3> = (1/2)\,(N - Z)|T, T_3>$$

Further, $T\pm|T, T_3> = \sqrt{(T \pm T_3)\quad(T \pm T_3 + 1)}\ |T, T_3 \pm 1>$, which can be proved in exactly the same way as the corresponding relation in the theory of angular momentum.

Note that T^2 can be written as

$$T^2 = (\sum_{i=1}^{A} \mathbf{t}^{(i)}) \cdot (\sum_{j=1}^{A} \mathbf{t}^{(j)}) = \frac{3A}{4} + \sum_{i=1} \sum_{\substack{j=1 \\ i \neq j}}^{A} \mathbf{t}^{(i)} \cdot \mathbf{t}^{(j)}$$

because $$\mathbf{t}^{(i)} \cdot \mathbf{t}^{(i)} = 3$$

Suppose that H_0 is the Hamiltonian of a system of nucleons, not including the Coulomb potential between protons. By conservation of charge, one must have $[T_3, H_o] = 0$. Thus T_3 is always a good quantum number for the system. Experience shows that $[T^2, H_0] \approx 0$, to a good approximation. Thus by implication the nucleon–nucleon potential V in H_o has the form

$$V = \sum_{i=1}^{A} \sum_{\substack{j=1 \\ i \neq j}}^{A} V(\mathbf{r}_{ij}) = \sum_{i=1}^{A} \sum_{\substack{j=1 \\ i \neq j}}^{A} \Big\{ V_0\,(\mathbf{r}_i, \mathbf{r}_j, \sigma_i, \sigma_j) + V_1\,(\mathbf{r}_i, \mathbf{r}_j, \sigma_i, \sigma_j)\,(\mathbf{t}^i \cdot \mathbf{t}^j) \Big\}$$

Then trivially $[T^2, V] = 0$, because $[T^2, T^2] = 0$.

The Coulomb potential between protons, written in the same formalism, is just

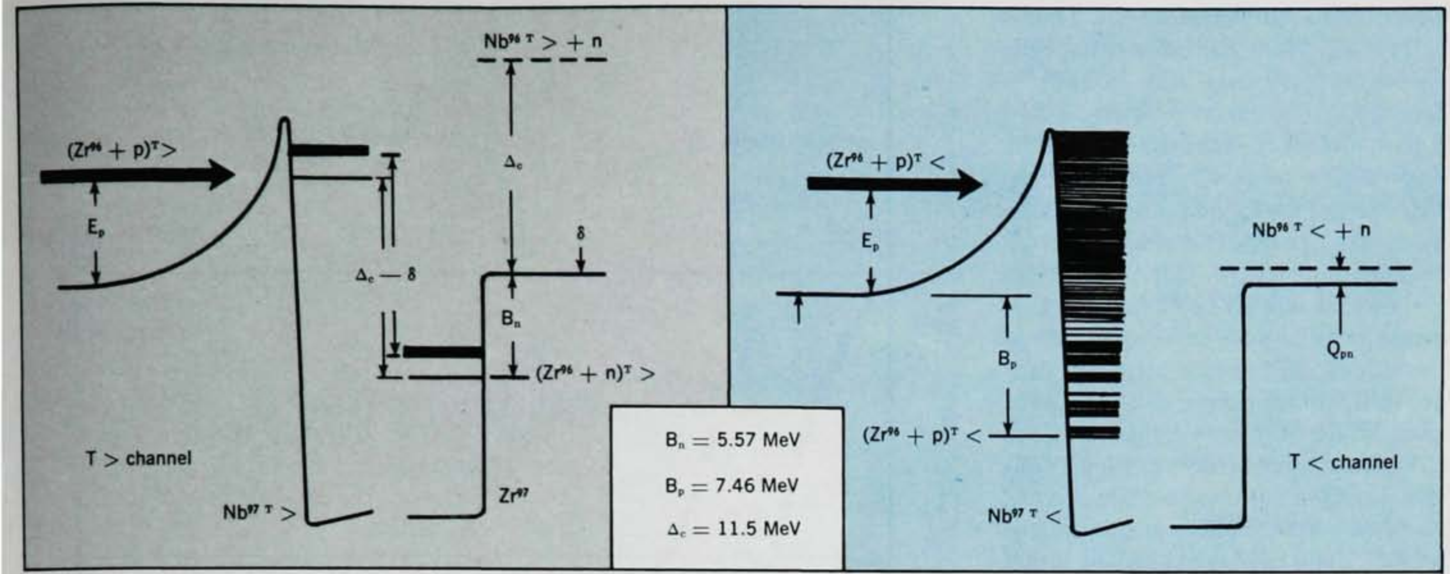
$$V_c = \sum_{i=1}^{A} \sum_{\substack{j=1 \\ i \neq j}}^{A} \frac{e^2}{r_{ij}}\,(1/2 - t_3^{(i)})\,(1/2 - t_3^{(j)})$$

It is clear that $[T^2, V_c] \neq 0$, because $[t_k^{(i)}, t_l^{(j)}] \neq 0$ if $k \neq l$.

