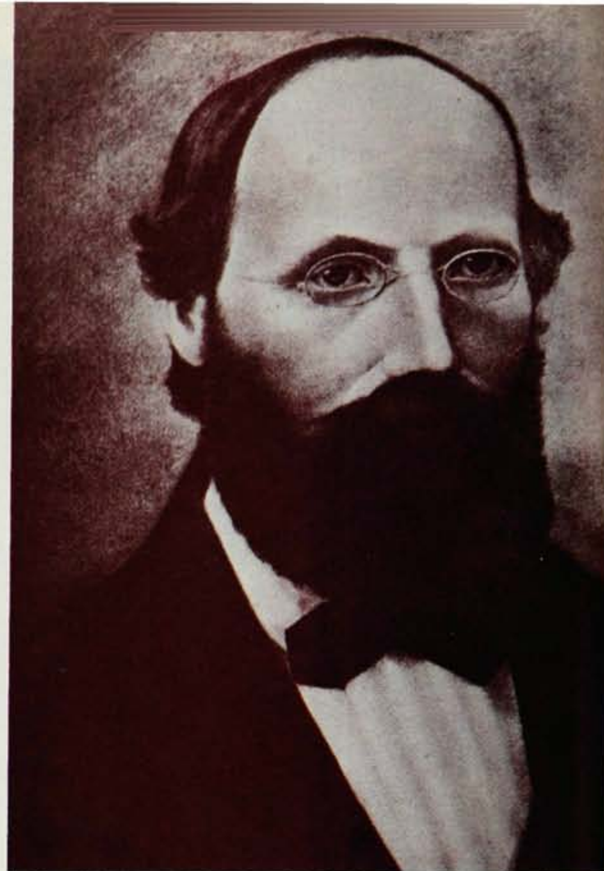


The Visionary Ideas of Bernhard Riemann

Although the life of this scientist was lamentably short, his insights into physics and mathematics are of lasting value. By applying a kind of religious devotion to his scientific work, Riemann left a legacy of concepts.

by Martin P. Jaggi

GEORG FRIEDRICH BERNHARD RIEMANN ranks among the most important mathematicians and theoretical physicists of the 19th century. Although he died at the age of 39, he managed to devote 17 years of his life to the visionary concepts of his work. His areas of interest were varied, and today we know that Riemann anticipated a large part of the research in mathematics and physics of our century. We can safely assume that the history of these two subjects would have taken a different course had he been granted the life span of Isaac Newton, Leonhard Euler or Karl Friedrich Gauss.

Searches and discoveries

In *calculus* he created the modern concept of the definite integral. By introducing geometrical concepts into the *theory of functions* he gave that field a highly pictorial interpretation that even today is capable of arousing a feeling of awe in its students. By this method he made possible the solution of important mathematical problems. Terms like "Riemann surfaces," "Riemann's sphere of (complex) numbers," etc. bear testimony to this.

The theory of functions was also stimulated by an analytical function discovered and examined by Riemann. This so-called zeta function later led to difficult and basic questions lying beneath the theory of numbers. It particularly led to a still unproved supposition already pointed out by Riemann himself concerning the "zeros" belonging to this function. The fundamental importance of this question may be illustrated with a remark by the great mathematician David Hilbert (1862–1943). When asked what question he would first ask if he returned to earth a hundred years after his death, he replied: "Whether Riemann's supposition has been proved."

Riemann's contribution to the *theory of differential equations* was also considerable. Since he concerned himself with equations that were of importance in physics and engineering, his work became the starting point of a great development in theoretical physics. His lecture "The Partial Differential Equations of Mathematical Physics" (edited by Heinrich Weber) served four generations of graduate students as the standard text.

Riemann also recognized the general principle behind the geometries of his day, that is, *Euclidean and non-Euclidean (hyperbolic) geometry.* He supplemented them by his *elliptic (non-Euclidean) geometry* and then combined them all into an integrated whole. He succeeded in this by attributing to "space-in-itself" a characteristic property, which varies from place to place and which he called "measure of curvature of space." As a mathematical expression of this local quantity he created a new mathematical concept: the tensor. This *Riemannian tensor* of space curvature became the outstanding working model in the development of general vector and tensor analysis, which today is one of the most powerful mathematical tools in theoretical physics. As Newton and Michael Faraday replaced "action at a distance" by "action going on in the

The author got his PhD from the University of Berne, Switzerland, and has done research and taught in Europe and the US. Since 1960 he has been at the Sate Technological Institute of Berne.

