only in certain limits represents one or the other. Quantum theory has become known to us as a complete whole since the end of the 1920's. We need not turn from a particle picture to a wave picture arbitrarily, and we need not be without real comprehension when using it. It is, on the contrary, possible to represent the states of a system in different ways, and these representations are connected by unique transformations.

Among these representations there is a space representation from which one can easily derive the probability of finding the particles at a certain point in space. There is a momentum representation as well from which one can easily read off the probability of the particles having certain momenta (that is, velocities). As the momentum, according to de Broglie, defines the wavelength one has found the wave properties of the system in this representation. These two representations correspond respectively to the "particle picture" and the "wave picture;" they date from the period when quantum theory as a whole had not yet been developed. But there is an unlimited number of other representations of the states of a system, for example the energy representation, from which one can easily calculate the energies of the system. Moreover, all properties of the system can be calculated in any of these representations (for example, the distribution in space of the particles can also be calculated in the momentum distribution). Thus with quantum theory one treats all systems in the same way, whether they consist classically of particles or are described classically by fields (waves).

Bosons and fermions

Much more fundamental than the distinction between particles and waves is the classification of particles between those that follow Bose statistics (bosons) and those that are ruled by Fermi statistics (fermions). Whereas bosons can be compared to classical particles and waves, as we have shown above by means of their fluctuations, this comparison is not altogether true for fermions, as we have already seen when looking at the fluctuation equation 3.

As to Landé's claim to have given a new derivation of quantum mechanics based on classical ideas though not as such taken from classical physics, it must of course be examined thoroughly; Shimony wrote of such an examination in his review.[3] We will add only the following remarks. Landé starts from statistical postulates when trying to give a new foundation to quantum theory. These postulates are unknown in classical physics, from which Landé takes all his other concepts. It appears to be not very surprising that one can derive theories similar to quantum mechanics from statistical postulates. Such a derivation can be interesting in itself and does not need to be accompanied by attacks on supposed enemies. The strangest aspect of Landé's treatment is his dogmatic use of classical and macroscopic concepts of particles and waves in atomic dimensions and his rejection of obvious explanations of simple experimental results on account of this dogma or prejudice.

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DIALOG ON DUALISM

Replies by ALFRED LANDE to points raised in the preceding article and further comments by MAX BORN and WALTER BIEM.

LANDE: Concerning "the historical origin of the dualistic interpretation" which I "have not realized:" I know of course of Einstein's light quanta in opposition to light waves. But I also know that there is a unitary quantum theory of radiation that has relegated the "photon" to the role of a quantum number attached to the periodic components of the continuous Maxwell field; thereby it has become unnecessary to attribute various *ad hoc* invented quantities—spin, interdependence of electric and magnetic properties of the photon—in order to save a particle picture dual to the wave picture of light. Light waves are real, matter waves are imaginary, in more than one sense.

Duality began to be taken seriously only after the experiment of electron diffraction *seemed* to allow no other explanation than the assumption that particles of matter pass through a wave interlude near a crystal or screen with slits: "An electron spreads out from its original size millions of times to cover both slits; thereafter it interferes with itself." This oddity, together with an associated "new conceptual situation" accounting for the unphysical transmutation magic, could have been avoided if quantum theorists around 1927 had been aware of the quantum rule for linear momentum (Duane, 1923). This rule explains the electronic diffraction patterns in a natural way as due to the quantized momentum activity of the diffractor, including coherence effects as shown in my books and articles. It has been quite a revelation to many younger physicists trained in the dualistic doctrine. To belittle the quantum rule for the momentum p beside those for E and p_φ as is done by Born, is as unphysical as if one would belittle the mechanical conservation law for p beside those for E and p_φ in classical theory. One here really must ask: "Why do quantum theorists ignore the quantum theory?" I would be delighted to be shown a single place in the literature on interpretation where Duane's unitary explanation of diffraction, applied to matter particles, is quoted. Born was one of the few who knew of Duane's March 1923 paper.

