FORM FACTORS OF ELEMENTARY PARTICLES

Scattering of electrons on nucleons and electron–positron scattering in colliding-beam experiments give a measure of the size and charge distribution of the proton, neutron and pion. Theoretical understanding is fairly advanced for the pion but still uncertain for the nucleons.

RICHARD WILSON

HOW BIG ARE the elementary particles? How is their charge distributed? These questions are tackled with form factors, which are measures of the charge and magnetic-moment distributions in the particles. Scattering of electrons on nucleons, and recent measurements made with electron–positron colliding beams, give form factors for the proton, neutron and pion. The behavior of the pion form factor can be fairly well understood by the "rho-dominance model," but the nucleon form factors are not so easy to understand. Different models are currently being examined in an attempt to solve this basic problem in strong-interaction physics.

Measuring the size of an object implies that we use some sort of probe. When the object is a small elementary particle the size of the probe must also be small. It is inherently impossible, according to the uncertainty principle, to position the probe precisely unless it simultaneously has a large momentum.

If the process of measurement is considered further, it is easily seen that we cannot measure the sizes (in coördinate space) of the elementary particles directly; what we measure is the distribution of the changes of momentum as a beam of probe particles is directed at the object. This distribution is called the *form factor*. It is then related to the size of the object by a Fourier transform.

X-ray and electron scattering

The concept of a form factor was introduced in x-ray scattering. It has long been known that the scattering through an angle θ of x rays of wavelength λ from a distribution of charge is proportional to the square of the Fourier transform of the charge distribution $\rho(r)$. This Fourier transform is the form factor $f(q)$; it obeys the relation

$$f(q) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(r)\, d^3\mathbf{r}$$

where $q = 2\lambda \sin(\theta/2)$ is the momentum transfer in wavelength units.

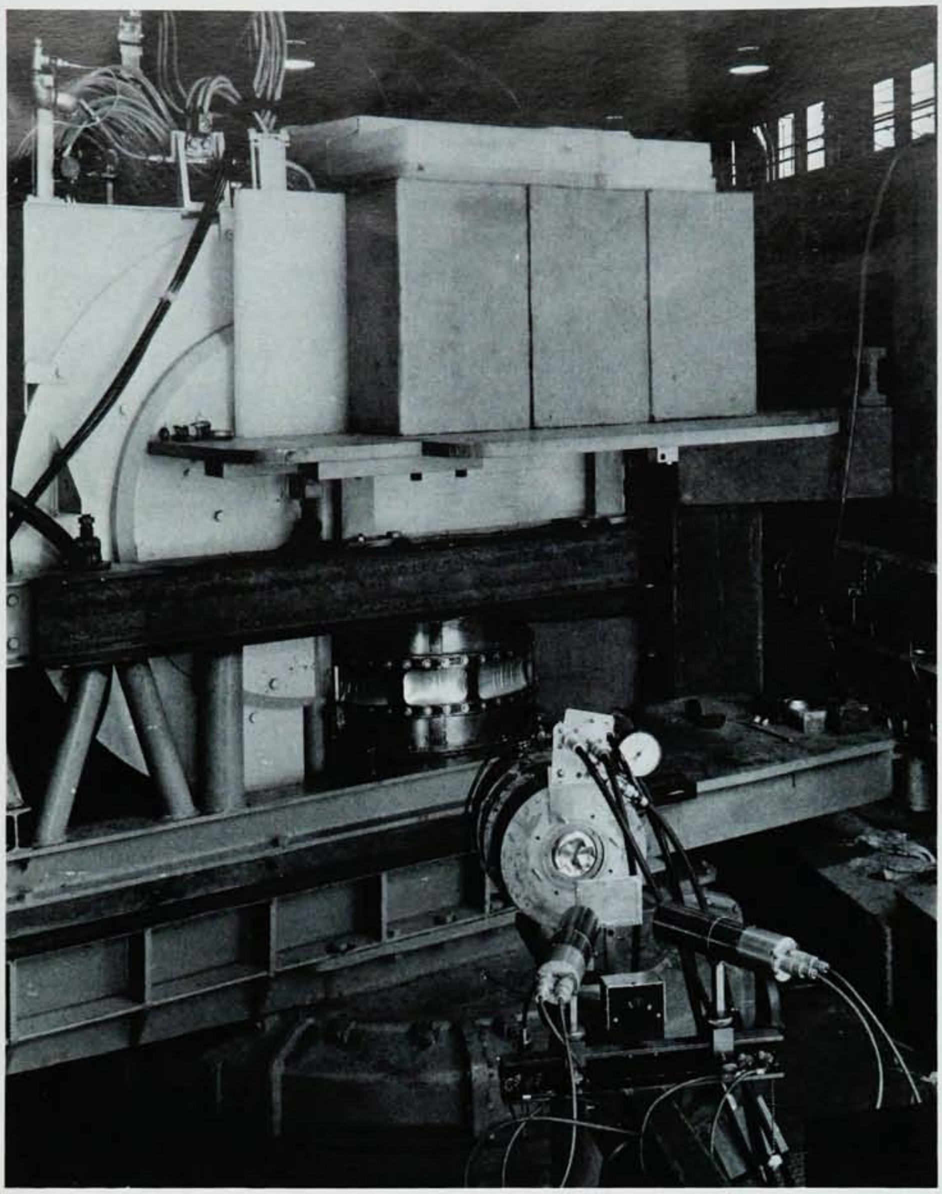
This relation may be inverted to yield the charge distribution from a set of form-factor measurements. Thus

$$\rho(r) = \int e^{i\mathbf{q}\cdot\mathbf{r}}f(q)\, d^3\mathbf{q}$$

The scattering of electrons from a charge distribution, in the first Born approximation, is also proportional to the square of the form factor. Electron scattering is the principal technique, first applied by Robert W. McAllister and Robert Hofstadter in 1950, for measuring the charge distribution of the proton. Since then other probes and targets have been used.

