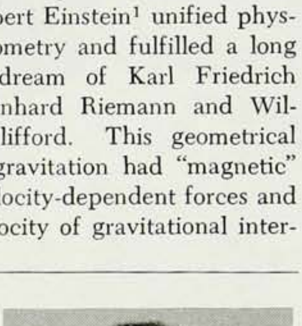
Gravitational Waves

The search for gravitational radiation is snowballing with innovations in technique and technology. To measure radiation one can now use masses small enough to fit in a laboratory or large enough to be the earth or moon.

by Joseph Weber

IN THE SECOND "golden" decade of this century Albert Einstein[1] unified physics and geometry and fulfilled a long cherished dream of Karl Friedrich Gauss, Bernhard Riemann and William K. Clifford. This geometrical theory of gravitation had "magnetic" types of velocity-dependent forces and a finite velocity of gravitational interactions; it apparently predicted gravitational radiation. The equations describe gravitation in terms of the curvature of space-time. Riemann gave the concept of the curvature of n-dimensional space as a logical generalization of the concept of curvature of a two-dimensional surface. Radii of curvature for such a surface are easy to visualize. There is a simple connection (given in the box on p. 39) between these radii and the sum of the angles of a small triangle made up of geodesic lines (figure 1).



The author received his bachelor's degree at the US Naval Academy (1940) and his PhD from Catholic University of America (1951). Since 1948 he has been a professor at the University of Maryland, except for two years in which he was at the Institute for Advanced Study during tenure of Guggenheim and National Research Council fellowships.

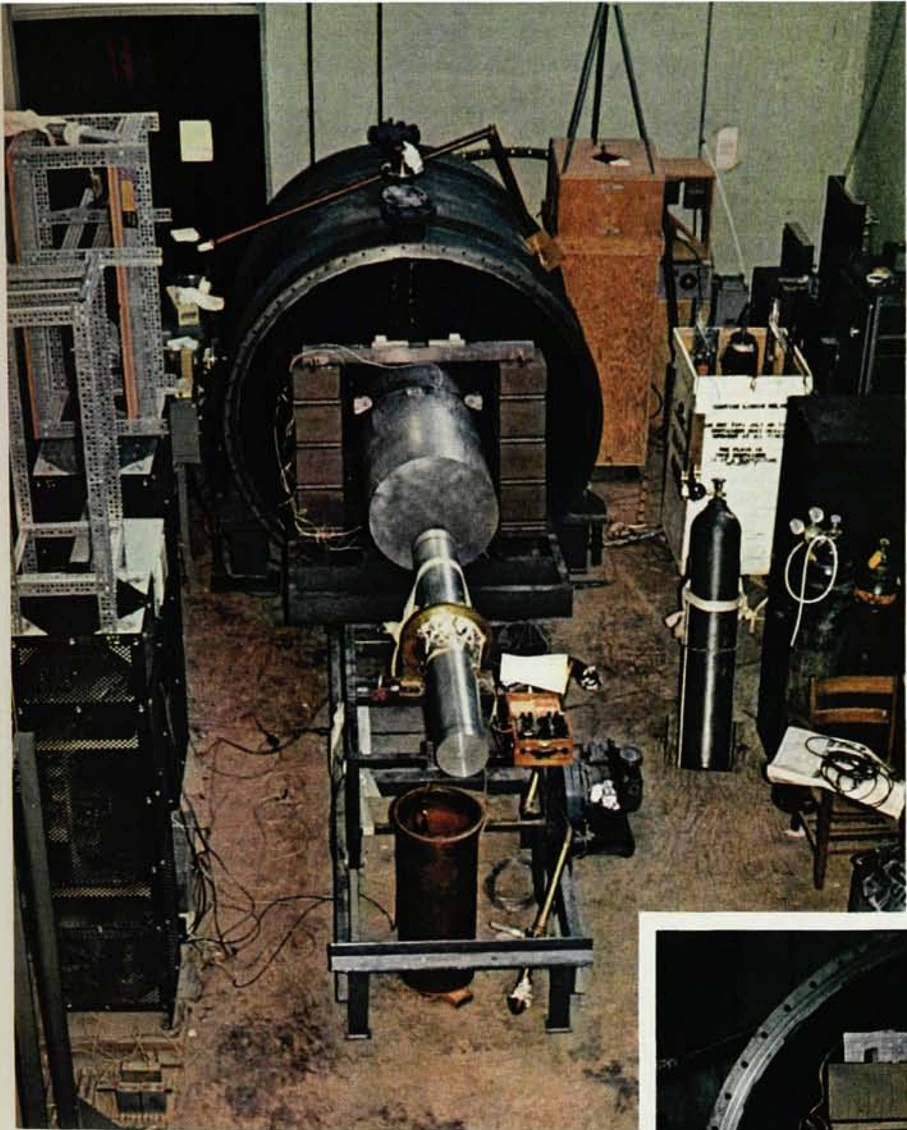
At each point there are two directions for which the radii of curvature are a maximum or minimum, and these directions are perpendicular. I will speak of these principal radii as the radii of curvature. The product of these radii plays a special role because it can be measured without going outside the surface. The reciprocal of this product is called the "Gaussian curvature."