BORN AND BIEM: Every physicist must accept Duane's rule,[6] which describes correctly all experiments of momentum exchange on periodic structures. But he has learned little if he accepts it. On the other hand de Broglie's paper of September 1923 contains the beginning of an insight. In this work Planck's and Duane's rules were connected. Using the theory of relativity, de Broglie associates with the four-vector of momentum and energy the four-vector of wave number and frequency. Duane's papers were well known in the 1920's.[7]

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LANDE: Depicting Einstein as a champion of dualism is utterly unhistorical. Every physicist knows of his persistent fight against the view of Bohr and Heisenberg that every single particle has wavelike properties such as an uncertain measurability and even a blurred existence in pq space, by translating $p = h/\lambda$ to become wavelike. His many thought experiments tried to prove that particles *do not* have wavelike uncertainties as in Bohr's dualism. Many observers assumed, as does Born, that Bohr "won" in the discussion. I am not so sure he did, because of mathematical impossibilities in Bohr's argument (see page 123 in my "New Foundations of Quantum Mechanics"). Furthermore, the side-by-side occurrence of corpuscular and wavelike fluctuations in a gas which, according to Born, proves that "duality is a fact," has been explained, partly by Einstein himself, as a result of the symmetry requirements in the pure-particle mechanics for bosons and fermions. The "fact" of dualism here has become no more than a contrast between obvious and not-so-obvious particle effects under quantum mechanics, so removing duality again except as a misleading word.

If matter consists (in elementary theory) of discrete particles surrounded by continuous fields, and both are dominated by the same fundamental quantum principles, we have an entirely different sort of duality. Even Einstein was not able to improve this sorry state of the world. It is not my fault that, in the simple three-dimensional case of diffraction, matter yields patterns similar to optical ones. However, one can very well distinguish material-particle and electromagnetic wave effects in most other circumstances. See Mario Bunge in "Analogy in Quantum Theory: From Insight to Nonsense,"[8] a most timely article. And where did I ever present Schrödinger as accepting the particle theory of matter against his own famous wave theory?

BORN AND BIEM: Landé says: "Matter consists of particles surrounded by continuous fields." There are many more entities than he mentions, and they are not classified into particles and continuous fields. There are better classifications, for example bosons and fermions or particles (or quanta) with finite mass and those with zero mass. Photons and mesons have something in common for both are bosons; neutrinos and electrons are both fermions. But photons and neutrinos are both quanta of zero mass and they are similar in some respects (for example, kinematically).

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LANDE: Historically, wave–particle duality received its death knell, to survive only under various assumed names, when Max Born himself in 1927 established his admirable statistical interpretation of the ψ function. He, more than anyone else, emphasized the pure statistical character of quantum mechanics against Schrödinger's deterministic waves. He thereby turned dualism into the illogical contrast of a single particle "interfering with itself" against one of its many qualities, namely that of displaying a probabilistic distribution in an ensemble with many other like particles, symbolized by a complex-imaginary wavelike function. This dualism, if it still deserves the name, is similar to calling an influenza patient dualistic because, on one hand, he is a single particle, and on the other hand, the ups and downs of the epidemic curve look wavelike. What Born today characterizes as "dogma and prejudice" and also as my "tilt against windmills" may originally have been like that of the lonely Don. But recently it has been joined by many prominent physicists and philosophers of science to become a sort of Volksbewegung. And nobody would have been delighted more than Einstein over the triumph of Born's statistical-particle interpretation supported against dualistic appearances by removal of the paradox of matter diffraction.

Incidentally, readers of my PHYSICS TODAY article[2] may take notice that the mathematical gap in my derivation of the probability-interference law from nonquantal postulates has been closed by the added postulate that the general probability law yields the ordinary addition law of probabilities in the average.

ALFRED LANDE, whose new foundation for quantum theory does not admit wave–particle dualism.

BORN AND BIEM: Would Einstein "have been delighted" over the new "Volksbewegung"? It appears quite clear to us that Einstein never accepted the statistical aspect of quantum theory. Born's statistical-particle interpretation was the part of quantum theory that Einstein could not disprove but that he hoped was only a provisional first step, to be eliminated later on from a better theory. Einstein was not "a champion of dualism," of course, but he found the fluctuation equation and tried without success to understand this fact in a way more acceptable to him than quantum mechanics.

Clearly it is possible to formulate quantum mechanics of particles avoiding all wavelike terms. Sometimes this is easy, sometimes it is very artificial. Since one loses much insight this way we must ask: Why the effort?

Additional references

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