The Emergence of Rheology

From the days of Newton and Hooke scientists from many fields have contributed to the study of the deformation of materials. But a lack of communication hindered developments until rheology emerged as a formal branch of mechanics.

by Hershel Markovitz

THE DEFINITION of the newly coined word "rheology" in the constitution of the Society of Rheology, when it was founded in 1929 under Eugene C. Bingham of Lafayette College, was "fundamental and practical knowledge concerning the deformation or flow of matter." As Markus Reiner recalled telling Bingham at the time, this word would appear to have the same meaning as the term *continuum mechanics*. An operational definition obtained by an examination of what most rheologists actually do would more likely lead to the conclusion that, as accurately as any scientific discipline can be defined, "rheology—a branch of mechanics—is the study of those properties of materials which determine their response to mechanical force," a statement that appears in a brochure recently issued by the Society of Rheology. This more restricted view, the one that I shall take in delineating the area covered in this historical review, removes from consideration the more complicated flow and deformation problems usually discussed in fluid mechanics, hydrodynamics and elasticity.

The material property that determines the response of a body subjected to force is usually expressed mathematically in terms of the dependence of the stress on the deformation. There are three classical models:

- the Hookean (or linear elastic) solid characterized by a stress that is a linear function of the strain
- the perfect (or inviscid) fluid characterized by a stress that is always isotropic
- the Newtonian (or linear viscous) fluid characterized by a stress that is isotropic when the fluid is at rest but which is a linear function of the rate of deformation when it is being deformed.

By Bingham's time it was established that these classical models were not adequate to describe the mechanical behavior of many materials. Among the experiments that were to have the greatest influence on the future of rheology were creep and related time-dependent phenomena in solids and fluids. A considerable amount of work had been done on the nonlinear relation between applied driving pressure and rate of flow through small tubes. These are the main developments that I will follow. To put them in their proper historical perspective, it is necessary to indicate

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how some of the classical theories developed and how confidence in their applicability to some materials was attained.

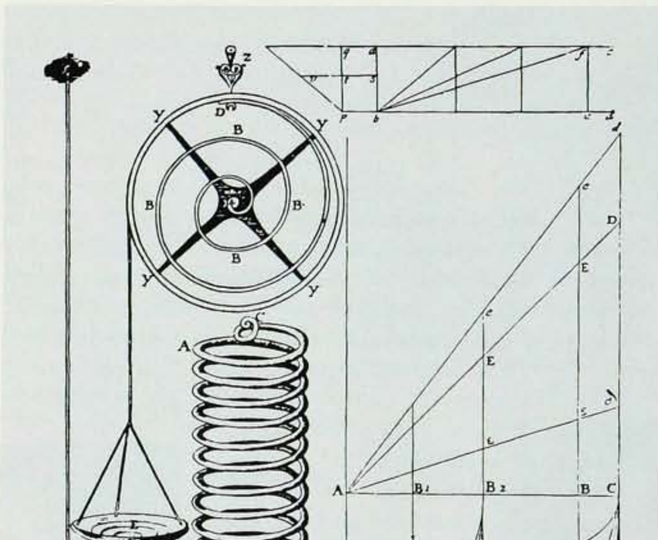
Hooke and Newton

The two classical model materials, the linear elastic solid and the linear viscous fluid, were introduced within a decade of one another in England towards the end of the seventeenth century.

In 1678 Robert Hooke announced his "true Theory of Elasticity or Springiness" . . . "The Power of any Spring is in the same proportion with the Tension thereof: That is, if one power stretch or bend it one space, two will bend it two, and three will bend it three, and so forward." (See figure 1.) His claim for his Law of Nature was quite general. It is true not only for any kind of deformation but "in all other springy bodies whatsoever, whether Metal, Wood, Stones, baked Earths, Hair, Horns, Silk, Bones, Sinews, Glass, and the like."

In 1687 Sir Isaac Newton stated his hypothesis concerning nonperfect fluids in his *Principia*. The main purpose of this work was to discuss the motion of the heavenly bodies. The prevalent theory at the time was René Descartes's theory of celestial vortices. It was Newton's main purpose in discussing the motion of bodies in resisting mediums to devastate Descartes before he proceeded with his own theory of celestial motion.

To this end he discussed "The circular motion of fluids." He began with the hypothesis: "The resistance arising from the want of lubricity in the parts of a fluid, is, other things being equal, proportional to the velocity with which the parts of the fluid are separated from one another." It is this "want of lubricity" that we today call *viscosity*. Newton immediately proceeded to discuss the problem of the rotation of an infinitely long cylinder in an infinite medium that offers a "resistance arising from the want of lubricity." At the conclusion of his discussion of this theorem he makes the statement: "All these things will be found true by making the experiment in deep standing water."



ELASTICITY EXPERIMENTS. Hooke investigated a great number of materials in various forms including: "the coyl or helix of Wire," "a Watch Spring coyled in a Spiral" and a "Wire string of twenty, or thirty, or forty foot long." His results indicated that the displacement of an elastic spring is proportional to the force applied to it. —FIG. 1

Some take this to mean that Newton had performed such experiments. But I doubt it. For one thing, Newton deduces that the angular velocity is inversely proportional to the distance from the axis of rotation—or that the linear velocity is independent of that distance. As George Gabriel Stokes was to point out, in 1845—more than a century and a half later—this answer is incorrect owing to an error in Newton's derivation.