Coördinates and dimensions

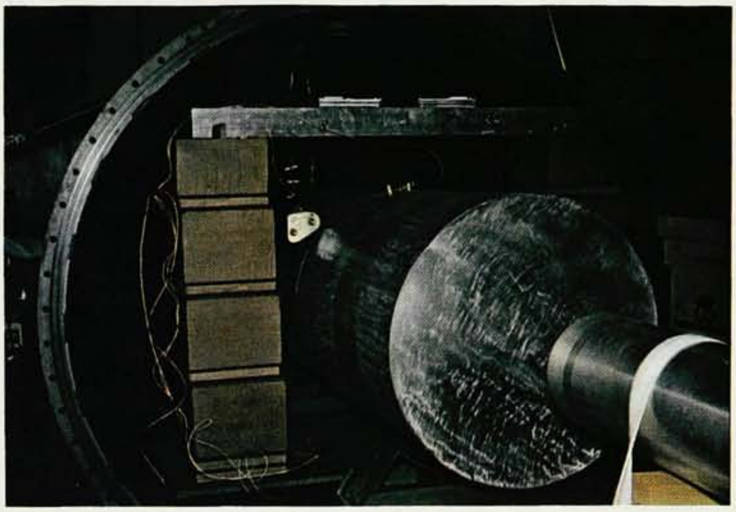
Let us employ coördinates u^1 and u^2 on our two-dimensional curved surface. The square of differential length between nearby points is

$$ds^2 = g_{11}(du^1)^2 + g_{22}(du^2)^2 + 2g_{12}du^1du^2 \quad (1)$$

The $g_{\mu\nu}$ are, in general, functions of coördinates u^1 and u^2. Gauss derived formulas for products of radii of curvature at any point in terms of the $g_{\mu\nu}$ and their first and second derivatives. Coördinate transformations will change the $g_{\mu\nu}$ but not the Gaussian curvature computed from them. These values of curvature are invariant quantities. Two-dimensional animals living on such a surface could—if their cleverness and talent for analysis were comparable with that possessed by Gauss—determine that they are living in a curved world. They would do this by setting up a two-dimensional coördinate system in a reasonable but nonetheless arbitrary way and measuring distances associated with changes in coördinates by means of measuring tapes that are bendable but not stretchable. These measurements, made entirely within the surface, without reference to the higher dimensional space in which the surface is embedded, determine the



ALUMINUM CYLINDER of 1400-kg mass is suspended by a wire on acoustic filters. Piezoelectric transducers are bonded to the top surface, as shown in the closeup.



DETECTOR and dynamic gravitational field generator are shown removed from their common vacuum chamber.

curvature everywhere. If a radius of curvature is infinite at some point, we say that the space is flat there.