When theorists consider why a particle has a certain charge distribution, they always return to a description of the form factor itself. The reason is simple; in all field-theory calculations, propagators are expressed in momentum space and not in coördinate space. Thus particle physicists have learned to respect the form factors for their own sake and do not usually compute

After taking BA, MA and DPhil degrees at Oxford, Richard Wilson moved to research positions at the University of Rochester, Stanford University and Oxford again. Since 1955 he has been a member of the Harvard faculty. He is a member of the National Science Foundation Physics Advisory Panel and is also a trustee of the Universities Research Association Inc.



MAGNETIC SPECTROMETER of the type used by Robert McAllister and Robert Hofstadter. Scattering chamber for electron elastic-scattering experiment may be seen in the center; 180-deg bend in magnetic field separates out the background flux of inelastically scattered electrons and other particles. Hofstadter built two scaled-up versions of this 250-MeV spectrometer to 500 MeV and 1 GeV. —FIG. 1

the Fourier transform to find the charge distribution.

Recoil and spin

Study of the form factors of the elementary particles is complicated by two factors. First, the particles recoil, and the charge distributions are therefore not static. In place of the momentum-transfer vector $\mathbf{q}$ we consider the form factor as a function of the relativistic four momentum; $q^2 = q \cdot q = \mathbf{q}^2 - q_0{}^2$. When the recoil of the proton is small, $q_0{}^2$ is also small. The square of the four-momentum transfer is then equal to the square of the three-momentum transfer. In general

$$q^2 = 4E_iE_f \sin^2(\theta/2)$$

where E_i and E_f are the incident and final electron energies. In this equation, units are chosen such that $\hbar = c = 1$.

The second complicating factor arises from spin. Some of the particles, in particular those most accessible to experiment (the proton and neutron) have spin and therefore magnetic moments, with a distribution $\mu(r)$ of magnetic moment. Thus we can define two independent form factors, $G_E(q^2)$ and $G_M(q^2)$, for the proton and the neutron; they correspond to the electric and magnetic distributions respectively. The decomposition into electric and magnetic distributions is a little arbitrary and depends upon the frame of reference. In a particularly simple frame (the Breit frame) the electromagnetic interaction separates into the two form factors noted above.

This arbitrariness need not trouble us. The concept of a "charge distribution" of an elementary particle has problems from a relativistic standpoint; if spherically symmetric in one frame, it becomes elongated in another. The form factors give us no such problems. Their definition in terms of scattering amplitudes can be rigorous, and the theoretical understanding in terms of other processes is, in any case, an understanding of the form factors themselves. Thus we may well say: Love the form factors for their own sake and not for the sake of their three-dimensional Fourier transforms in a particular reference frame.

Electron-scattering experiments

The number of people who have contributed experimentally since Hofstadter's first experiments is so great that to list them all is excessive; table 1 shows the laboratories involved and the measurements performed in them. So far only the proton, neutron and pion have been studied, and only the proton has been studied in detail—for the obvious reason that only the proton can be made into a stable target. My purpose here is to discuss the conclusions of all this extensive work.

The experiments are, in principle, very simple. A beam of electrons of known intensity impinges on a hydrogen target of known length and density; electrons scattered into a known solid angle are counted. These measurements determine a scattering cross section. There is an important, complex, but straightforward correction for those events in which appreciable radiation is emitted by the accelerated charges. This radiative correction raises questions of pedagogical interest in electrodynamics but, fortunately, no practical problems in electrodynamics or strong interactions.

Electrons that are elastically scattered must be separated from the large background flux of inelastically scattered electrons and other particles. This separation has conventionally been done by measuring momentum in a magnetic spectrometer. In figure 1 we show a photograph of the first of these spectrometers with which Hofstadter set the convention; it bends 250-MeV electrons through 180 deg. Scaled-up versions of this spectrometer have been built. When the cross section for elastic electron–proton scattering has been measured, it is compared with the theoretical calculation for a proton with no structure—the "point" proton. The ratio gives a form factor. At backward angles the scattering depends primarily on the magnetic structure; at forward angles the scattering depends upon the sum of the squares of both form factors.

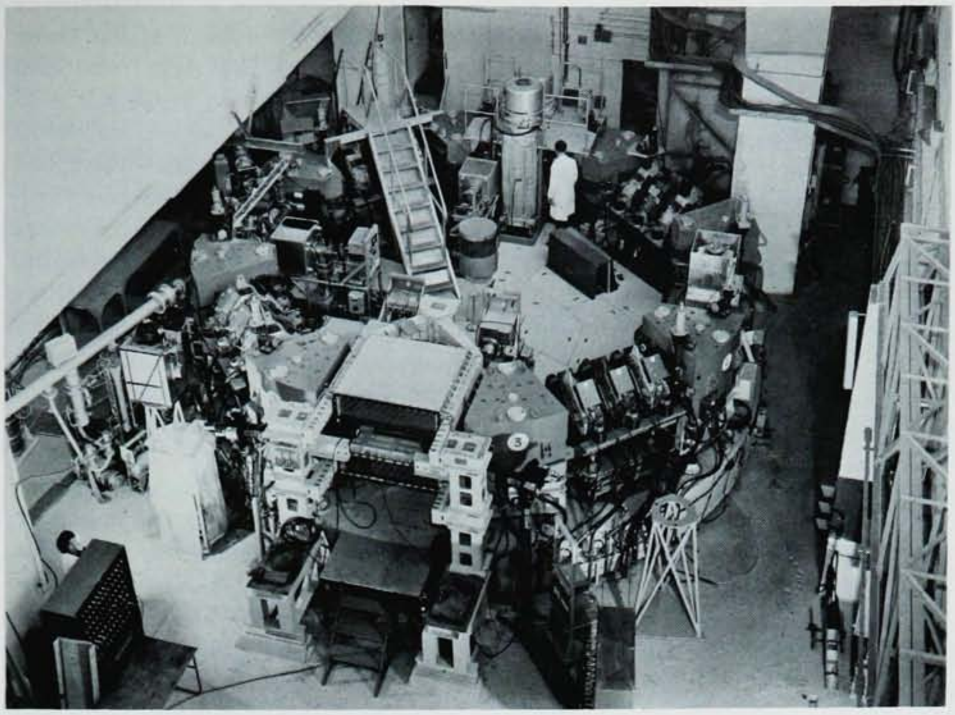
The electric form factor of the neutron may also be determined by scattering neutrons from stationary electrons; the first such measurement was by Enrico Fermi and Leona Marshall.[2] It turns out that this experiment measures $d/dq^2[G_{En}(q^2)]$.