If one does not worry about the indistinguishability of nucleons, the interaction of a single nucleon with a nucleus containing A nucleons is in its simplest form, as suggested by the equations above,

$$V_{nA} = V_0\,(\mathbf{r}_{nA}) + V_1\,(\mathbf{r}_{nA})\,(\mathbf{t} \cdot \mathbf{T}) + V_c\,(\mathbf{r}_{nA})$$

where $\mathbf{r}_{nA}$ is the separation between the nucleon and the center of mass of nucleus A, $\mathbf{t}$ is the nucleon isobaric spin and $\mathbf{T}$ the nuclear isobaric spin.



THE TWO ISOSPIN CHANNELS through which the reaction Zr^{96}(p,p)Zr^{96} can proceed. In the $T_>$ channel (left) are the analogs in Nb^{97} of the parent Zr^{97} states. In the $T_<$ channel (right) is the continuum of Nb^{97} states of "ordinary" isospin, which as shown can decay by neutron emission. Inelastic proton channels are ignored for simplicity. —FIG. 3

$T_3 \equiv T_0$, consider for definiteness a transition to a state of specific total isobaric spin T', in which the core has been excited to $T_c \neq 0$.

For example, take $T_c = 1$. In constructing a final state of definite T' according to the specific vector coupling $\mathbf{T}_c + \mathbf{T}_0 = \mathbf{T}'$, we must know the probability of obtaining a given value of T'. Here there exist only the possibilities $T' = T_0$ or $T_0 + 1$ ($T_0 - 1$ is impossible, because then $T'_3 > T'$). Those familiar with the Clebsch–Gordan algebra of angular-momentum coupling in quantum theory can easily show that the likelihood of $T' = T_0$ is T_0 times that of having $T' = T_0 + 1$. Thus the final isobaric spin state can be written, in correctly weighted form, as

$$|F> = \left(\frac{T_0}{T_0+1}\right)^{1/2} |T_0, T_0> +$$

$$\left(\frac{1}{T_0+1}\right)^{1/2} |T_0+1, T_0>$$

The agent that produces transitions from the initial state to such final states, and thus mixing of isobaric spin, is just the Coulomb potential, as we have said. But final states with, for example, isobaric spin $T' = T_0 + 1$ are seen to be less probable by a factor

$$T_0^{-1/2} = \left(\frac{2}{N-Z}\right)^{1/2}$$

in comparison to states having "ordinary" isobaric spin

$$T' = T_0 = \frac{N-Z}{2}$$

Indeed, the heavier the nucleus, the purer the isobaric spin, because the neutron excess of stable nuclei increases with increasing A. But the increase in neutron excess is itself a consequence of the Coulomb force between protons; more precisely, the neutron excess of stable isobars is a consequence of the balance required between the nuclear-symmetry energy (favoring $N = Z$) and the Coulomb energy (favoring $Z = 0$) in producing the greatest possible binding energy. Isobaric analog states can be said to exist because of, not in spite of, the Coulomb force.

Isobaric analog resonances

Beginning in 1962, a series of experiments carried out by John Fox and his coworkers yielded another surprising result: Low-energy proton elastic scattering from heavy nuclei displayed very strong resonant behavior.[8] The resonances were at energies consistent with the interpretation that they were isobaric analog states in the compound nucleus p + N_c.

Such a compound system made from an incoming proton and a heavy target with isobaric spin $T = T_0$, $T_3 = T_0$, does not itself have pure isobaric spin, but contains "ordinary" states with isobaric spin $T_0 - 1/2$ *and* also states with isobaric spin $T_0 + 1/2$, which are the isobaric analogs of the $T_0 + 1/2$ states of the neutron-plus-target system. To understand this more easily, consider a specific reaction, Zr^{96} (p,p) Zr^{96}. The nucleus $^{56}_{40}Zr^{96}$ has isobaric spin $T_0 = 8$, $T_{03} = 8$. We also suppose that the incident proton has an energy less than the Coulomb-barrier height, here ≈ 9 MeV, as in figure 3.

If isobaric spin is conserved, the reactions induced by the incident proton can involve only states or "channels" with $T_3 = T_0 - 1/2$, and $T = T_0 \pm 1/2$; that is 17/2 and 15/2 in this case. The reaction channel with $T = T_0 + 1/2$ is often called the $T_>$ channel, that with $T = T_0 - 1/2$ the $T_<$ channel. These two channels would be uncoupled—they could not influence one another—without the Coulomb force.