Indeed Newton himself did not take his hypothesis seriously: "And though, for the sake of demonstration, I proposed, at the beginning of this Section, an Hypothesis that the resistance is proportional to the velocity, nevertheless, it is in truth probable that the resistance is in a less ratio than that of the velocity." It remains a puzzle to me why Newton invited the experiments in deep standing water.

However, even the idea that deviations from perfect-fluid behavior are due to a property of the bulk fluid was an advance. For example 50 years after Newton, Daniel Bernoulli attributed such deviations to "adhesion of water to the sides of the tube, which adhesion can certainly exert an incredible effect in cases of this kind."

Newton's hypothesis was apparently ignored by contemporaries and successors for more than a century. The theoretical hydrodynamic investigations of this time continued to deal mainly with perfect fluids while experimental work concentrated on the flow of liquids through pipes and open channels where inertial effects dominated. For example shortly after the French Revolution in 1801 when Charles Augustine de Coulomb published his paper on the resistance of fluids in very slow motion, he refers to Newton's experimental results and accompanying theory but not to the "want of lubricity" hypothesis. Coulomb here introduced the widely used freely oscillating torsion pendulum shown in figure 2. (He had described experiments on the torsional oscillation of a wire in 1784, 17 years earlier.) Moreover in this paper he tried to resolve the question of the boundary condition at the surface, where the fluid is in contact with a solid. He noted that the addition of neither tallow nor sandstone affected the motion of the pendulum and concluded that the fluid moves with the solid surface.

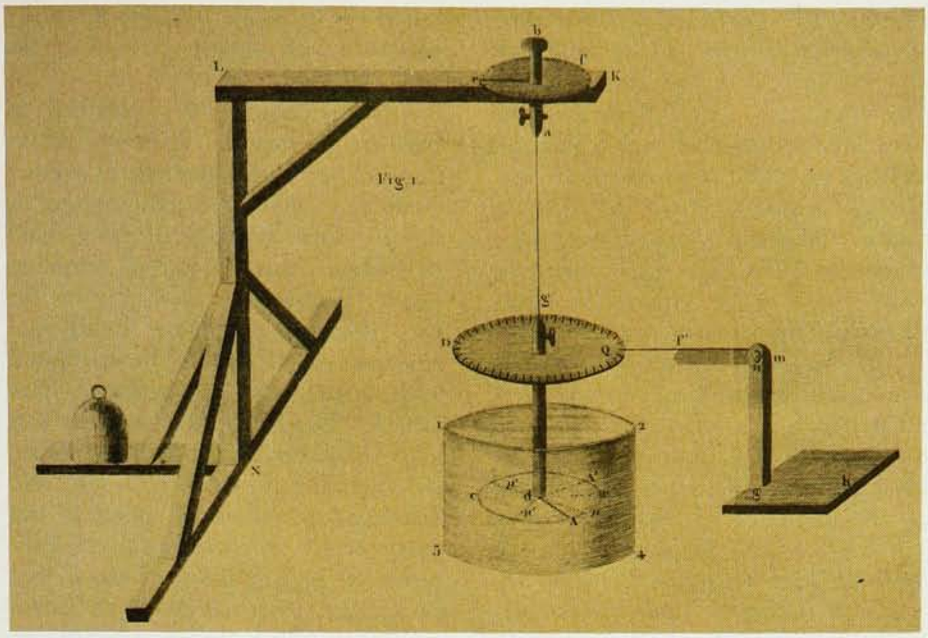
Linear viscous fluid

The advance from the experimental side towards a basic law of viscous flow came from the study of flow through narrow tubes. The empirical determination of the law of slow flow through tubes was established about 1840 by the researches of G. H. L. Hagen and Jean Louis Poiseuille. Hagen's experiments led him to the conclusion that the rate of flow was proportional to the pressure gradient and to the fourth power of the radius (figure 3).

Poiseuille had been working simultaneously in France; his reports appeared between 1840 and 1846. He undertook this investigation because he was interested in the flow of blood in the body and had already published a prize-winning paper on that subject in 1835. His experiments on flow through small-diameter tubes were classical not only in their conception and execution but also in their reporting (figure 4). Poiseuille was able to achieve amazing precision—usually better than 0.1% and not infrequently greater than 0.01%.

We note that the experiments of Hagen and Poiseuille were not performed to study Newton's, or any other, law. They were empirical investigations in narrow tubes.