Colliding-beam measurements

In scattering experiments the square of the four-momentum transfer can only be positive. The region for which q^2 is greater than zero is called "space-like." However, the form factor may be defined for *negative* values of q^2 where the four-vector q^μ is time-like. Experimentally the form factors in the time-like region may be measured by measurements in which electrons and positrons collide head-on and produce particle–antiparticle pairs; we have to replace the electron momentum by the negative of the positron momentum. In the colliding-beam experiments there is no three-dimensional momentum transfer in the laboratory frame, as it is the center-of-mass frame. There is only energy transfer. Then

$$-q^2 = (E_+ + E_-)^2 = 4E^2 \geqslant 4M^2$$

where $E_+ = E_- = E$ is the *total* energy of the electron and positron (or outgoing particles). This energy must be greater than $2M$, the mass of the outgoing particles. Thus there is a nonphysical region where form factors cannot be measured, defined by $-4M^2 < q^2 < 0$.



STORAGE RING for colliding beams of electrons and positrons at Orsay. This ring stores beams of over 500-MeV energy each. Annihilation of electrons and positrons with pion production measures the pion form factor. —FIG. 2

Experiments on the annihilation of antiprotons to lepton pairs also give time-like four momenta. Also, related and time-like, are experiments on leptonic branching ratios of vector mesons.

Experiments with colliding beams are completely independent of scattering experiments because the invariant square of the four-momentum transfer, q^2, has the opposite sign. However, we compare the cross section to the calculation for "point" protons in the same way, to yield a form factor. In all presently existing theories these form factors are continuous, as q^2 changes from positive to negative, with certain well defined singularities.

Form-factor measurements with colliding beams are very new, exciting developments. The experiments are done in circular accelerators, in which the beams lose energy by synchrotron radiation, and energy is replaced by radiofrequency cavities. Great care is taken to ensure stability of the beams. In figure 2 we show a photograph of the storage ring for colliding-beam experiments at Orsay, where the group under Pierre Marin announced their first experimental result in September 1967. Comparison of this photograph with figure 1 illustrates the change from "old" to "new" in form-factor measurement (a change in bending from a single bend through 180 deg to many rotations through 360 deg). We shall see that there is more exciting structure in the time-like (colliding-beam) experiments, compared with a smooth structure in the space-like re-

Table 1. Form factor experiments

Laboratory	*Parameter*	*Range of q^2(GeV/c)^2*	*Comment*	*Last reference*
Stanford (HEPL)	$G_{Ep}G_{Mn}$	0–1		*9*
	$G_{Ep}G_{Mn}$	0–1		*10*
Stanford (SLAC)	G_{Mp}	1–25		*11*
Cornell	$G_{Ep}G_{Mp}$	0–2		*12*
	$G_{En}G_{Mn}$	0–2		*13*
	F_π	0–0.5	Electropion production	*14*
Harvard (CEA)	$G_{Ep}G_{Mp}$	1–7		*15*
	$G_{En}G_{Mn}$	1–5		*16*
	F_π	0–0.5	Electropion production	*17*
Northeastern (CEA)	F_π	−0.55	Rho branching	*18*
Orsay	$G_{Ep}G_{Mp}$	0–0.5		*19*
	F_π	−0.55	Storage ring	*20*
Novosibirsk	F_π	−0.4 to −0.7	Storage ring	*21*
DESY	$G_{Ep}G_{Mp}$	1–9		*22*
	F_π	−0.55	Rho branching	*23*
Harvard (AGS)	F_π	−0.55	Rho branching	*24*
Northwestern (Chicago)	F_π	0–0.1	Pion–alpha scattering	*25*
Cal Tech (AGS)	$G_{Ep}G_{Mp}$	−4 to −7		*26*
CERN	F_π	−0.55	Rho branching	*27*
	$G_{Ep}G_{Mp}$	−7		*28*
Argonne	dG_{En}/dq^2	0	Neutron–electron	*29*
Columbia	dG_{En}/dq^2	0	Neutron–electron	*30*
Dubna	F_π	−0.55	Rho branching	*31*

gion. We hope for, and expect, much new data in the time-like region from the new storage rings at Orsay, Novosibirsk, Frascati and the Cambridge Electron Accelerator.

In these colliding-beam experiments at Orsay and Novosibirsk pairs of pions have been observed, particularly near the rho-meson mass ($E_+ = E_- = E = 380$ MeV); it is the pion form factor, $F_\pi(q^2)$, that is measured. The pion has no spin and therefore only one form factor. Unfortunately, *direct* experiments on the pion form factor in the space-like region are not available; we must use some inaccurate indirect measures from inelastic electron scattering. However, the theory of the nucleon form factors, which are well measured in the space-like region, is closely coupled to the theory of the pions.

Observed features

The features of these experimental results that we must explain are:

• Those form factors that have been measured in the space-like region vary smoothly with momentum transfer. There are no diffraction maxima and minima or resonances. In the time-like region only the pion form factor in the vicinity of the rho resonance and the upper limits to the proton form factors at large momentum have been measured.