Riemannian geometry enlarges these concepts. We introduce a curvature tensor with components that, in a two-dimensional space, are inverse products of the principal radii of curvature. The vanishing of all components of this tensor is then a necessary and sufficient condition for the space to be flat.

Between 1905 and 1915 Einstein sought a generalization of the Minkowski geometry of special relativity that would be valid for observers undergoing accelerations. Since an accelerated frame is indistinguishable from a gravitational field by local measurements, the more general geometry also described gravitation. Development of tensor calculus led to curvature expressions having the same form in all coördinate systems, and Einstein's use of it for a theory of gravitation resulted in covariance under arbitrary coördinate transformations.

Einstein's geometrization of physics consists, then, of taking the $g_{\mu\nu}$ as gravitational potentials. Validity of formalism for arbitrary coördinates freed physics from the special relativity requirement that observers be in inertial frames. A set of nonlinear partial differential equations determines the $g_{\mu\nu}$ with nongravitational stress energy as source terms.

Space-time derivatives of the $g_{\mu\gamma}$ give the geometry of space-time its curvature. Paths of planets and the curved trajectories of photons are geodesics. The acceleration of a body is determined by local geometry that completely replaces Newtonian gravitational force. In a curved space the sum of the angles of a small geodesic triangle will depart from 180 deg, and all components of the Riemann curvature tensor will not vanish.

Dynamical curvature-tensor components are the most important field quantities in the theory of gravitation. It is strange and perhaps remarkable that until 1963 no apparatus existed for measurement of these quantities for which we had a complete theory with programs for quantization and renormalization. Thus before this time the technology and experimental techniques had lagged very far behind the theory.

Measuring the tensor

I shall outline the conception and technique for measuring the Riemann curvature tensor[2] by considering detection of a gravitational wave. One might begin by having a very small test particle of mass m interact with the wave. A local observer would not be conscious of any motion of m because his own mass "falls freely" with the same acceleration as m. Indeed the forces are the first derivatives of the potentials, and these are "transformed away" by free fall. Suppose that instead of a small mass we employ an extended body. The gravitational force is transformed away by free fall only at the center of mass. At different points the integrated gradient of the force would result in gravitationally induced stresses within the body. Thus dynamical second derivatives of potentials will give rise to observable internal motions of the body. In different terminology one can say that an extended body will be deformed if it is placed in a curved space. A dynamical curvature will result in relative internal motions of different parts of the body and may excite its normal modes. Measurement of these motions gives certain components of the Riemann curvature tensor.

Gravitational radiation

In consequence of nonlinerarity, few solutions of Einstein's equations are known, and no satisfactory spherical wave solutions have been found either within the region of the sources or in matter-free space.

Einstein[3] and Arthur Eddington[4] linearized the equations to study gravitational radiation. They predicted quadrupole radiation in lowest order. Lack of dipole radiation can be understood in the following way. Consider an isolated system of masses m and M coupled by a spring (figure 2). Linear momentum conservation requires that

$$mv_m + Mv_M = 0 \qquad (2)$$

If velocities are small, the time dependence of m and M may be neglected, and equation 2 can be differentiated to give

$$m\frac{dv_m}{dt} + M\frac{dv_M}{dt} = 0 \qquad (3)$$

Equation 3 states that the sum of mass times acceleration over a system vanishes because of momentum conservation, and this sum corresponds to the sum of charge times acceleration, which gives dipole radiation in electrodynamics. We may think of the effects of the two masses as cancelling each other, the effect of the small mass being compensated by its larger velocity. If retardation is considered, or effects quadratic in the velocity, cancellation will not take place, and this failure of cancellation leads to radiation with a quadrupole character. Power radiated by a spinning rod is calculated to be

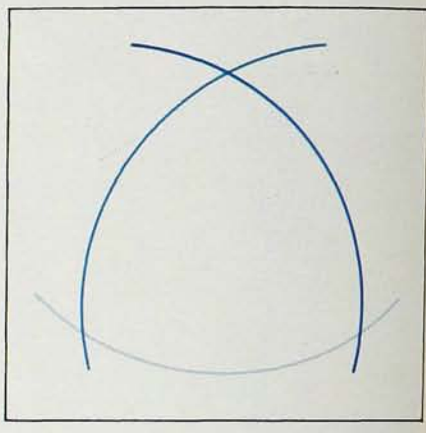
$$P = \frac{32GI^2\omega^6}{5c^5} \qquad (4)$$