The general theory for linear viscous flow owes its development to a number of investigators: Augustin Louis de Cauchy (1823), Simeon Denis Poisson (1831), L. M. H. Navier (1821), and Jean-Claude B. de St Venant (1843). The theory of Stokes (1845), which is quite modern in its approach, is based on the principle: "The difference between the pressure on a plane in a given direction passing through any point P of a fluid in motion and the pressure which would exist in all directions about P if the fluid in its neighborhood were in a state of relative equilibrium depends only on the relative motion of the



COULOMB'S APPARATUS to study the resistance of fluids in very slow motion. He measured the amplitude of a freely oscillating torsion pendulum: The disc is suspended from a torsion wire and submerged in a large cylinder of fluid. —FIG. 2

fluid immediately about P; and that the relative motion due to any motion of rotation may be eliminated without affecting the differences of the pressures above mentioned." From this assumption, after a linearization on the basis of a molecular argument, Stokes derived the Navier–Stokes equation. He then solved for the velocity distribution for flow between rotating coaxial cylinders and for flow through a cylindrical tube. He also derived the expression of the discharge from the tube, but he did not record it: "But having . . . compared the resulting formulae with some of the experiments of Bossut and Dubuat, I found that the formulae did not at all agree with experiment." Apparently in 1845 Stokes was not aware of the experimental results already published by Hagen and Poiseuille.

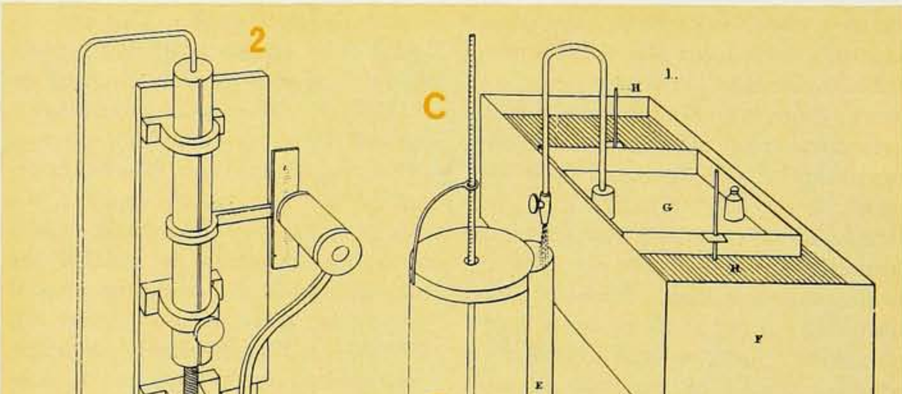
The first publication of the discharge equation for the rate of flow through a small tube was postponed another decade, when in 1856 G. Wiedemann published Edward Hagenbach's derivation. This derivation permitted the interpretation that Poiseuille's experiments verified a prediction drawn from the Navier–Stokes theory.

The acceptability of that theory was furthered by studies of flow between rotating coaxial cylinders. Stokes in 1845 had derived the velocity distribution and had suggested that experimental verification "would probably be best done by observing motes in the fluid." In 1881 Max Margules calculated the torque required to turn a cylinder at a given rate and concluded that, for water, the torque would be very small. The first account of an experimental investigation was published in 1888 in England by A. Mallock. After making a correction for end effects he verified the viscosity values calculated from Poiseuille's experiments. M. Couette reported the results of his very painstaking experimentation in which he avoided the end-effect difficulty with the elegant guard-ring construction (figure 5). He compared these coaxial-cylinder results with those he obtained in tube flow and found good agreement.

Constant controversy

Thomas Young, in 1807, was the first to introduce a property characterizing a material undergoing deformation when he defined the elastic constant—Young's modulus. In the succeeding decades the full three-dimensional theory for elasticity was produced by the same architects who developed hydrodynamics. On the basis of a molecular hypothesis, Navier (1827) arrived at a theory that involved only one elastic constant. Cauchy (1823), George Green (1839), Stokes (1845) and Gabriel Lamé (1852), using phenomenological approaches, produced theories that indicated the need for two constants. The controversy between the uniconstant and biconstant theories raged for some decades. Decision by experiment was not easily attained. Most elastic experiments were done on wires and these materials were known to be anisotropic. Furthermore the uniconstant Poisson–Navier theory corresponded to the biconstant theory with a Poisson ratio of 0.25, which is close to the value for many metals. Proponents of the biconstant theory offered evidence based on the properties of rubber and gels, but the uniconstant group rejected it because these materials were not accepted as isotropic. James Clerk Maxwell summarized the situation in 1853: "There are few parts of mechanics in which theory has differed more from experiment than in the theory of elastic solids." The final resolution of this controversy came as the result of the careful experiments of men such as G. Wertheim (1848), A. Kupffer (1853) and W. Voigt (1887). Thus, like the theory of viscous fluids, the applicability of the biconstant theory for isotropic elastic solids was not generally accepted until almost the end of the 19th century.

An experiment was described in 1808 that forms an important cornerstone of our understanding of rubber elasticity. The entropic nature is shown when suddenly a rubber band is stretched and the rise of temperature is observed by contact with the lip. The early revelation of this phenomenon can, I believe, be attributed to the blindness of its discoverer, John Gough. Despite the fact that he had been blinded by smallpox at age three, he decided on a scientific career. He developed his sense of touch to the point where he was able to identify plants by feeling them. I think that because he made full use of all his remaining faculties, he discovered that heat was liberated when rubber was stretched. Incidentally, his explanation of the phenomenon was that it "depends upon the mutual attraction of Caloric and Cautchouc, the former of which penetrates the latter, and pervades every part of it with the greatest ease and expedition." This explanation and experiment were not bettered for a half century when James P. Joule performed some quantitative experiments and Kelvin provided an explanation in terms of thermodynamics.