• The proton and neutron form factors follow approximately the relation

$$\frac{G_{\mathrm{Mp}}(q^2)}{\mu_{\mathrm{p}}} = \frac{G_{\mathrm{Mn}}(q^2)}{\mu_{\mathrm{n}}} = G_{\mathrm{Ep}}(q^2) = \left(\frac{1}{1 + q^2/0.71}\right)^2 = (\text{maybe})\ \frac{G_{\mathrm{En}}}{\mu_{\mathrm{n}}} \left[\frac{(q^2/4M^2)}{1 + 4(q^2/4M^2)}\right]$$

with q^2 measured in units of $(\mathrm{GeV}/c)^2$. The relation for the magnetic form factor of the proton has been checked to 25 $(\mathrm{GeV}/c)^2$; the relation for the electric form factor has been checked up to 3 $(\mathrm{GeV}/c)^2$ for the proton and to 3 $(\mathrm{GeV}/c)^2$ for the neutron magnetic form factor. Data for the form factors are shown in figures 3 and 4.

• The pion form factor has been measured in the time-like region by colliding-beam experiments on the reaction $e^+ + e^- \rightarrow \pi^+ + \pi^-$. So far these measurements have been only near 380-MeV energy in each beam—conditions that produce the pion pair at the ρ^0 resonance with total energy 760 MeV. A resonance has been found of about the right mass and width to enable us to identify it with the ρ^0 meson found in π–p collisions. Of course, if the mass and width had come out appreciably different, colliding-beam experiments, with no other strong interactions in the final state, would be taken as correct, and we would assume that measurements of resonance parameters in strong-interaction experiments are subject to error—with dire consequences for the whole field of elementary-particle physics. In addition the pion form factor has been found indirectly in this region by measuring the branching ratio for leptonic decay of rho mesons, produced by pion collisions with nuclei or photo-produced. This branching ratio is the quotient of the partial width for decay into electron pairs $\Gamma_{e^+e^-}$ to that for pion pairs $\Gamma_{\pi^+\pi^-}$ and it is connected with the cross section for electron–positron colliding beams at the peak of the resonance ($q^2 = M_\rho{}^2$) by the simple relation

$$\sigma_{(e^+e^-\rightarrow\pi^+\pi^-)} = \frac{12\pi\hbar^2}{M_\rho{}^2c^2}\frac{\Gamma_{e^+e^-}}{\Gamma_{\pi^+\pi^-}}$$

All measurements are in reasonable agreement with a value of $(\Gamma_{e^+e^-}/\Gamma_{\pi^+\pi^-})$ approximately equal to 6×10^{-5}.

Dispersion relations

The most powerful technique for understanding nucleon form factors is that of dispersion relations. We cannot measure the form factors for $-4M^2 < q^2 < 0$ where M is the mass of the particle under study. Can a graph of the form factors be analytically continued across this gap? For pions this interpolation has been rigorously proved. For nucleons we assume it is possible, subject to various resonances (poles and cuts) in the region $-4M^2 < q^2 < -4M_\pi{}^2$. Then we can write the dispersion relation

$$\mathrm{Re}\ F(q^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\mathrm{Im}\ F(q')^2}{q^2 - (q')^2}\, d(q')^2$$

provided that $F(q^2) \rightarrow 0$ as $q^2 \rightarrow \infty$ (to make the integral converge). The analyticity can be proved for the pion form factor, and Im $F(q^2)$ equals zero for $\infty > q^2 > -4M_\pi{}^2$. Thus we find

$$\mathrm{Re}\ F(q^2) = \frac{1}{\pi}\int_{-4M_\pi{}^2}^{-\infty} \frac{\mathrm{Im}\ F(q')^2}{q^2 - (q')^2}\, d(q')^2$$

These, and similar, relations tell us that if $F(q^2)$ is measured in the time-like region ($q^2 < 0$), the value in the space-like region ($q^2 > 0$) can be derived and vice versa. As the nucleon form factors are well measured in the space-like region, attempts have been made to derive the form factor in the time-like region. In the latest attempt J. S. Levinger[6] proved that the data must be 1000 times more accurate than it is at present if we are to derive correctly the position and width of the dominant rho resonance. Extrapolation in the other direction—deriving a form factor for $q^2 > 0$ from the knowledge of those for $q^2 < -4M^2$—would be easier if we had data, as the structure exists in the time-like region and not in the space-like region. To obtain an understanding we must, therefore, make approximations and assumptions.

Let us consider, for a start, the pion form factor. The function $F(q^2)$ is the vertex function for the transition $\gamma\pi\pi$. We can use the unitarity relation

$$\mathrm{Im}\ F_\pi(q^2) = \sum_N \langle\gamma|T^*|N\rangle\langle N|T|\pi\pi\rangle$$

where the summation is over all intermediate states, N. We now approximate by saying that the sum is dominated by the lowest energy states. The states of two pions are clearly of lower energy than those of the four,

Form factors and charge distributions

Charge distribution $\rho(r)$		*Form factor* $F(q^2)$	
Point	$\delta(\mathbf{r} - \mathbf{r}')$	Constant	1
Yukawa	$\frac{\mathbf{m}^2}{4\pi r}e^{-mr}$	Single pole	$1/(1 + q^2/m^2)$
Exponential	$\frac{\mathbf{m}^3}{8\pi}e^{-mr}$	"Dipole"	$1/(1 + q^2/m^2)^2$
Gaussian	$\frac{\mathbf{m}^3}{(2\pi)^{3/2}}e^{-(m^2r^2/2)}$	Gaussian	$e^{-(q^2/2m^2)}$

The pion appears to be given by a single pole (Yukawa). The proton is approximated by a "dipole" (exponential).

six, eight or more pions, and we therefore assume that only two pion states contribute. Then we find

$$\mathrm{Im}\, F_\pi(q^2) = F_\pi^*(q')^2 \langle \pi\pi | T | \pi\pi \rangle$$

where the second term is the amplitude for pion–pion scattering. We see in this equation another reason why the pion form factor is particularly simple; it is related to pion–pion scattering (in this approximation). The pion–pion scattering amplitude is particularly simple because all four particles, two incoming and two outgoing, are identical. Both dispersion relations and current algebra find this simplicity particularly attractive.