PORTRAIT OF RIEMANN painted three years before his death and tombstone bearing an appropriate inscription for the deeply religious scientist: "All things serve for the best for those who love the Lord."

medium itself," that is, the integral law by the differential law, so Riemann accepted Euclidean geometry only in the infinitely small. He thereby created for space the a priori possibility of all conceivable geometries. Seventy-three years later this point of view, going beyond Kant's conception of space as a mere form and necessary mode of human perception, found its sensational application in Albert Einstein's general theory of relativity. Today, in the history of mathematics and theoretical physics, Riemann's conceptual achievement in regard to the interpretation of the notion of space ranks on a par with the discoveries of Faraday and James C. Maxwell.

Riemann's tendency to seek the fundamental laws of nature "in the infinitely small" rather than "in the whole" proved to be generally successful. It is also recognizable in his essay in which he predicted the existence of *electromagnetic waves* and introduced the retarded potential that Heinrich Hertz used 35 years later in calculating the electromagnetic oscillator.

His emphasis upon the differential, the local (in contrast to the whole, the function) was already apparent during his student years in Berlin, where he held the view (opposed by Ferdinand G. M. Eisenstein, his teacher) that the definition of the concept of the complex function was to be found in the "infinitely small" (the Cauchy-Riemann differential equations).

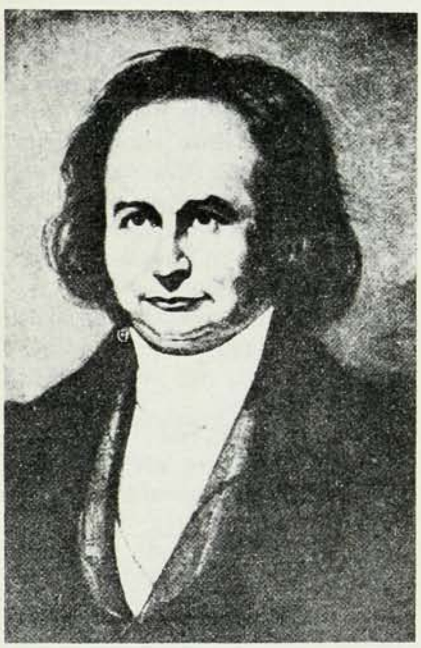
Somehow Riemann always found the right approach in his research. He had a visionary mind that enabled him to select from several conceivable possibilities the one that, in the course of scientific and technical development, would prove to be correct. His prediction of the existence of the *shock wave* in gases is typical. But this investigation is also remarkable from an historical and philosophical point of view since it is one of the rare examples in which a calculation undertaken by purely mathematical interest brought to light a hitherto unknown phenomenon of physics. This essay, which appeared in 1860, is accepted as the beginning of modern gas dynamics; thus Riemann happens to be the discoverer of supersonic flow.

From theology to science

Who was Riemann? He was born in 1826 into the middle-class family of a Lutheran minister. The first 14 years of his life were spent in Quickborn, a small town located near the Elbe River in the former Kingdom of Hannover. He was by nature a shy and reserved child. These characteristics were only strengthened by his strict upbringing and the relative isolation of his childhood. Receiving his schooling as a private pupil of his father, he had scant communication with other children of his age until his enrollment in the Lyceum in the city of Hannover. Here Riemann found it difficult to adapt to his new surroundings. His inability to deal with strange persons caused him to spend much of his time alone, engrossed even at this early age in problems of mathematics.