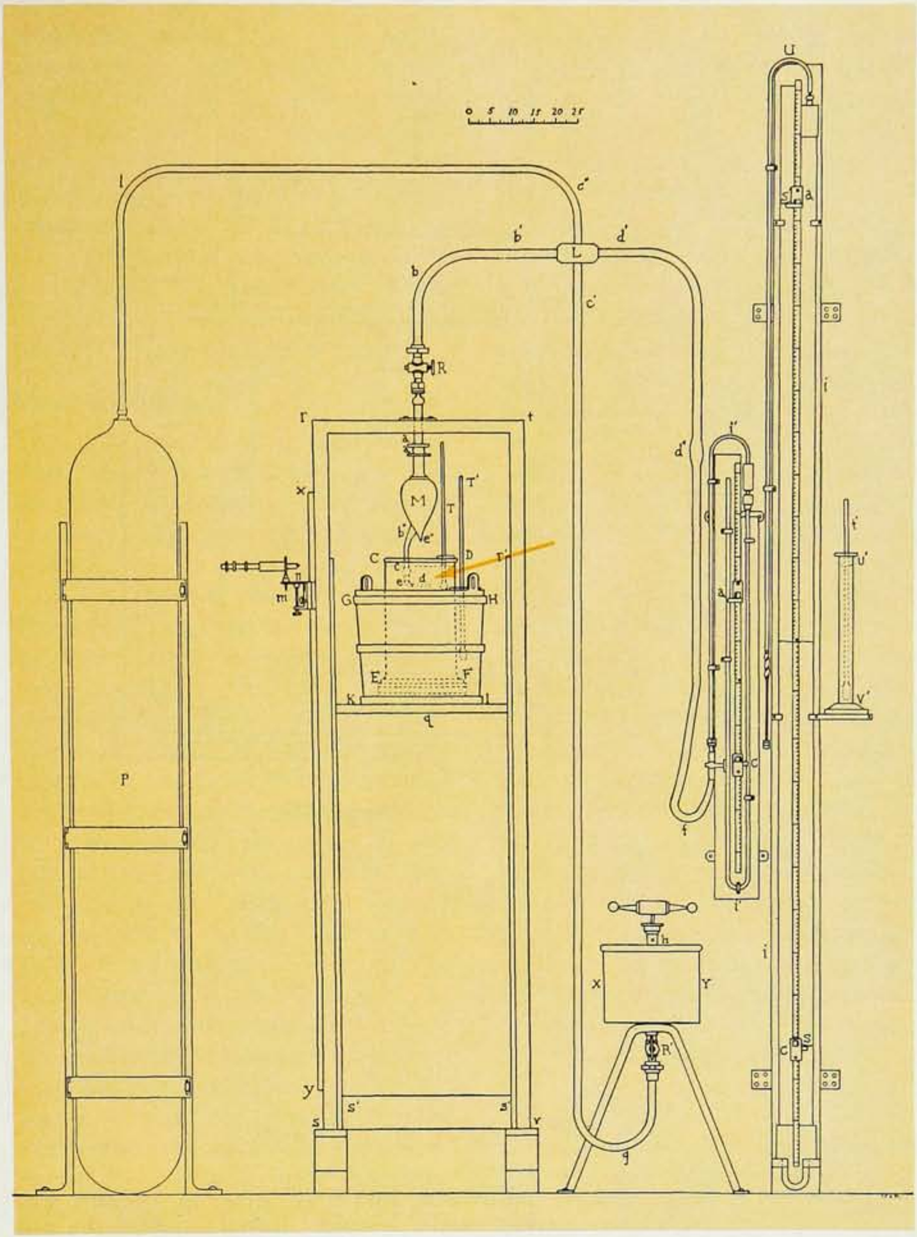


FLOW THROUGH NARROW TUBES. To study viscous flow Hagan caught and weighed the efflux from tube A. He determined the driving head from the difference between the height of fluid in efflux vessel K (measured with scale and pointer 2) and in cylinder B (indicated on scale C fixed to float D). —FIG. 3

Linear viscoelasticity

Very early investigators reported time-dependent phenomena that could not be explained by either of the classical models, the Hookean solid or Newtonian fluid. The first serious study of this phenomenon was that of Wilhelm Weber in 1835 on silk threads. This fiber interested him since it was frequently used in electrical and magnetic instruments because of its small torsional rigidity. Employing an ingenious experimental technique suggested by Karl F. Gauss, Weber found that raw silk was not perfectly elastic. A longitudinal load produced an immediate extension that was followed by a further lengthening with time. On removal of the load, an immediate contraction equal to the initial immediate extension took place. This was followed by a gradual further decrease of length until the original length was reached. Thus Weber knew that he was not dealing with a permanent set. He summarizes with the statement: "There is an extension which can be calculated by the law of proportionality from the modulus of elasticity, and in addition a further extension which takes place over a long period of time. This further extension is not defined in the law of elasticity and is to be regarded as an action or function of the *duration* of the loading." The author has denoted this extension by the term "Nachwirkung" (aftereffect). Furthermore, he deduced that the corresponding phenomenon of stress relaxation should exist and that the same phenomenon is responsible for the damping of vibrations in materials and for determining which materials produce sounds on being struck. There is no doubt that, qualitatively at least, Weber had captured the essence of viscoelasticity before Poiseuille had reported his results on flow through tubes and a decade before Stokes wrote down his version of the laws of viscous fluids and elastic solids.