We now ask how to extend this result to the nucleon form factors. We form isotopic scalar and vector combinations

$$2G_{EV}(q^2) = G_{Ep} - G_{En}$$

$$2G_{ES}(q^2) = G_{Ep} + G_{En}$$

The isovector intermediate state has the same quantum numbers as a two-pion state. Thus, by a similar approximation, we find that the isovector nucleon form factors are given by

$$\mathrm{Im}\, G_{MV}(q^2) = G_{MV}^*(q^2) \langle \pi\pi | T | \mathrm{NN} \rangle$$

The second term is the amplitude for two pions to annihilate and produce a nucleon and antinucleon. There are two amplitudes T: For the form factor G_{MV} the nucleon's spin (helicity) is flipped in the transition and for G_{EV} the nucleon spin remains the same. These amplitudes have been calculated and may include, in addition to any resonant terms in the pion–pion interaction, nonresonant terms related to the pion–nucleon scattering by a dispersion relation. If the amplitudes $\langle \pi\pi | \mathrm{T} | \pi\pi \rangle$ and $\langle \pi\pi | T | \mathrm{NN} \rangle$ are dominated by one or more narrow resonances, we can approximate Im F by a delta function and find

$$F_\pi(q^2) = \frac{G_{MV}(q^2)}{G_{MV}(0)} = \frac{G_{EV}(q^2)}{G_{EV}(0)}$$

$$= \sum \frac{\beta_i m_i^2}{m_i^2 + q^2}$$

where the summation is over all resonances.

Rho-dominance model

Any resonance, to contribute, must have the quantum numbers of the electromagnetic field, $J = 1^-$. We are also considering isovector resonances. The only one known is the rho resonance at 765 MeV, with Γ approximately equal to 100 MeV. Thus we are led to a unique model for the pion form factor; the one constant, β, is determined to be unity by fitting the pion charge (F_π at $q^2 = 0$). This model we call the "rho-dominance" model. That the rho meson dominates the form factor near its resonance is obvious. The model assumes that it dominates everywhere and predicts the root-mean-square radius of the pion to be $\sqrt{6/(M_\rho)^2}$; the fall-off at infinity goes as $1/q^2$. Subtractions are assumed to be absent (in the disagreement we note below, they make matters worse). Also the process $e^+ + e^- \rightarrow \pi^+ + \pi^-$ is given completely by the square of the pion form factor and a resonance shape is expected near $q^2 = -M_\rho^2$ (and has been found at Novosibirsk and Orsay). The cross section at the peak of the rho resonance is

$$\sigma = \frac{\pi \hbar^2 \alpha^2}{3c^2 \Gamma_\rho^2} \left(\frac{M_\rho^2}{M_\rho^2 + 4M_\pi^2} \right)^{3/2}$$

and the branching ratio of the resonance states for leptons is

$$\frac{\Gamma_{e^+e^-}}{\Gamma_{\pi^+\pi^-}} = \frac{\Gamma_{\mu^+\mu^-}}{\Gamma_{\pi^+\pi^-}}$$

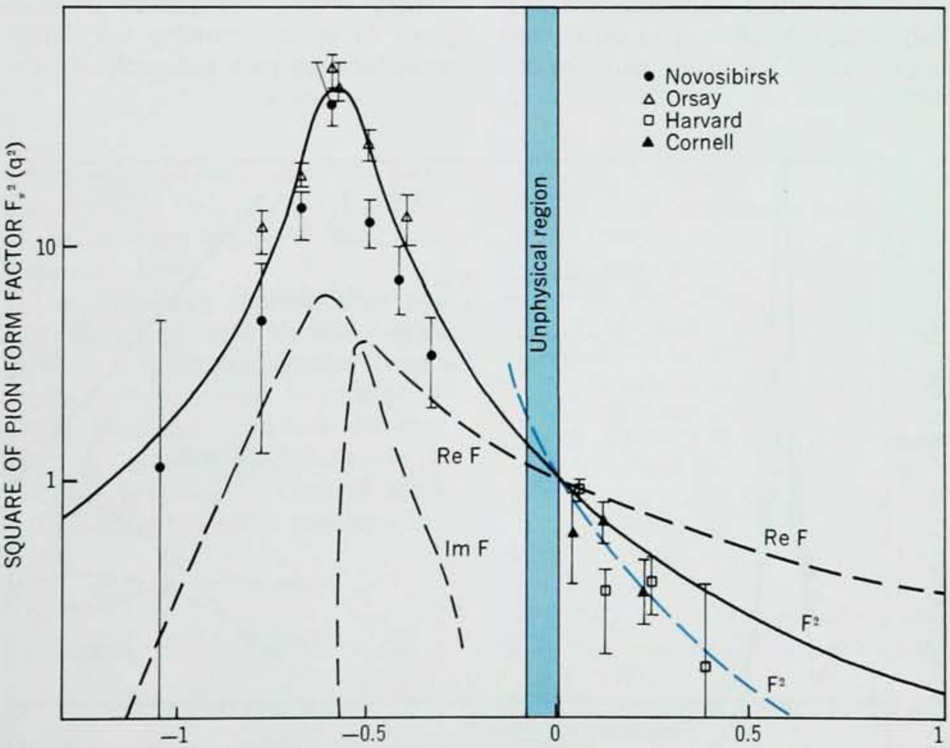
$$= \frac{\alpha^2}{36} \left(\frac{M_\rho^2}{M_\rho^2 + 4M_\pi^2} \right)^{3/2} \left(\frac{M_\rho}{\Gamma_\rho} \right)^2$$

We note that the theoretical values all depend on the width Γ_ρ for the rho meson, which is almost equal to the partial width for decay into two pions, $\Gamma_{\pi^+\pi^-}$. This partial width is presently uncertain. We find agreement with the measurements in table 1 to within the accuracy of 10% and therefore have a test of this model of the pion form factor.