After finishing his preliminary studies at Hannover and Lüneburg, the 20-year-old Riemann entered the University of Göttingen, the cultural center of his homeland. Following the dictates of his father, he first enrolled as a student of theology. However in the drafts of his sermons he could not refrain from applying a geometrical approach to religion, unconsciously following the method known from the *Ethics* by the Dutch philosopher and apostate Benedict de Spinoza. His father, realizing that this trait

CARL FRIEDRICH GAUSS (1777–1855), the head professor at the University of Göttingen during Riemann's student days. He produced fundamental advances in mathematics, physics and astronomy. His biographer, Eric Temple Bell, called him the "prince of mathematicians."



CARL G. J. JACOBI (1804–51), the brilliant algorist, was one of Riemann's teachers at the University of Berlin. Jacobi developed the theory of elliptic functions and contributed to variation calculus and the theory of differential equations.

together with his almost pathological shyness would not allow him to go far in his intended profession, reluctantly gave permission for his son to turn to what he considered to be a thoroughly impractical subject—mathematics.

The change from theology to mathematics was a step that the young Euler had taken almost a hundred years before. Euler's background was similar to Riemann's: He also had come from a straightlaced family of a minister, had entered the University (Basle) as a student of theology and, like Riemann, had maintained throughout his life the piety that had been instilled in him in his youth. In his last years Riemann was influenced by his experiences as a Protestant in Catholic Italy and by his own failing health. He came to regard a daily self-examination and respect for those of different beliefs as the main tasks of the Christian church.

At Göttingen, Riemann fared no better than his fellow students. His head professor, Gauss, was already in his own lifetime a legendary figure in the field of mathematics, but his brilliance brought little advantage to his students. He was primarily the director of the astronomical observatory in Göttingen. Routine instruction did not interest him and was something that he left to other members of the faculty. His students heard little of the more interesting developments in higher mathematics or of Gauss's own discoveries. From time to time another well known mathematician would visit the university. Gauss would then sit with him and a select group of students around the table. On these rare occasions those present would gain an insight into the mathematical and pedagogical superiority of the master. Since Riemann was not one of the chosen few, it was impossible for him to be seen in the proper light by the great mathematician. After four semesters, Riemann reached the conclusion that he was not receiving his due in Göttingen and decided to transfer to the University of Berlin.

Student days in Berlin

In Berlin Riemann studied under a group of bright young professors: Carl G. Jacobi, Peter Gustav Lejeune-Dirichlet, Eisenstein and Jakob Steiner. These men, unlike Gauss, demonstrated their discoveries to their students. In this fertile atmosphere, Riemann developed a scientific self-confidence and mathematical judgment of his own. Here he acquired the unbridled scientific ambition and urge toward research that were so out of keeping with the rest of his character.

In Berlin the 22-year-old Riemann was swept up in the revolution of 1848. A staunch supporter of Friedrich Wilhelm IV of Prussia, he took turns with fellow students in standing watch before the royal palace. His devotion to the king may have indirectly saved his life; for during one of his tours of duty at the palace gate, a stray bullet entered through the window of his unoccupied room and plowed through his bed.

Riemann's attitude toward the King of Prussia was dramatically opposed to his attitude toward his own king, Ernst August of Hannover. For Riemann, Friedrich Wilhelm was the model of the modern, enlightened monarch, while in Ernst August he saw the embodiment of narrow-minded absolutism of the past.

After four semesters at the University of Berlin, Riemann returned to Göttingen where he was befriended by the electrodynamicist and codiscoverer of the telegraph, Wilhelm Weber. Weber, who had earlier incurred the displeasure of King Ernst August and had been banned from the University, had now resumed his physics lectures. Riemann was impressed by these lectures and by Weber as well. Weber, for his part, recognized Riemann's genius and remained for the rest of his life (Riemann's, that is, since Weber outlived him) a friend and adviser.

Riemann assumed charge of Weber's mathematical-physics seminar. Although this position hindered him in working toward his doctorate in mathematics, he did not consider this a disadvantage. Dealing with problems of physics sharpened his senses and aroused his interest in those mathematical questions that either were already current or were to become important in their physical application. This is partly the basis of Riemann's success.