First concentrate on the $T_>$ channel. There are two nuclear configurations possible with isobaric spin $T_>$, the reaction state itself, p + $Zr^{96} \rightarrow Nb^{97*}$, which contains the isobaric analog resonances ($T = T_0 + 1/2$, $T_3 = T_0 - 1/2$), and the low lying states of the nucleus Zr^{97} ($T = T_0 + 1/2$, $T_3 = T_0 + 1/2$. These low lying states of Zr^{97} are the "parent" states of the isobaric analog resonances. The resonances should differ in energy from the parent states by about the Coulomb energy of a single proton, $\Delta_c = Q^a_{pn}$ which for Zr^{97} is ≈ 11.53 MeV. The ground state of Zr^{97} is bound by ≈ 5.57 MeV, so that the isobaric analog of the ground state is at an energy close to $11.53 - 5.57 = 5.96$ MeV. A resonance is indeed found when the incident proton has a center-of-mass energy equal to 5.9 MeV, as seen in figure 4. At this energy a free proton approaching the nuclear interior can become a neutron, through the t^+T^- part of V. Its escape, as a neutron, is impossible (without isobaric spin mixing) because it is now ($\Delta_c - E_p$) MeV below the state in which a neutron and the nucleus T^- (Zr^{96}) = Nb^{96*} are infinitely separated, that is, the Anderson-type quasielastic p,n threshold. Thus it will remain inside the nucleus until the system is converted into (proton + Zr^{96}) again, by the t^-T^+ term. When E_p is such that the nucleon can spend a considerable time, $\triangle t$, as a neutron—which is just the E_p that provides a neutron energy close to that of a true bound neutron state in Zr^{97}—then there occurs a resonance of width $\Gamma \approx \hbar/\triangle t$. Γ is of course not the total observed width of the resonance, because there are various other contributions, to be discussed later. So far only the resonance due to the ground state of Zr^{97} has been considered. Other resonances are seen at energies corresponding to the various excited states of Zr^{97}, as shown in figures 3 and 4.