The pion root-mean-square radius is predicted, by the rho-dominance model, to be 0.64×10^{-13} cm; experiments do not yet give this value definitively, but suggest the value 0.8×10^{-13} cm. No measurements are made at large momentum transfers.

In figure 3 we show the experimental data, in both the time-like and space-like region, fitted to this model (or at least a slightly improved version of it that gives a Breit–Wigner shape to the rho resonance). We see that the data fit fairly well, although there is a tendency for the sparse and inaccurate measurements in the space-like region to give too steep a slope to the form factor (implying too large a radius). These data in the space-like region are from inelastic electron–proton scattering and involve a lot of interpretation.

We see from this graph that, if we prefer the Orsay data to that from



PION FORM FACTOR (squared) in both time-like and space-like regions. Data from four laboratories is compared to the vector-dominance model (black lines; solid line is F^2, dashed lines are Re F and Im F). Colored line is proportional to the nucleon form factors. Vector-dominance model has a single resonance and has small adjustments to satisfy analyticity requirements. —FIG. 3

Novosibirsk, we can fit the data on the colliding-beam work and the pion charge simultaneously *without* a kink in the curve. Such a kink would imply that Im $F(q^2)$ is approximately zero near the kink, which would be inconsistent with vector dominance.

Discrepancy in nucleon form factors

However, the isovector *nucleon* form factors do *not* agree with this simple approach. The root-mean-square radius is 0.8×10^{-13} cm. If we rewrite the dispersion relation for the factor, we obtain

$$\text{Re } F(q^2) = \frac{1}{\pi q^2}\int \frac{\text{Im } F(q')^2}{1-(q')^2/q^2}\, dq'^2$$
$$= \frac{1}{\pi q^2}\int \text{Im } F(q')^2\, d(q')^2$$
$$+ \frac{1}{\pi q^4}\int (q')^2$$
$$\times \text{ Im } F(q')^2\, d(q')^2 + \ldots$$

The fall-off at infinity is measured to be about $1/q^4$, so we find

$$\int \text{Im } F(q')^2\, d(q')^2 = 0$$

This result is clearly not satisfied by a single resonance. To explain the discrepancy we can first follow the extrapolation technique of reference 6. Im $G_{MV}(q')$ for $q^2 < 0$ is derived from a knowledge of $G_{MV}(q^2) > 0$ and an assumption of a rho-meson contribution, $A[q^2/(m^2 + q^2)]$ plus other terms. Good fits to the data are obtained for various values of the contribution of the rho meson A (figure 4). A equal to unity, and no other terms, corresponds to the rho-dominance model. Two other contributions appear to be necessary; first, a positive contribution to Im $G_{MV}(q^2)$ below the rho mass, $-q^2 < M_\rho^2$, and a large negative contribution above the rho mass. This large negative contribution has an area under the curve equal to the area under the positive contribution, as required to satisfy $\int$Im $G_{MV}(q^2)\, dq^2 = 0$ and gives a fall-off as $1/q^4$.

A second resonance has often been conjectured but not found (although the nonrelativistic quark model suggests one somewhere in the right region). There have been three papers discussing how to explain these added contributions to Im $G_{MV}(q^2)$. Susuimi Furuichi and K. Watanabe[3] claim that a complete treatment of the amplitude $\langle \pi\pi|T|NN\rangle$, with a resonance of finite width gives about this form. N. G. Antoniou and J. E. Bowcock,[4] on the other hand, find that the finite width, while giving effects in the right direction, is *not* adequate by itself; they have to add the two extra contributions *ad hoc*. Peter Signell's and J. W. Durso's[5] results lie in between. Many other theorists are now working on this approach. G. Höhler, Strauss and Wunder have a paper in press discussing the differences between pion and nucleon form factors along the lines of Furuichi[3] and Signell[5] but with improvements; and M. A. B. Beg, Bernstein and Tausner relate the approach to field-current identities.

Particularly appealing in the work of Signell and Furuichi is an attempt to relate the $\langle\pi\pi|NN\rangle$ amplitude to the two-pion exchange part of nucleon–nucleon scattering as well as to pion–nucleon scattering. Thus, although extra terms are added, they are explained by extra experiments.

Alternative explanations

The resonance theory of form factors can thus be reconciled with the data in several alternative ways and no one is yet sure which is right. The different possible explanations are:

• The pion form factor follows the simple rho-dominance model, and the nucleon form factor differs from it because of differences in the $\langle\pi\pi|NN\rangle$ amplitude (alternatively assigning a momentum dependence to the ρNN coupling).

• The pion and isovector-nucleon form factors vary roughly together and the rho does not dominate everywhere. Alternatively we say the $\rho\gamma$ or $\rho\pi\pi$ couplings vary with momentum transfer.

The pion and isovector-nucleon form factors vary together and electrodynamics (the eeγ vertex or γ propagator) breaks down.

A different approach to nucleon form factors goes back to the naive concept of a nucleon size. It has long been noted that proton–proton scattering angular distributions and the nucleon form factors are related by