In equation 4 G is the constant of gravitation, I is the moment of inertia, ω is the angular frequency, and c is the speed of light. Equation 4 is approximately what one would calculate when using the electromagnetic quadrupole radiation formula and replacing squared charge q^2 with Gm^2. (This fact tells us also why graviton spectroscopy from atoms and molecules is difficult. The ratio Gm^2/e^2 is 10^{-43} for electrons, thus leading to graviton emission 43 orders smaller than photon emission in lowest order.) For a spinning laboratory-sized rod, high velocities result in rupture owing to excessive strain. This phenomenon is related quantitatively to the elastic modulus and density. The limiting peripheral velocity would be expected to be sound velocity. Radiated power is therefore exceedingly small, involving a volume far smaller than a sound wavelength cubed. Power radiated by a spinning 1-meter rod is about 10^{-37} watts. For laboratory-size sources one can gain substantial improvement[2] with electromagnetic stresses over volumes with dimensions of the order of a gravitational wavelength. Recently Freeman J. Dyson[5] observed that, for (astronomical) gravitationally bound systems moving with speeds approaching that of light, extremely large amounts of power might be radiated as gravitational waves. For such a case, if v approaches c, we have for a binary system

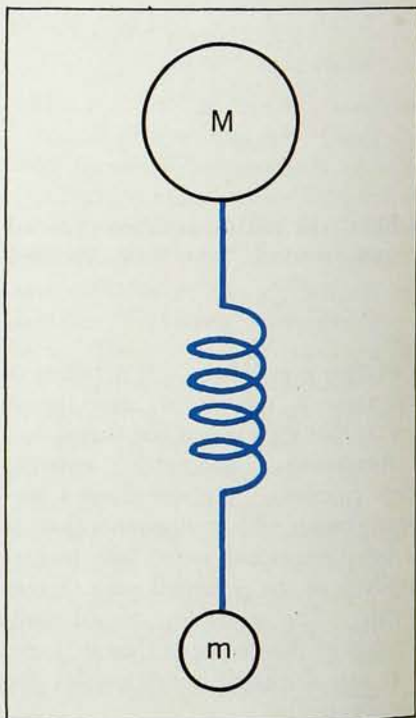
$$\frac{mc^2}{r} \approx \frac{Gm^2}{r^2} \qquad (5)$$

$$c \approx \omega r \qquad (6)$$

Substituting equations 5 and 6 in



GEODESY. A line triangle in a curved space. The internal angles of such a triangle do not total 180 deg. —FIG. 1



ISOLATED MASSES coupled by a spring. Einstein and Eddington predicted that gravitational quadrupole radiation would be emitted when the masses are made to oscillate. —FIG. 2

equation 4 gives

$$P \rightarrow c^5/G \qquad (7)$$

A more careful analysis leads to the result $P = 2048\ V^{10}/5c^5G$, where V is the velocity.[5]

We might expect these unusual powers, 10^{50} ergs/sec, to be radiated away if a star like our sun, having a modest angular momentum, should collapse. As collapse proceeds, peripheral velocity increases, approaching the speed of light. The resulting deformation or bifurcation may lead to quadrupole radiation.

To search for gravitational radiation in some band of frequencies we can select an elastic body with normal modes within that band and attach instruments to it. Two quite different approaches[2] have been followed. One involves laboratory apparatus together with extreme isolation from earth movements. A second approach utilizes normal modes of the earth and moon as detectors.

High-frequency detectors

One detector is an aluminum cylinder, with a mass of approximately 1400 kg, developed by David M. Zipoy, Robert L. Forward, Richard Imlay, Joel A. Sinsky and me. If we take its axis in the x direction, the lowest compressional mode is driven by the Riemann tensor component R_{oxox}.