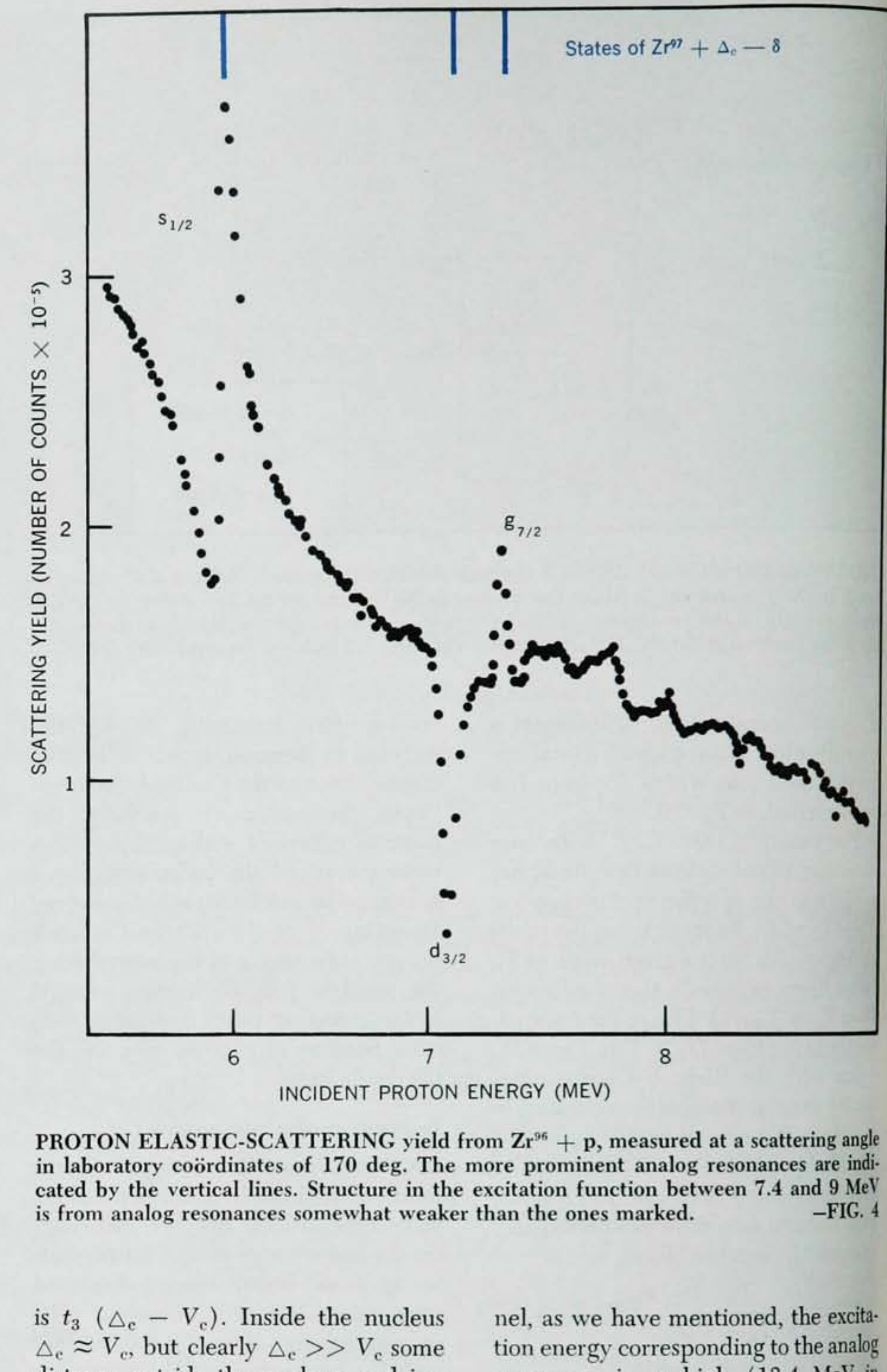
Spacing and structure

The remarkable and useful features of isobaric analog resonances are thus, first of all, their very existence as sharp, isolated resonances at such high excitation in the compound nucleus—some 13.5 MeV for the ground-state analog resonance in Nb^{97}. The ordinary levels of the compound nucleus at such an energy are so closely spaced as to be essentially continuous. Secondly, the relative energy spacing of the isobaric resonances is practically identical to that of the parent bound neutron states, and their structure—as gathered from measurement of their total angular momentum and parity—is identical to that of the parent states.

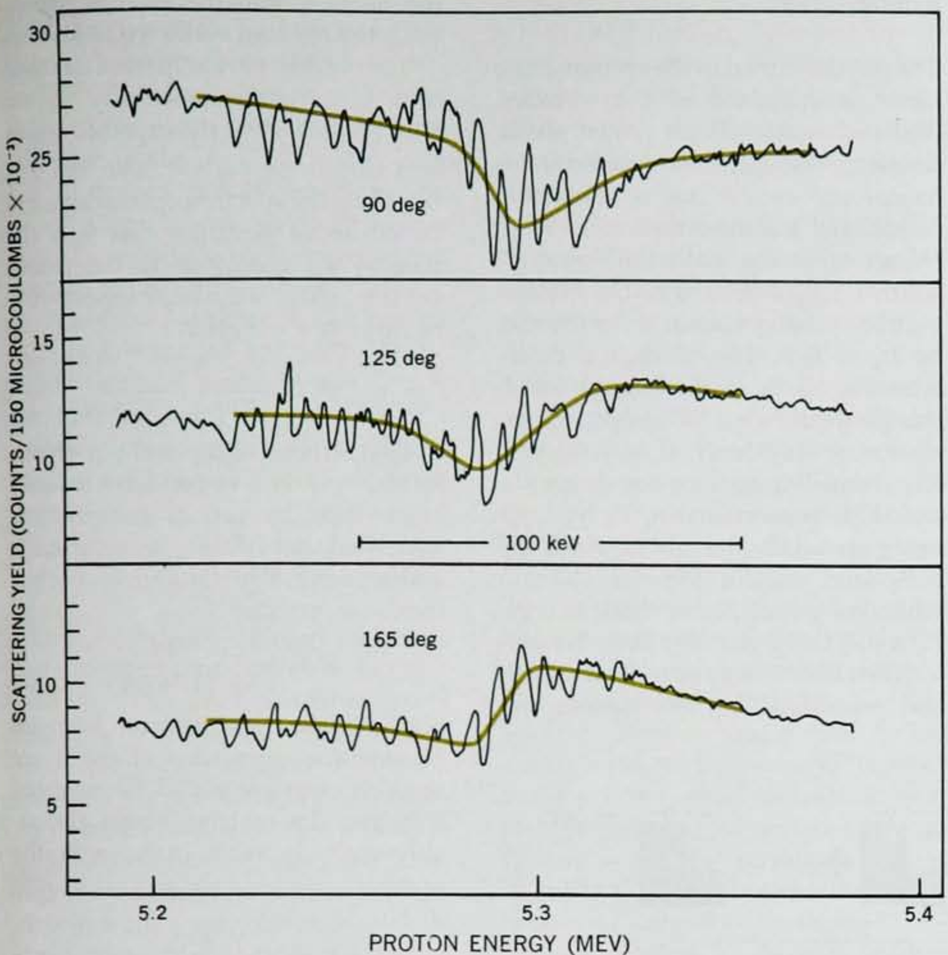
These observations may seem perplexing; protons and neutrons near a nucleus would be expected to have quite different wave functions, because protons experience a Coulomb force and neutrons do not. Apparently, inside the nucleus, the Coulomb force gives rise to a constant energy shift $\triangle_c$ between the parent n + N_c state and the part of the compound p + N_c state which is its isobaric analog, but does not mix the isobaric spin of the analog state with the continuum of surrounding states of lower isobaric spin. Some distance away from the nucleus, however, the situation is quite different. We can see this quickly by writing the nucleon–nucleus interaction, remembering that the interaction between the proton and N_c in the $T_>$ channel should be essentially the same as the n + N_c interaction increased in energy by $\triangle_c$. (This is accomplished if $N_c \rightarrow N_a$; consult figure 3.) Then $V = V_0 + V_1 \mathbf{t} \cdot \mathbf{T} + (1/2 - t_3)\, V_c + (1/2 + t_3)\, \triangle_c$, and we see that the part of V that fails to commute with the total isobaric spin is $t_3\ (\triangle_c - V_c)$. Inside the nucleus $\triangle_c \approx V_c$, but clearly $\triangle_c >> V_c$ some distance outside the nucleus, and isobaric spin is not a useful quantum number. This results in what is often called "external mixing" of the isobaric spin, as first pointed out by Donald Robson in a landmark paper[9] published in 1965, which gave for the first time a realistic theoretical treatment of analog resonances.