In 1847 during an investigation of the capabilities of an electrometer for quantitative measurements of charge, R. Kohlrausch investigated problems that arose from the aftereffect in the silk-fiber torsion element. Following



PRECISE MEASUREMENTS on viscous flow were carried out (simultaneously with Hagen) by Poiseuille with apparatus that included an extensive pressure control and measuring system. Viscometer is at d(arrow). —FIG. 4

his death these investigations were greatly extended by his son, Friedrich Kohlrausch, in Weber's laboratory in Göttingen (figure 6). In 1863 he established the linearity of the phenomenon.

The experiments of Weber and the two Kohlrausches led Ludwig Boltzmann in 1874 to present his principle of superposition. He arrived at his integral representation of linear viscoelasticity in its full three-dimensional generality and discussed a number of problems. The more limited representations (for example the differential operators, relaxation spectra, spring–dashpot combinations) were not introduced until after Boltzmann formulated his more encompassing approach.

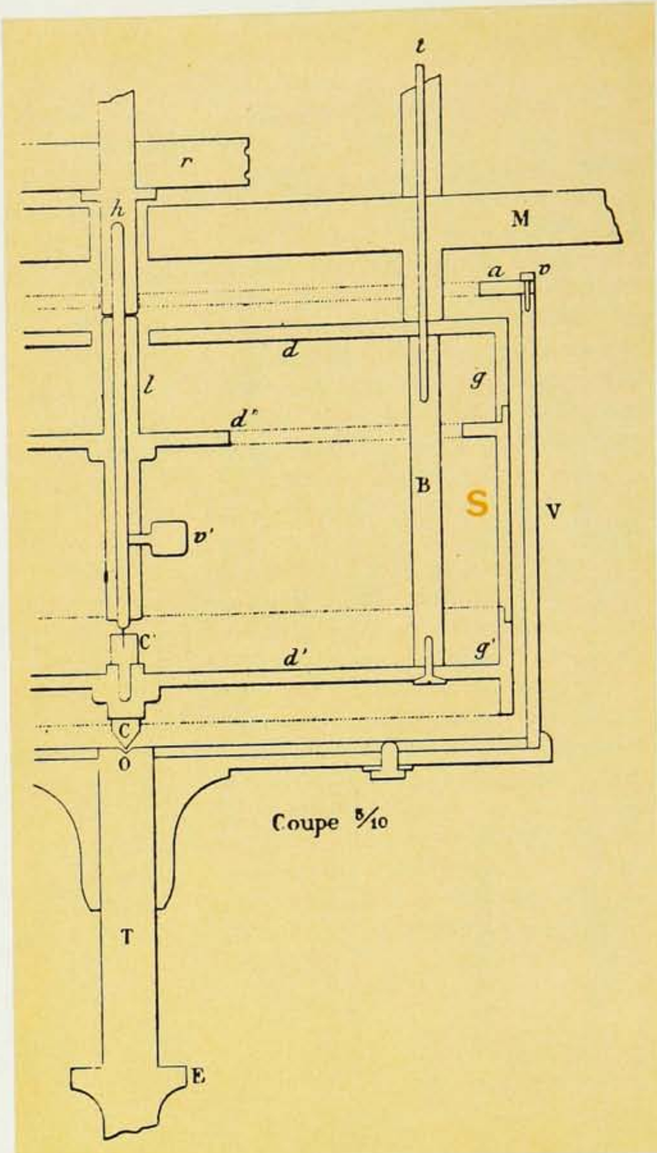
The younger Kohlrausch immediately put Boltzmann's theory to the test by a number of superposition experiments on glass and rubber. In perhaps the most dramatic of these he obtained reversals (sometimes a double reversal) of torsion direction in a rubber thread with a certain torque history. He measured the torque-history creep and recovery data for rubber, some of them over a 24-hour period. Thus he covered almost four decades of the time scale, a range rarely exceeded even today. He also demonstrated the large effect of temperature on creep.

Maxwell's influence

During these developments in Germany, an influential contribution to rheology came from a surprising direction. Maxwell in a paper "On the Dynamical Theory of Gases" (1867), an empirical explanation for the existence of shear stresses in flowing gases, introduced his famous first-order differential equation relating stress and deformation and the accompanying exponential stress relaxation. He furthermore proposed that this equation represents the phenomenon of viscosity in all bodies "independent of hypothesis." The simple exponential decay of stress does not explain the broad time scale frequently found in actual experiments. Maxwell knew about this from the work of Weber on silk fibers, Kohlrausch on glass and his own work on steel wires. He suggested that an explanation of these results could be found if the relaxation time was a function of the stress. However, he abandoned this explanation as soon as he heard of Boltzmann's work. In his 1878 article on the "Constitution of Bodies" he quoted Boltzmann's integral representation and ignored his own. Perhaps because the article was published in the famous 9th edition of the *Encyclopedia Brittanica,* Maxwell's first-order differential equation continued to exert a much greater influence on the future development of rheology than did Boltzmann's more encompassing theory.