For isolation the cylinder is suspended by a wire on acoustic filters in a vacuum chamber. Quartz piezoelectric crystals bonded to its surface convert normal-mode oscillations to an output electric field. To understand its operation and achieve good noise performance we must solve the boundary-value problem of a large isotropic body with piezoelectric crystals bonded to its surface as it interacts with gravitational radiation. The solution can be understood in terms of the equivalent electrical circuit of figure 3. Only terminals X and X are available to the observer. The driving "voltage" is the Riemann curvature tensor. The huge inductance L is the equivalent of mass, and the incredibly small capacitance C is the equivalent of stiffness. Output voltage is roughly proportional to the impedance appearing between X and X. One way to obtain a large impedance is to employ resonance with a superconducting inductance. If such a device is instrumented to operate at sensitivity limited by cylinder thermal fluctuations, we can expect when averaging over one relaxation time to see relative displacement δ of the end faces with kinetic energy exceeding thermal energy kT

$$\tfrac{1}{2}m\omega^2 \langle\delta^2\rangle \geq \tfrac{1}{2}kT \qquad (8)$$

Here m is the mass and ω is the normal-mode angular frequency. Equation 8 predicts that end-face displacements of a few hundredths of a nuclear diameter can be seen, that is, strains of a few parts in 10^{16}.

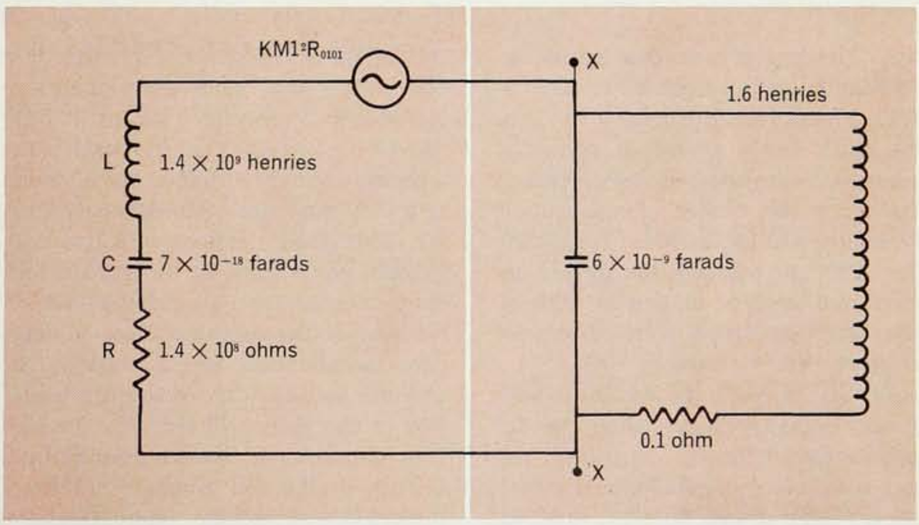
To verify the sensitivity of the detector and to test a new type of gravitational generator Sinsky carried out a high-frequency Cavendish experiment. A second cylinder was suspended in a vacuum chamber. Electrically driven piezoelectric crystals bonded to its surface built up large, resonant, mechanical oscillations. The dynamic gravitational field of the generator was observed at the detector as a function of longitudinal and lateral displacements. Agreement of theory and experiment confirmed that strains of a few parts in 10^{16} were being observed. We achieved gravitational communication at center-to-center distances close to 2 meters with a relatively high frequency of $\omega = 10^4$ radians/sec. This communication represents a major advance in the technology of gravitation. This experiment is also noteworthy because electromagnetic means generate the gravitational field that in turn produces electromagnetic output at the detector. Thus both inputs and outputs are photons with gravitational interaction taking part as an intermediate step.

Equation 8 implies that electrical output power exceeding kT over the relaxation time is detectable. Even with extreme precautions we can expect occasional violent earth motions or intense electromagnetic disturbances to couple such small energies into the detector. To study effects of earth motion we have at the detector site a number of low- and high-frequency seismometers together with tilt meters that give tilt data about north-south and east-west axes.