In the $T_<$ channel, the p + Zr^{96} state ($T = 15/2$, $T_3 = 15/2$) has no parent analog, because such a state would have $T_3 > T$, that is, $T_3 = 17/2$. However, the $T_<$ states of Nb^{97} formed by p + Zr^{96} are mixed by the Coulomb force with the overlying "ordinary" isobaric spin states of Nb^{97}, which result from adding a neutron to "ordinary" ($T = 7$, $T_3 = 7$) Nb^{96}. In the $T_<$ channel, as we have mentioned, the excitation energy corresponding to the analog resonance is so high (13.4 MeV in Nb^{97}, in this case) that the $T_<$ levels are nearly a continuum. The $T_<$ states of Nb^{97} can decay by neutron emission—$Q_{pn} = -0.57$ MeV for the "ordinary" reaction Zr^{96} (p,n) Nb^{96}—leaving Nb^{96} in its ground state. But we have seen that the analog resonance cannot decay in this way, because conservation of isobaric spin would leave the nucleus as highly excited, $T = 8$, Nb^{96} in an Anderson-type quasi-elastic p,n reaction, energetically forbidden until $E_p \geq \triangle_c$, which in this case is 11.5 MeV.

It is thus somewhat amusing that much of the early study (around 1964) of the $T_>$ analog resonances involved the "forbidden" $T_<$— channel p,n reaction. "External mixing," which cou-



PROTON ELASTIC-SCATTERING yield from Zr^{96} + p, measured at a scattering angle in laboratory coördinates of 170 deg. The more prominent analog resonances are indicated by the vertical lines. Structure in the excitation function between 7.4 and 9 MeV is from analog resonances somewhat weaker than the ones marked. —FIG. 4



THEORETICAL ISOLATED-LEVEL FIT compared with data for the proton elastic-scattering yield from $Mo^{92} + p$, measured at scattering angles in laboratory coördinates of 90 deg, 125 deg and 165 deg near the s-wave isobaric analog resonance at E_p = 5.3 MeV. A smooth curve has been drawn through the data points. The colored curve is the theoretical isolated-level fit, assuming Γ_p = 7 keV and Γ_{total} = 27 keV. —FIG. 5

ples the $T_<$ and $T_>$ channels outside the nucleus, is usually asserted to be responsible. Thus if one wishes to discuss analog states in terms of approximately *uncoupled* Lane equations, one must work with good eigenstates of the total isobaric spin, say $\Psi_<$ and $\Psi_>$, inside the nucleus, and mixed isobaric-spin functions Ψ_{pc} and Ψ_{na} outside. Ψ_{pc} and Ψ_{na} describe the analogous physical systems $p + N_c$, and $n + N_a$. $\Psi_>$ is seen to be the part of Ψ_{pc} analogous to the parent neutron state.

Fine structure

The connection between the states Ψ_{pc}, Ψ_{na} and the states $\Psi_>$, $\Psi_<$ is given by

$$\Psi_{pc} = (2T_0 + 1)^{-1/2}\ (\Psi_> + 2T_0\ \Psi_<)$$
$$\Psi_{na} = (2T_0 + 1)^{-1/2}\ (2T_0\ \Psi_> - \Psi_<)$$

The tightly packed $T_<$ states, such as $\Psi_<$, supply the smooth proton elastic scattering one would expect, the background on which the $T_>$ analog resonance is superimposed. The importance of the $T_<$ states in elastic scattering is clear because they supply practically all of Ψ_{pc}

$$\left(\frac{2T_0}{2T_0 + 1}\right)^{1/2} \approx 1$$

except near resonance. The effect of the individual $T_<$ states is observed in proton elastic-scattering data taken in very small (for nuclear physics) energy steps, ≈ 0.5 keV or less, as a very complex "fine structure" superimposed on the analog resonance, several tens of keV wide. This fine structure was first observed by Patrick Richard[10] in the elastic scattering of protons from Mo^{92}, as shown in figure 5. The presence of the analog resonance is seen to induce a kind of "giant resonance" of the $T_<$ states, much in the same way that a single driven oscillator can produce oscillations in a system of undriven oscillators, each of which is coupled to the driven oscillator.