The gap between the approaches of Maxwell and Boltzmann was partially spanned in 1888 by E. Wiechert and by Joseph J. Thomson who independently introduced the concept of the distribution of relaxation times. The most commonly used representation of viscoelastic behavior did not make its appearance until the turn of the century; in 1903 John H. Poynting and Thomson introduced the "spring-and-damper" analogy to illustrate the elastic aftereffect (figure 7).

Maxwell's differential equation inspired Theodore Schwedoff, a professor and rector of the University of Odessa, to seek direct evidence for elasticity in fluids. In 1889 he performed stress-relaxation experiments on gelatin solutions with a coaxial-cylinder instrument and found that the



COAXIAL CYLINDER VISCOMETER. In studying flow between rotating coaxial cylinders, Couette avoided the end-effect difficulty with guard rings. The outer cylinder is driven at a fixed angular speed, and the torque is measured only in the central portion S of the inner cylinder by the twist of the torsion wire to which it was attached. —FIG. 5

stress did not relax to zero in this material; it was a gel and not really a fluid. Thus its behavior could not be represented by Maxwell's equation. He was able to fit his data with a modified equation in which he assumed the existence of a yield value. On the basis of this equation, in the following year he published his conclusion that in steady-state flow, the viscosity depends on the rate of the deformation. He had in fact, arrived at the equation for plastic flow—an equation more usually associated with Bingham. For his experimental work, he constructed a copy of Couette's viscometer that had just been described (1888), and he obtained data that confirmed his theory. Despite the fact that he reported this work at the 1900 International Congress on Physics in Paris and a summary appeared in *Physikalische Zeitschrift,* it apparently remained unnoticed for several decades.

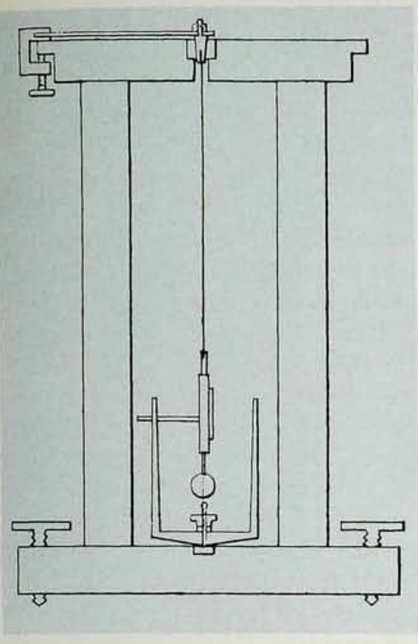
In his writings on states of matter, Maxwell drew a careful distinction between solids and fluids and, in many of his publications, pointed out that materials such as pitch are fluids: "If, therefore, we define a fluid as a substance which cannot remain *in permanent equilibrium* under a stress not equal in all directions, we must call these substance (cold pitch and asphalt) fluids, though they are so viscous that we can walk on them without leaving any footprints. What is required to alter the form of a soft solid is a sufficient force, and this, when applied, produces its effect at once. In the case of a viscous fluid it is *time* which is required, and if enough time is given, the very smallest force will produce a sensible effect."

Remarks such as these apparently were the inspiration for a number of experimenters. Around the turn of the century they undertook the study of very viscous fluids that could readily be mistaken for elastic, even brittle, solids. Carl Barus of the US Geological Survey Physics Laboratory, A. Pochettino in Italy and Wilhelm C. Röntgen, Rudolf Reiger and others in Germany studied similar materials.