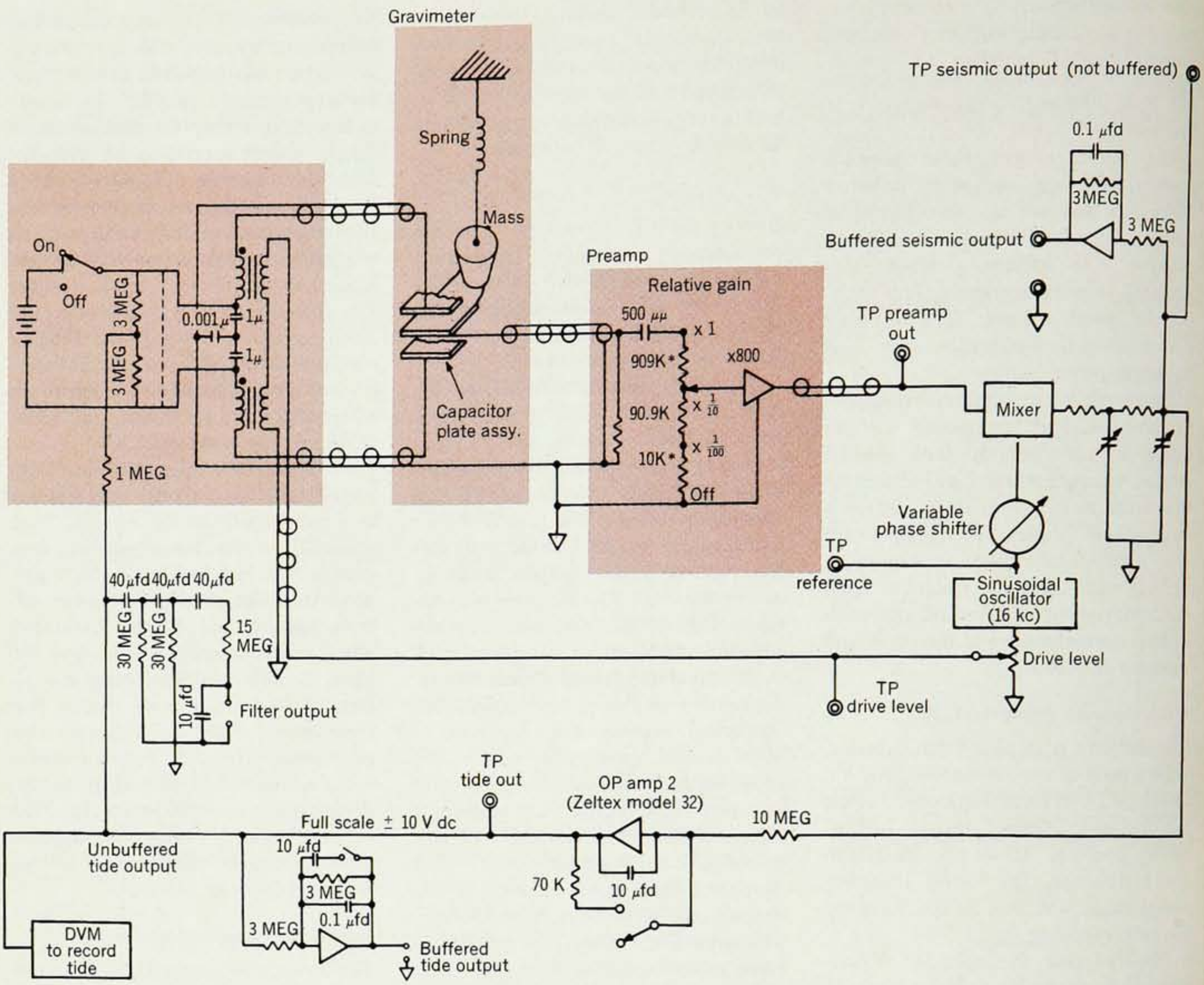
After Sinsky's experiment was completed, the second driving cylinder was converted to a detector and mounted on a concrete pier approximately 1.5 km from the main site. Since then about ten coincident events have been observed with these two detectors, unaccompanied by any obvious seismic or electromagnetic effect. It is by no means certain that these events are of gravitational radiation origin. However, the time resolution was recently improved by two orders and is now considerably less than the propagation time for acoustic waves through the earth. Coincidences still occur.