It is time we returned briefly to the question of the observed total width of analog resonances. The resonance has the same width Γ characteristic of the parent neutron states, but the observed width $\Gamma_T > \Gamma$ in general (apart from any contribution due to the spread in energy of the incident proton beam). Such a situation holds even when elastic scattering is the only process energetically allowed. The excess width $W = \Gamma_T - \Gamma$ is traditionally called the "spreading width," and it can be considered another result of the external mixing of isobaric spin. Such mixing populates $T_<$ states, and the $T_<$ (p,n) threshold is usually low enough for such states to decay by way of neutron emission. We would then expect to see resonances in p,n near the p,p resonance energies, and indeed, as we mentioned, isobaric analog resonances were first studied with p,n reactions. If other reactions are possible, through which proton flux may be lost, they make up an additional width

$$\Gamma_R = \sum_f \Gamma_f$$

the sum being over all possible final states other than the elastic-scattering state. The width $\Gamma_p = \Gamma + \Gamma_R$ is often called the "natural" width of the analog state.

Spectroscopy

Isobaric analog resonances have provided a remarkable boost to nuclear spectroscopy. One can investigate the states of a given nucleus by measuring the proton elastic-scattering cross section of the isotope with one less neutron as a function of energy. Data are customarily taken at several scattering angles. From such data, the orbital quantum number l of the resonance, and therefore of the parent state, can be easily extracted—in many cases, almost by inspection. These results provide a valuable cross check on the information obtained by independent methods, such as d,p and t,d neutron-stripping reactions, on the same targets. We can deduce total angular momentum, $\mathbf{j} = \mathbf{l} + \mathbf{s}$, of the resonance from experiments that distinguish the two possible spin states of the outgoing proton's intrinsic spin $\mathbf{s}$. If the number of outgoing protons with spins parallel to a specified direction is N_+ and the number with spins antiparallel is N_-, the polarization is given by $P_R = (N_+ - N_-)/(N_+ + N_-)$. The quantity P_R can be obtained by again scattering the already scattered protons from a second target, usually C^{12}, and measuring the number of protons moving, through the same angle, to the right and to the left of the beam axis in the scattering plane. Then we can show that $P_R = P_C^{-1}\ (I_R - I_L)/(I_R + I_L)$

where I_L and I_R are the numbers of protons scattered left and right by the second scatterer, and P_C is the polarization produced by the second scatterer, which is generally known from other experiments. The behavior of the proton polarization P_R at the resonance is distinctive and unambiguous; if the polarization peaks at resonance for $j = l + 1/2$, it dips for $j = l - 1/2$.

Extraction, from analog-resonance data, of the so-called "spectroscopic factor," essentially the fraction of a state that has the configuration of single particle plus inert core, is somewhat more difficult. It has, however, been attempted for levels in various heavy nuclei by several independent groups in the past year, using various methods. For what it is worth, such values as have been obtained agree well with those found using d,p and other direct reactions.

Reduced width

The physics buried in the spectroscopic factor is contained in the so-called "reduced width." From proton elastic scattering through isobaric analog resonances one can, if one is reasonably careful and has some theoretical help, extract from the data the "natural" width Γ_p. Γ_p is related to the lifetime τ of the analog resonance, of course, by $\Gamma_p \approx \hbar/\tau$. The lifetime is determined by all the modes through which this particular type of compound nuclear state may decay. If only the proton channel were open for decay, Γ_p would be proportional to $P_p\ \gamma_p{}^2$, $\gamma_p{}^2$ being essentially the rate at which the compound nucleus forms a state in which one proton has its elastic energy. P_p is just the probability that this proton then finds its way past the Coulomb and centrifugal barriers surrounding the nucleus, into the exterior region. $\gamma_p{}^2$ is the reduced width we seek.