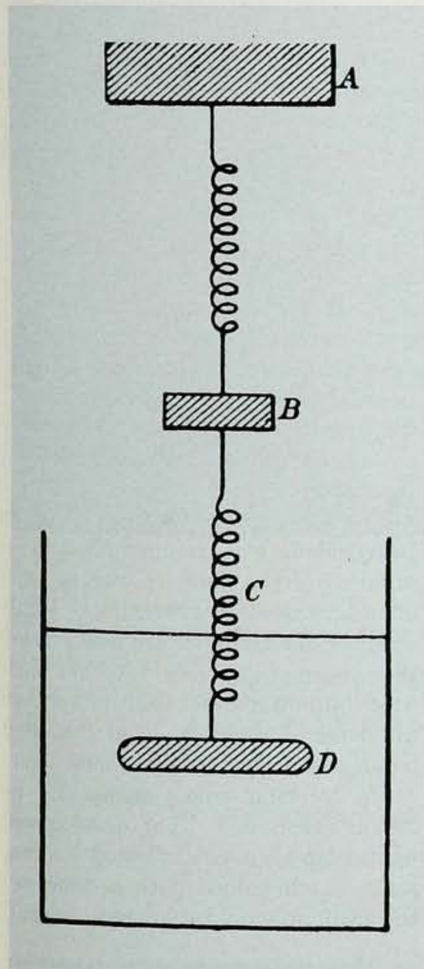
Experimental advances

The most remarkable study of such materials was done in England. F. T. Trouton and E. S. Andrews in 1905 reported on their studies on hot glass, pitch and shoemaker's wax. In their apparatus they mounted a cylindrical sample of the material and performed torsional experiments. On applying a load they were surprised to note first that the coefficient of viscosity of bodies such as pitch is a function of the time and "second, that on removing the stress there is a flow back in the opposite direction." Thus, for the first time, we have a creep and recovery experiment on a fluid. They noted further that when they plotted the applied torque against the steady-state angular velocity they did not obtain a straight line through the origin; this was the first evidence of a shear-dependent viscosity in a fluid. Then they proceeded to perform a stress-relaxation experiment by removing weights to keep the angular deformation constant. Even now few rheological investigations are that thorough.

In this same year (1905) Poynting presented an extension of his work on pressure of electromagnetic waves by applying it to the calculation of the pressure of elastic waves in a solid. For this application he had to introduce terms in the square of the strain. This conclusion led him, in 1909, to perform careful experiments in which he determined the lengthening of a



DECAY OF TORQUE. F. Kohlrausch held a silk fiber in torsion for a fixed time, and observed its angular motion when it was released. —FIG. 6



SPRING AND DAMPER of Thomson and Poynting. This model demonstrates the elastic aftereffect and damping of oscillations in metals. —FIG. 7

steel wire when it is twisted. He found that loaded wires increased in length by a small amount that was proportional to the square of the twist. In a still more elegant experiment, three years later, he actually measured the change in volume during a torsion experiment; for steel wires there was a contraction that was proportional to the square of the angle of twist. These experiments in nonlinear elasticity have not been improved as far as I know.

Shear dependence

The phenomenon of a shear-dependent, steady-flow viscosity was discovered independently by several researchers with quite different motivations.

About 1900, at the University of Heidelberg, experiments were being performed that, although of limited value, apparently did have considerable influence. To test his "foam-cell" theory of colloids, G. Quincke experimented on the decay of oscillations of a disc in various collodial suspensions. In their doctoral researches in this laboratory, Henry Garrett, with solutions of gelatin, silicic acid and albumin, and A. du Pré Denning with colloidal iron hydrate, found that the logarithmic decrement depended on, among other things, the amplitude of the oscillation. Although these were quite complex experiments, both in the nature of the material and the type of measurement, these results were interpreted by others as indicating nonlinearities in steady-flow behavior and apparently led them to study simpler flows.

Denning returned to England and there he performed experiments on the flow of blood through tubes. From the effect of the bore size he and John H. Watson concluded that blood containing a large number of corpuscles does not obey Poiseuille's law if the tube diameter is less than 3 mm; that is, the viscosity is apparently a function of the radius for smaller capillaries. Furthermore it also depends on the driving pressure for these small tubes.

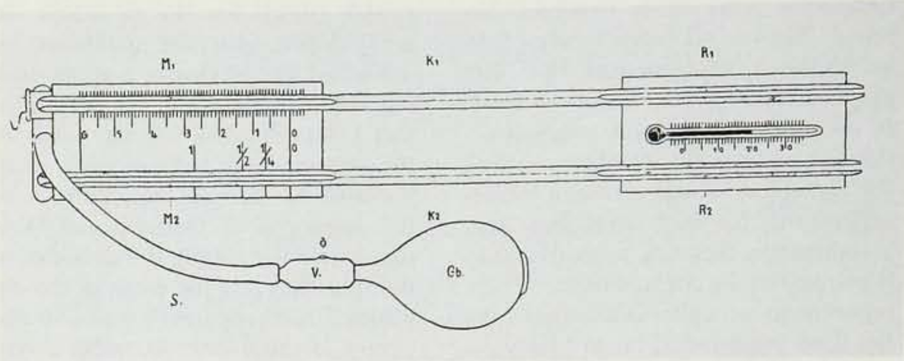
In the second decade of the twentieth century there were already a number of laboratories investigating deviations from Navier–Stokes behavior in steady-flow situations.

The physiologist Walter R. Hess, first at Rapperswil and continuing at Zürich, studied the flow of blood and gelatin solutions in tubes. He noted (1910, 1912) that the measured viscosity depended on the apparatus and convinced himself that he was not dealing with turbulence. He concluded that Poiseuille's law was not valid for these materials in laminar flow. Furthermore he showed that, contrary to the contention of Denning and Watson, it was not simply the diameter of the tube that was the cause of the deviation. Hess apparently made his discovery because he was using a viscometer that he had designed much earlier for other considerations (figure 8). In this apparatus the same pressure could be applied simultaneously to force fluids through two parallel tubes. He attributed the failure of Poiseuille's law to the existence of *cohesive forces*, which should lead to elastic behavior. He demonstrated the latter (in 1915) by performing free-oscillation experiments in a Couette configuration. In his work on gelatin solutions he was careful to note that, not only were they not gels, but also that they did not solidify even on standing. Thus this appears to be the first demonstration of memory effects in fluids of ordinary viscosity. Apparently because Hess published in medical journals, for a few years his discovery lay unnoticed by another group who were about to be involved in this problem.