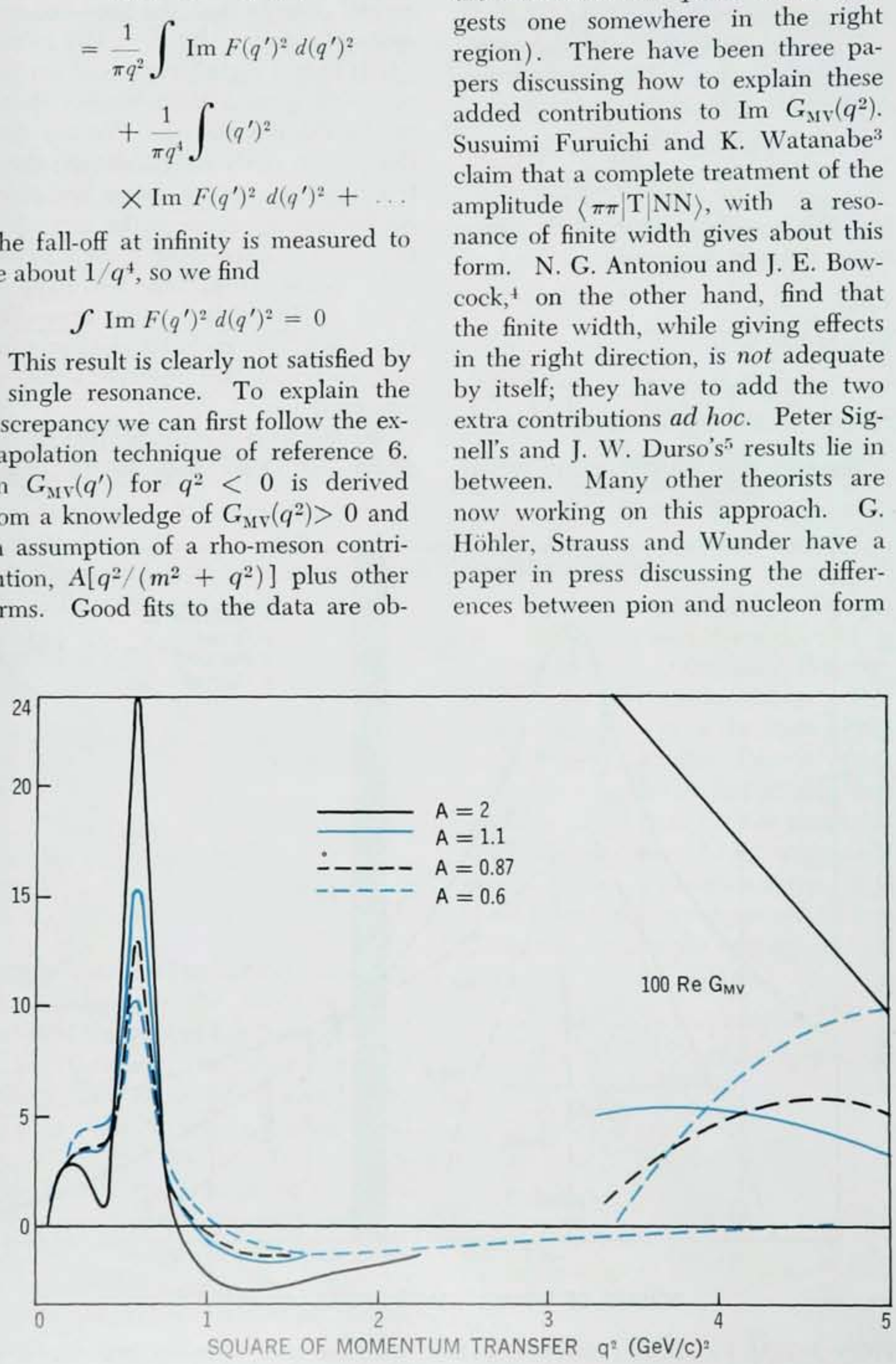
$$(d\sigma/dt)_{pp} \propto [G_{Mp}(q^2)]^4$$

A tempting argument is that both experiments measure the size of the proton; proton–proton scattering measures it with strong interactions and the form-factor measurements with electromagnetic interactions. Allowing for the two colliding protons in the proton–proton case we find equal sizes. This result is now often regarded as evidence for a nonrelativistic quark model.

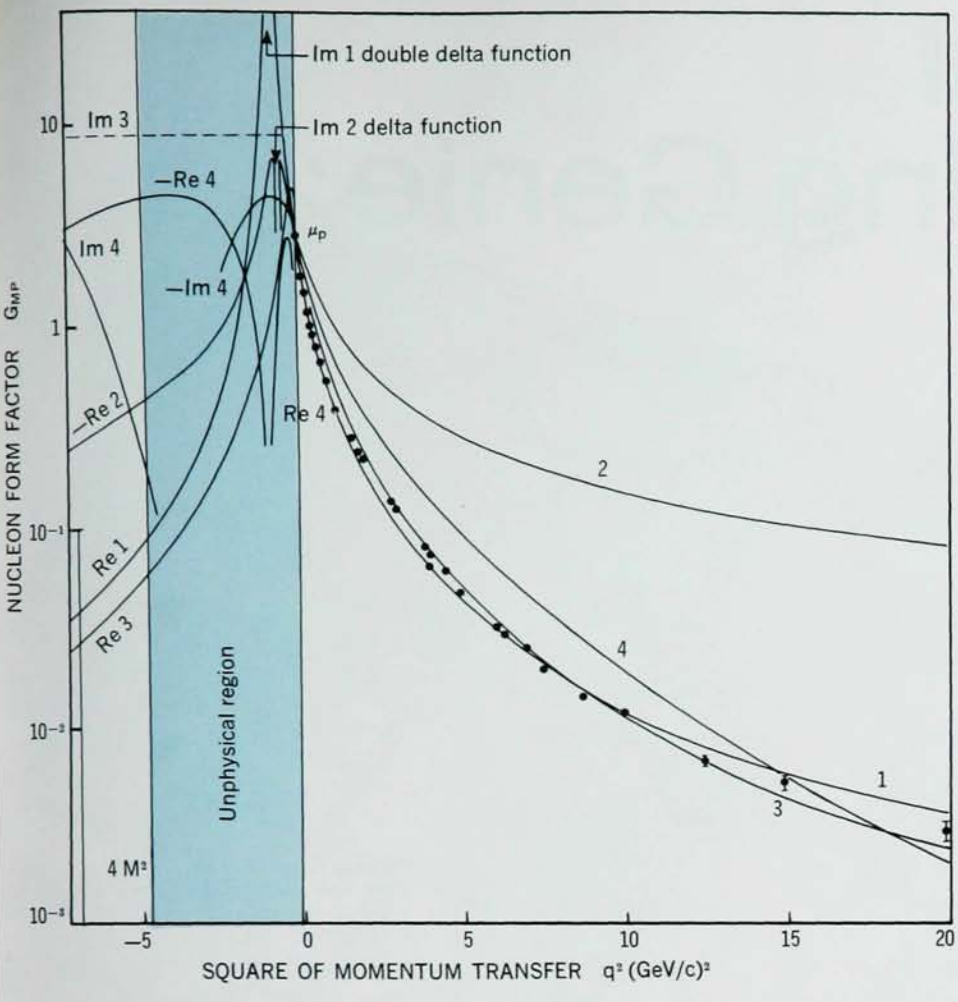
If the constituents of the proton themselves have a size, we find for the proton form factor a fold of two form factors, one for the constituents and the other the Fourier transform of the spatial extent of the constituents