The width of the parent neutron state is in general given by $\gamma_n{}^2 = S\,\gamma^2_{sp}$, where γ^2_{sp} is the expected width of a true single-particle state and S is hopefully the spectroscopic factor mentioned above. Recalling that $\Psi_>$ is the part of Ψ_{pc} analogous to the parent neutron state, we would expect (because $\gamma^2_{sp} \approx \Psi_>{}^2$) that

$$\gamma^2_p \approx \frac{S}{2T_0 + 1}\,\gamma^2_{sp}$$

Comparisons of d,p and p,p experiments on various targets have recently been made by various groups. It is somewhat remarkable, awe inspiring and amusing, that for a given nuclear state, the number

$$S' = \frac{\sigma_{dp}\ (\mathrm{exp})}{\sigma_{dp}\ (\mathrm{DWBA})}$$

obtained via distorted-wave Born-approximation calculation of the direct reaction cross section of the neutron-stripping d,p reaction agrees reasonably well, in most cases, with the number

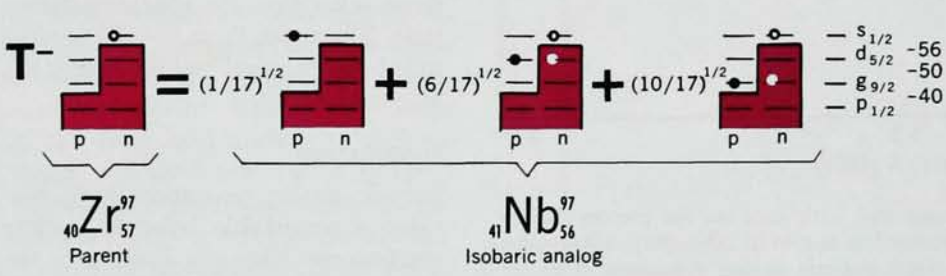
$$S = \frac{\gamma^2_p\ \mathrm{exp}}{\gamma^2_{pR}}$$

where γ^2_{pR} has customarily been calculated from some simple resonance theory of isobaric analog states. Some of these theories would be expected to work only for $S = 1$. In view of the present near-total lack of connection between theories of direct reactions and some of the theories of resonant reactions, this agreement is refreshing.

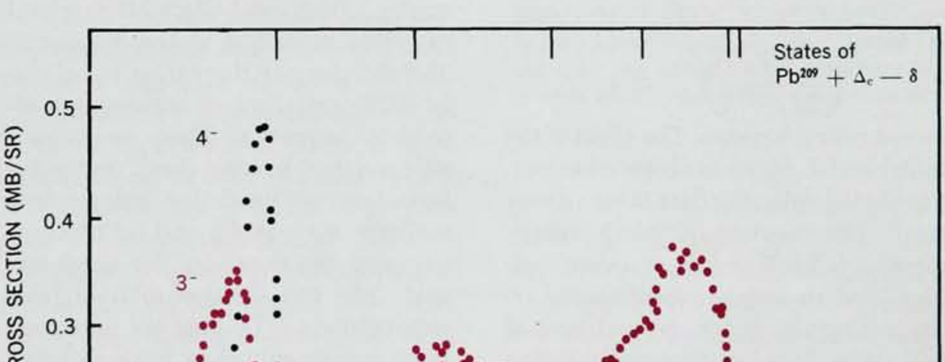
Numerous theories[11] of isobaric analog resonances have been put forward in the past few years, by Robson, by William MacDonald, by Hans Weidenmüller, by Richard Stephen, and others. All lead, in practice, to not very different numerical results exactly at a resonance. However, proton elastic scattering is not a very rigorous test of such theories, and studies of inelastic scattering, which have now begun, will permit a better evaulation.

Inelastic scattering

The usefulness of analog states in obtaining nuclear-structure information was greatly enhanced by the resonance behavior observed in inelastic proton channels by George Jones, Anthony Lane and George Morrison in 1964.[12] Such experiments are an almost unique means of investigating the excited-core configurations of nuclear states. The intimate connection be-



SYMBOLIC REPRESENTATION for the configurations making up the $p + Zr^{96}$ analog resonance, assuming the parent state in Zr^{97} is a pure single-particle state and that the analog state is the configuration obtained by applying the total isobaric-spin lowering operator, which changes the excess neutrons into protons, one by one. The weighting coefficients are obtained from the number of particles in any given shell. —FIG. 6



PROTON INELASTIC-SCATTERING yield from $Pb^{208} + p$, measured at scattering angles in laboratory coördinates of 170 deg (to the first 3^- state, color) and 90 deg (to the first 4^- state, black). The proton energy range covers the region of the prominent analog resonances in Bi^{209} as indicated by the vertical lines. —FIG. 7

tween analog states and excited core states can be appreciated from a second look at the equation $\Psi_a = T^- \Psi_c$.