Rate of shear

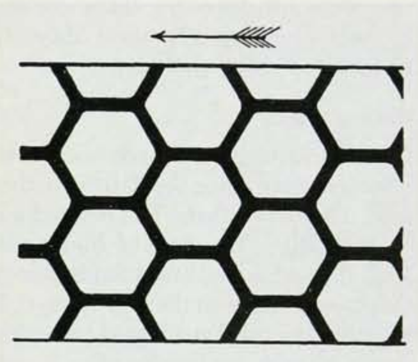
Colloid chemists had been using viscometers ever since the father of that field, Thomas Graham, had pointed out in 1864 that "The flow of liquid colloids through a capillary tube is always slow compared with the flow of crystalloid solution, so that a liquid-transpiration tube may be employed as a colloidoscope."

On 12 March 1913 the Faraday Society conducted a General Discussion on colloids and their viscosity. Wolfgang Ostwald, in his opening statement, listed ten factors that could influence the viscosity of colloidal systems. Rate of shear was not one of them. Famous colloid chemists such as Herbert Freundlich and Emil Hatscheck spoke, but no mention of either Schwedoff or Hess is recorded. Hatscheck mentioned Garrett's dissertation in the discussion of his theory of emulsions. He pictured a dispersed phase, represented as polyhedra, separated by a film of continuous phase (figure 9). On shearing, these polyhedra are dis-



VARIATION OF APPARENT VISCOSITY with the nature of test apparatus was discovered by Hess with this viscometer. The same suction pressure could be applied simultaneously to a test fluid and a standard fluid in the two separate tubes. Driving pressure could be varied by changing applied suction. —FIG. 8

torted. Hatscheck then concluded that for every system there must exist a critical value of the rate of shear at which a significant change in the system's viscosity begins. He stated, without giving data, that gum arabic and gelatin sols possess different viscosities at different rates of shear. Only two months after the Faraday Society discussion, Hatscheck reported his first experimental results, obtained with a Couette viscometer at the Sir John Cass Institute in London. Unlike Hess and many later workers, but like Schwedoff, Hatscheck thought that he was dealing with a viscosity that was a function of the *rate of shear*. He is probably entitled to credit for an independent discovery. Despite this work, there were still many papers to come where analogous data were viewed simply as deviations from Poiseuille's law rather than as reflecting nonlinear relation between shear stress and rate of shear.



SHEARING PHENOMENA. Hatscheck pictured the continuous phase of emulsions as polyhedric films A that on shearing would assume an altered shape B. He concluded that below a critical value the viscosity would depend on the rate of shear. —FIG. 9

One-dimensional approach

In the 1920's a number of laboratories were engaged in work related to studies of deviations from Poiseuille's law. Among the most active were those of Freundlich at the Kaiser-Wilhelm Institute for Physical Chemistry and Electrochemistry in Berlin, Ostwald at Leipzig and H. R. Kruyt in Holland. A number of capillary instruments were developed, particularly in Ostwald's laboratory, to cover a wide range of shear rate, and a great number of materials were investigated.

The method of handling these steady-flow data developed by stages. A one-dimensional approach was adequate to describe the results of the experiments: It was assumed that the shear stress was not simply proportional to the rate of shearing but rather a more complicated function. This approach allowed the data to be processed without touching on the basic problem, which was the failure of the Navier–Stokes equation to describe the flow behavior of these materials. Therefore the consequent three-dimensional stress-deformation relation was not sought. At first various functional forms were assumed for the one-dimensional relation and then expressions were obtained for measured quantities in steady-flow experiments in terms of the parameters in these functional forms. For the plastic material, E. Buckingham (1921) derived the equations for flow through tubes while Markus Reiner and Rassa Riwlin obtained the corresponding expression for Couette flow in 1927. For the power-law relation between shear stress and rate of shearing, H. L. Dryden (1926) exhibited the first deviation for capillary flow and F. O. Farrow, G. M. Lowe and S. M. Neale (1928) for Couette flow. The treatment of tube-flow data reached its culmination with the derivation by K. Weissenberg of an expression that made it possible to deduce the shear-stress-rate-of-shear relation from the experimental data without a priori assuming any functional form.

The above-mentioned paper by Farrow, Lowe and Neale is also noteworthy because they reported measurements made on their materials (a number of starch pastes) in Poiseuille flow with five different capillary tubes as well as in Couette flow and showed that the results were concordant. Thus they established that a shear-dependent viscosity is a useful concept. They also pointed out that, when they plotted their data linearly, the Couette data could hardly be seen since they were so close to the origin. In order to give them their due weight, they plotted the data logarithmically and thus set the mode widely followed thereafter.

This presentation lays no claim to being scholarly. My intention is to remind us of the debt we owe to some of our scientific progenitors. While some of our creditors are well known, their fame frequently rests on their contributions to other fields rather than on those to rheology. On the other hand, important innovations were made by men whose names are no longer mentioned. The development of rheology has not followed a monotonic, single-valued path in time—nor the road we would think most logical.

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