$$G_{Mp}(q^2) = F_{\text{const}}(q^2)\, F_{\text{spatial}}(q^2)$$

In a bootstrap theory we find that



RHO-MESON CONTRIBUTION to the isovector nucleon form factors varies with the parameter A in this spectral function Im G_{MV} with which J. S. Levinger fits the known data on G_{MV}. Rho-dominance model is given by $A = 1$. —FIG. 4



NUCLEON FORM FACTOR G_{MP}. The experimental points are from many laboratories and agree to within the accuracy of the graph. The lines show: (1) the dipole fit; (2) the vector-dominance fit (in principle applies to the isovector form factor); (3) a fit given by Gerhard Mack; (4) the Wu–Yang fit. Note that some curves, although similar in the space-like region, differ widely in the time-like region. —FIG. 5

the form factor for the constituents, in turn, has a similar multiplicative property. We soon find an infinite product and a prediction that, at high momentum transfers, the form factors must fall exponentially. Data at the highest momentum transfers so far available neither prove nor disprove this prediction.

There appears to be an attractive simplicity and smoothness to the data. Sidney Drell, particularly, feels that the complexity of the cancellations required by the vector-dominance model is unappealing and that we must search for a $1/q^4$ law independent of vector dominance. Tai Wu and C. N. Yang[8] argue that the basic assumption of the dispersion theory, that we are dominated by the nearest singularities, is completely wrong and that we must search elsewhere. They predict a fall-off as exp $(-1.72q^2)$. Figure 5 shows how some curves that are similar in the space-like region can give wildly different behavior in the time-like region. Only the vector-dominance curve has any justification from dispersion theory.

The Wu–Yang formula does not have the correct analytic structure at $q^2 = 0$; a slight modification (curve 4), $G_{Mp} = 4.5 \exp\ (-1.725q^2 + 4m_\pi{}^2)$ gives the right behavior near $q^2 = 0$. Another formula, curve 3, has been proposed by Gerhard Mack and has some success in the space-like region

$$G_{Mp} = 2.79 \times \exp\{-0.216\ [\log^2 (15.7\sqrt{q^2}) - \log^2 15.7\sqrt{q^2 + 4m^2}]\}$$

However, curves 3 and 4 do not satisfy the dispersion-theory postulate that $G(q^2) \rightarrow 0$ as $q^2 \rightarrow -\infty$. As we have so much flexibility it appears preferable to retain forms that follow the dispersion-theory requirements.

The behavior of these formulas is very different in the time-like region, illustrating how little is really known. It is interesting to note that this problem is probably one of the simplest in strong-interaction physics, and therefore must be solved.

References

1. R. W. McAllister, R. Hofstadter, Phys. Rev. **102,** 851 (1956).
2. E. Fermi, L. Marshall, Phys. Rev. **72,** 1139 (1947).
3. S. Furuichi, K. Watanabe, Prog. Theor. Phys. **35,** 174 (1966).
4. N. G. Antoniou, J. E. Bowcock, Phys. Rev. **159,** 1257 (1967).
5. P. Signell, J. W. Durso, Phys. Rev. Letters **18,** 185 (1967).
6. J. S. Levinger, Phys. Rev. **162,** 1589 (1967).
7. S. D. Drell, Comments on Nuclear and Particle Physics **2,** 36 (1968).
8. T. T. Wu, C. N. Yang, Phys. Rev. **137 B,** 708 (1965).
9. T. Janssens et al, Phys. Rev. **142,** 922 (1966).
10. E. B. Hughes et al, Phys. Rev. **139B,** 458 (1965).
11. D. H. Coward et al, Phys. Rev. Letters **20,** 292 (1968).
12. K. Berkelman et al, Phys. Rev. **130,** 2061 (1963).
13. C. W. Akerlof et al, Phys. Rev. **135B,** 810 (1966).
14. C. W. Akerlof et al, Phys. Rev. **163,** 1482 (1967).
15. M. Goitein et al, Phys. Rev. Letters **18,** 1016 (1967).
16. R. Budnitz et al, Phys. Rev. **173,** 1357 (1968).
17. C. Mistretta et al, Phys. Rev. Letters **20,** 1523 (1968).
18. J. K. de Pagter et al, Phys. Rev. Letters **17,** 767 (1967).
19. B. Dudelzak et al, Nuovo Cimento **28,** 18 (1963).
20. J. E. Augustin et al, Phys. Rev. Letters **20,** 126 (1968).
21. V. L. Auslander et al, Phys. Letters **25B,** 433 (1967).
22. W. Albrecht et al, Phys Rev. Letters **17,** 1192 (1966).
23. J. G. Asbury et al, Phys. Rev. Letters **19,** 869 (1967).
24. A. Wehmann et al, Phys. Rev. Letters **20,** 1523 (1968).
25. M. M. Block et al, Phys. Rev. **169,** 1074 (1968).
26. A. Tollestrup, private communication.
27. B. D. Hyams et al, Phys. Letters **24B,** 634 (1967).
28. M. Conversi et al, Nuovo Cimento **40,** 690 (1965).
29. V. E. Krohn et al, Phys. Letters **18,** 297 (1965).
30. E. Melkonian, B. M. Rustad, W. W. Havens Jr, Phys. Rev. **114,** 1571 (1959).
31. M. N. Khatchaturyan et al, Phys. Letters, **24B,** 349 (1967).