After passing his doctoral examination under Gauss (1851), Riemann applied for the vacant post of astronomer at the observatory. Gauss, who was hardly generous in the recognition of his students' accomplishments, had in the meantime become aware of the extraordinary capabilities of his youngest doctoral candidate. Realizing that Riemann's promise lay in another direction, he saw to it that the application was denied. Gauss's intention was fulfilled: Riemann remained bound to pure mathematics and theoretical physics, the fields for which his genius was created. He eventually assumed the post held by Gauss and his successor, Lejeune-Dirichlet.

Riemann's best known publication is his inaugural lecture given before the faculty at Göttingen (1853). In this lecture he managed to present the subject of his space-curvature hypotheses in terms that were even understandable to those of his colleagues who were unfamiliar with higher mathematics. He closed with the statement: "Investigations starting from general notions can be useful to prevent research from being restricted by the limitation of concepts and to prevent the recognition of relationships from being hindered by traditional prejudices."

Sickness and death

At the height of his career and fame, Riemann was stricken with tuberculosis. The long rests in Italy in the care of his young wife, Eliza Louisa Frederica Koch, brought relief but no lasting cure. He whose mind was still filled with great ideas, waiting to be released, was himself finally subjected to the implacable laws of Nature. He died on 20 July 1866.

His death occurred during his third journey to Italy. For this reason his grave does not occupy its rightful place of honor in Göttingen beside that of his predecessor Gauss. His final resting place was in the cemetery of the church of Selasca, located above Lake Maggiore, about half an hour by foot from the Italian resort Verbania. After the removal of the grave, the head stone that had been donated by his Italian colleagues was set in the cemetery wall. It is probable that only a very few of the tourists and visitors from the north are aware that the resting place of one of their greatest abstract thinkers is located here. When I photographed the marble slab, neither the local priest nor the authorities of Verbania had any knowledge of its existence, nor did the name of Riemann have any meaning for them. Even the tenants of the former Villa Pisoni, where Riemann had lived, had never heard of him. "Tempora mutantur . . ."

Last year in commemoration of the 100th anniversary of his death, the mayor of Verbania, Stefano Ammenti, ordered the restoration of the inscription on the gravestone. Notice of the memorial will also be made in the tourist guide for this region.

Riemann was already seriously ill when he set out on his last journey to Italy. He had suffered through the winter of 1865–66 and once more decided to seek relief in the sun of his beloved Italy. He was aware that he might not survive the trip and left his two-and-one-half-year-old daughter, Ida, behind in Göttingen. He and his wife began their journey southward but succeeded in going no further than Kassel when their trip was interrupted. The German-Prussian War had broken out, and one of the first acts of war had been the destruction of the railway. They continued the journey as far as Giessen by carriage and from there to Lucerne (Switzerland) by train. The trip was completed by paddle steamer across Lake Lucerne and by coach over the Gotthard pass. The journey put such a strain on Riemann's failing health that he was forced to interrupt the trip in the small town of Selasca, 20 miles south of the state line between Switzerland and Italy. He and his wife stayed at a large and beautifully situated house. (This house, then called Villa Pisoni, is today a hospital and rest home.)

Until the day before his death Riemann worked under a fig tree in the garden of the villa on a record of his only physiological research. Inspired by Helmholtz's theory of sound perception (and perhaps also by Euler's work on the subject), Riemann conceived a theory of sound transference based on hydraulic principles. The fragment that was later published was incomplete, and there is no way of determining in what manner Riemann's and Helmholtz's theories differ from each other. Yet this work symbolizes the viability of thought that characterizes this important scientist. □

JAKOB STEINER (1796–1863) introduced Riemann into modern geometry. Steiner had been one of the first students and assistants of the Swiss educator, philosopher and philanthropist Heinrich Pestalozzi. The well known "theorem of Steiner" in the theory of the moment of inertia, however, has nothing to do with Steiner: It is due to Christian Huygens (1656). The misnaming happened at the turn of the century and proliferated into most physics textbooks.

WILHELM WEBER (1804–91) was the first to recognize Riemann's scientific competence. He became his friend, counselor and promoter. Together with Gauss he invented the electric telegraph and worked on a unifying theory of electromagnetism. In Germany he was the first to introduce regular classroom demonstrations in physics.