Earth and moon as detectors

The absorption cross section of a gravitational wave detector is proportional to its mass. How large can this mass



EQUIVALENT CIRCUIT of the gravitational wave detector. To the left are the properties of the cylinder and its suspension system. To the right is the resonant circuit that provides a high impedance between X and X. —FIG. 3



GRAVIMETER SYSTEM. Change in the capacitance between parallel plates in the instrument unbalances the bridge (left), and the voltage from the servo loop (right) applies a restoring force to the plates. —FIG. 4.

be? The largest mass that can be instrumented at this time is the earth itself. The gravest quadrupole mode of the earth has a period of about 54 minutes. General relativity predicts that only the modes of quadrupole symmetry will be excited. To employ the earth[2] we measure the surface acceleration or force of gravity with instrumentation that is sufficient to see changes of a few parts in 10^{11}.

We use a harmonic oscillator with a very weak spring constant as our gravimeter (figure 4). Small changes in *g* are therefore accompanied by relatively large displacements. Such instruments with optical means for observing the displacement of mass, are used in oil prospecting. For observation of the smallest possible earth accelerations we have developed a closed-loop servosystem shown in figure 4.[9,10] Imagine that the mass is in a position corresponding to some value of *g*. A capacitor plate is secured to *m*. This plate, together with two additional plates fixed to the gravimeter case, constitutes a radiofrequency bridge. If the earth's surface undergoes acceleration or if surface *g* changes for any other reason, the position of the mass will tend to change, thus unbalancing the driven radiofrequency bridge and giving a radiofrequency output voltage. Amplification and phase-sensitive detection lead to a dc output voltage with polarity depending on the sense of displacement; the output dc voltage is fed back to the plates, exerting restoring forces on them until the mass is returned to a position of equilibrium. The voltage required for this restoration measures the change in surface *g*. Thus we measure changes in *g* by comparison of gravitational and electrostatic forces.

The mechanical portion of the Maryland instruments was developed by LaCoste and Romberg Inc. The noise performance of these instruments permits observation of changes in *g* 10 000 times smaller than those associated with tidal deformations of solid earth. Thus far, quiet-period observations have not shown evidence of residual-earth-mode excitation. We cannot be certain that all distur-

FORMULAS FOR CURVATURE IN GENERAL RELATIVITY

For completeness I include here some of the formulas that define the curvature quantities employed by Einstein in formulating general relativity. A repeated index is a sum over three space coördinates and one time coördinate. A comma denotes partial differentiation with respect to the coördinate index following the comma.

Line element

$$-ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

Christoffel symbols

$$\Gamma^\mu{}_{\alpha\beta} = \tfrac{1}{2}g^{\mu\iota}\left(g_{\iota\alpha,\beta} + g_{\iota\beta,\alpha} - g_{\alpha\beta,\iota}\right)$$

The raised index $g^{\mu\nu}$ are components of the matrix inverse to $g_{\mu\nu}$.

Riemann-Christoffel curvature tensor

$$R^\mu{}_{\alpha\beta\gamma} = \Gamma^\mu{}_{\alpha\gamma,\beta} - \Gamma^\mu{}_{\alpha\beta,\gamma} + \Gamma^\sigma{}_{\alpha\gamma}\Gamma^\mu{}_{\sigma\beta} - \Gamma^\sigma{}_{\alpha\beta}\Gamma^\mu{}_{\sigma\gamma}$$

Ricci tensor

$$R_{\alpha\gamma} = R^\mu{}_{\alpha\mu\gamma}$$

Curvature scalar

$$R = g^{\mu\nu}R_{\mu\nu}$$

In a two-dimensional space we have at any point

$$\frac{R_{1212}}{g} = \frac{R}{2} = \frac{1}{r_1 r_2} = \lim_{S\to 0}\frac{\Delta\theta}{S}$$

where g is the determinant of $g_{\mu\nu}$, r_1 and r_2 are the principal radii of curvature, S is the area of a small geodesic quadrilateral, and $\Delta\theta$ is the excess of the sum over 2π radians.