Again for clarity consider the specific case of the $A = 97$ isobar. Zr^{96} has 40 protons and 56 neutrons. The proton states are all filled, up to the $1g_{9/2}$ single-particle shell, whereas the neutron states are filled up to the $3s_{1/2}$ shell, as shown in figure 6. The ground state of Zr^{97} indeed has total angular momentum and parity $J^\pi = 1/2+$. If we formally apply T^- to the Zr^{97} ground state to obtain its analog in Nb^{97}, we must recall that by the Pauli principle,

$$T^- = \sum_{i=1}^{N-Z} t^{(i)-}$$

That is, only the $N - Z$ excess neutrons have vacant proton states to be lowered into. The result, assuming Zr^{97} is indeed a good shell-model nucleus, is shown graphically in figure 6. It is the first term that resonates in proton *elastic* scattering, but the other terms are of immediate interest when one notes that they involve excited core configurations with a neutron–neutron hole structure, such as may be formed in proton *inelastic* scattering, and that they will resonate in inelastic scattering to residual states with the same particle–hole structure.

A simple case, pointed out by Peter von Brentano, is illustrated in figure 7 for the 4^- (2.87 MeV) excited state of Pb^{208}. The energy region covered contains several analog resonances, but the remarkable simplicity of the 4^- excitation curve tells us at once its particle character; it resonates only at the $g_{9/2}$ analog state and thus has the structure of a $g_{9/2}$ neutron coupled to a hole. As the topmost filled neutron shell in Pb^{208} is $3p_{1/2}$, the natural candidate is a $p_{1/2}$ hole.

Nature, however, rarely serves us an inert core. The ground states of closed-shell nuclei are often complicated configurations of particles and holes already. The T^- operator then gives an analog state that is even more complicated. In general, though, the parent state may be expanded in terms of a single neutron in a linear superposition of potentials generated by the core in its ground and all possible excited states. When the situation is not hopelessly complicated, inelastic scattering through isobaric analog resonances makes possible at least qualitative identification of the contributing configurations. An example, also in figure 7, is the 3^- (2.60 MeV) state in Pb^{208}, which is collective (presumably an octopole vibration). It resonates in a very complicated but informative way, at the various analog energies, indicating the various configurations that contribute to it.

The detailed theoretical description of inelastic scattering through analog resonances is one of the more exciting problems facing nuclear physicists, requiring a remarkable and powerful wedding of the shell-model theory of nuclei and formal scattering theory. The product of this marriage will be a most potent tool indeed, but we do not have the space to discuss it further.

Light nuclei revisited

The lively interest in application of isobaric-spin conservation in heavy nuclei that has awakened in the past half decade has also had a reviving effect on work in light nuclei. An example is the work of Georges Temmer and numerous others on isobaric spin-forbidden compound-nucleus reactions in light nuclei. For a reaction such as $A + a \rightarrow C \rightarrow B + b$, if total isospin is conserved, we obviously require $\mathbf{T}_a + \mathbf{T}_A = \mathbf{T}_C = \mathbf{T}_b + \mathbf{T}_B$. Because isobaric spin is not precisely conserved, several more possibilities for T_C can be realized in experiment than are "allowed", with interesting consequences.

Consider [13] the reaction $p + O^{16} \rightarrow F^{17*} \rightarrow p + O^{16}$. The isobaric spin in the initial and final states is $T = 1/2$ (the ground and low lying states of O^{16} have $T = T_3 = 0$), so only $T = 1/2$ states in the compound nucleus F^{17} can be excited "legally." To excite a state in F^{17} with $T = 3/2$ the isobaric-spin selection rule must be broken *twice*, if the energy of the incident proton is not sufficient to excite $T + 1$ states inelastically in the residual O^{16}. As such $T = 3/2$ states are indeed excited in these experiments, we obtain information on both the $T = 1$ admixture into the O^{16} ground state, and the $T = 1/2$ admixture into the $T = 3/2$ resonance in F^{17*}. The violation of conservation of isobaric spin, because of Coulomb mixing, is of course very slight in such light nuclei. The result is that the resonances observed are narrow, with widths of about 1 keV, as compared to the approximately 100-keV observed width of analog resonances.

Things to come

Experiments involving isobaric analog resonances are now being carried out in many laboratories throughout the world. In the spring of 1966 a conference devoted entirely to isobaric spin in nuclear physics was held at Florida State University. At that time, we were still not sure whether or not analog resonances could be used to give spectroscopic information comparable or superior in quality and quantity to that supplied by direct reactions. Even so, it was pointed out that there are many cases where the desired nuclear-structure information can be obtained only by analog experiments (for example the previously mentioned inelastic-scattering studies).

The past two years have seen extremely rapid progress in the study and application of analog states and resonances in nuclear spectroscopy. It is not unusual to find several sessions of a large American Physical Society meeting devoted exclusively to analog resonances and their role in the understanding of nuclear structure. A yearly international conference on nuclear isobaric spin seems in the offing. Judging by the amount of work being done in this field at present, the next few years should provide an interesting and exciting sequel to the very limited introduction we have given here.

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