Einstein's field equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

G is the constant of gravitation, c is the speed of light, $T_{\mu\nu}$ is the matter-stress energy tensor.

These equations can be obtained from an action principle employing the action function

$$I = \frac{c^3}{16\pi G}\int\left(R + \frac{16\pi G}{c^4}L_F\right)\sqrt{-g}\,d^4x$$

Here the integration is extended over the whole space-time, and L_F is the Lagrangian density of all fields in nature other than the gravitational field. The coupling constant in front of L_F couples all other fields to gravitation in the same way.

The equation of motion for one-dimensional acoustic waves driven by the Riemann tensor is

$$y\frac{d^2\epsilon}{dx^2} - \rho\frac{d^2\epsilon}{dt^2} - b\frac{d\epsilon}{dt} = c^2\rho R_{0x0x}$$

Here y is the elastic modulus, ρ is the density, b is a damping constant, and ϵ is the strain.

bances are of geophysical origin. During the next few years we plan to emplant a similar instrument on the lunar surface. Observation of coincidences would provide good evidence for gravitational radiation.

Light fluctuations

Light interacts with a gravitational field and gives rise to deflection of starlight by the sun and the possibility of lens-like[11,12] intensification effects. It is presently believed that objects such as quasars are some billions of light years away. Could the presence of gravitational radiation in the intervening medium be responsible for phenomena such as intensity fluctuations?

This possibility has been explored by Charles Misner, Zipoy[13] and me. Independent and earlier investigations of effects of gravitational waves on light fluctuations were carried out by Bruno Bertotti and John A. Wheeler.

Analyses indicate that there are no first-order effects giving light fluctuations that increase with distance. Imagine a tube containing light passing through a gravitational-wave-filled medium. The quadrupole character of the gravitational wave results in equal but opposite changes in the two dimensions of the tube. In second order there are small effects, associated with the energy density of the gravitational radiation, as indicated by Zipoy's analysis.[13] The effects are too small to observe at this time.

* * *

This paper was prepared while the author was a visiting scientist at the theoretical physics division of the Aspen Institute, Aspen, Colorado. His research was supported by the National Science Foundation. Sinsky's experiment was supported by the Air Force Office of Scientific Research, and the gravimeter research was sponsored by NASA.

References

1. A. Einstein, Ann. Physik **49,** 769 (1916).
2. J. Weber, *General Relativity and Gravitational Waves,* Interscience, New York (1961), chap. 8.
3. A. Einstein, Sitzungsberichte Pruss. Acad. Wiss., p. 688 (1916); p. 154 (1918).
4. A. S. Eddington, Proc. Roy. Soc. (London) **A102,** 268 (1923).
5. F. J. Dyson, chap. 12 in *Interstellar Communication,* A. G. W. Cameron, ed., W. A. Benjamin, New York (1963).
6. J. Weber, Phys. Rev. Letters **17,** 1228 (1966).
7. J. Weber, Phys. Rev. Letters **18,** 418 (1967).
8. J. Sinsky, J. Weber, Phys. Rev. Letters **18,** 795 (1967).
9. B. Block, R. Moore, J. Geophys. Res. **71,** 4361 (1966); N. Clarkson, L. LaCoste, Trans. A.G.U. **38,** 8 (1957).
10. J. Weber, J. Larson, J. Geophys. Res. **71,** 6005 (1966).
11. A. Einstein, Science **84,** 506 (1936).
12. S. Liebes, Phys. Rev. **133,** B835 (1964).
13. D. M. Zipoy, Phys. Rev. **142,** 825 (1966). □