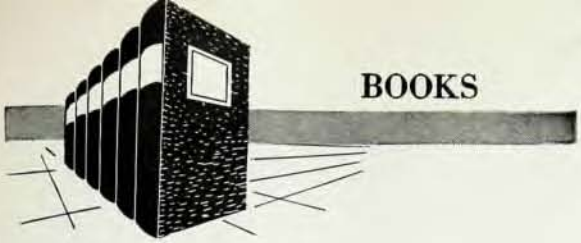

BOOKS

Continuum

PHILOSOPHY OF MATHEMATICS AND NATURAL SCIENCE. By Hermann Weyl. Revised and augmented edition based on a translation by Olaf Helmer. 311 pp. Princeton University Press, Princeton, 1949. $5.00.

This book, whose theme is the rapprochement of science and philosophy, is by a philosophically-minded mathematician, eminent both in pure mathematics and in mathematical physics. In brief space we find amalgamated and elucidated with masterly elegance a treasure of deep, central ideas from the fields of mathematics, physics, biology, and methodology. The author not only reports the more important philosophical results and viewpoints stemming from scientific research in recent decades; he also points out their connections with the great philosophical systems of the past.

The numerous literal quotations from a wide range of original sources add grace to the brilliant style, and these, together with the many illustrative examples, make the book pleasant to read. But it is not a popular book for, as Weyl is careful to state, "knowledge of the sciences themselves must be upheld as a prerequisite for anyone engaging in the philosophy of science." The references appended to various sections, though far from complete, seem to include mostly works deserving careful study. As the author observes, "One of the principal tasks of this book should be to serve as a critical guide to the literature listed in the references."

The Philosophy of Mathematics and Natural Science is based on a translation of the article Philosophie der Mathematik und Naturwissenschaft, which originally appeared in Oldenbourg's Handbuch der Philosophie in 1927. The author has revised the translation and added six appendices, bringing the work up to date. Mathematics occupies roughly Part I of the translation and the first two appendices, which together constitute nearly half of the book. The rest deals largely with physics while touching on biology in some of the appendices. As it is not possible to discuss (or even mention) each of the many interesting things in this book, the reviewer will confine himself to a few points which happen to strike him with special interest and summarize the remaining portions briefly.

The most important issue in the philosophy of mathematics since the turn of this century seems to have been that of intuitionism versus realism, and the author, himself one of the two leading intuitionists of our time, tells the story with ease and clarity.

The central problem is that of the infinite, and especially, that of the continuum. Roughly, the task is to found mathematical analysis and set theory on an intuitively clear and reliable basis. Those interested in the philosophy of mathematics now seem to agree that the foundations of analysis and set theory are not completely unquestionable. The difficulties are connected with certain modes of reasoning in these fields which, though leading to the desired consequences in a simple manner, appear on close examination to lie beyond the range of our clear understanding. In order to resolve these difficulties, the intuitionists, together with constructivists of other brands, attempt to reconstruct part of classical mathematics on more stringent bases, and sometimes to discard the rest as superfluous (for instance, in so far as applications to natural science are concerned) while the so-called realists want to support the status quo of the edifice of analysis and set theory, appealing to arguments which make it necessary for us to modify considerably the traditional notion of the a priori character of our mathematical knowledge.

More specifically, the usual way of defining real numbers by convergent sequences or by Dedekindian cuts cannot be considered as putting analysis on a completely clear and reliable basis once and for all. Thus, suppose we have somehow managed to pick out an aggregate B of properties or sets of natural (or rational) numbers. We may then define a set P to which a number m belongs if and only if there exists a set c of B such that m belongs to c. There is no intuitive reason to think that such a P must in general also belong to the given aggregate B. On the contrary, since P has been defined in terms of the totality of the sets of B, it is more natural to think that P in general does not belong to B. Yet in analysis, the proof (for instance) of the very fundamental theorem of least upper bounds makes use of precisely the assertion that for every such B (if bounded), the P constructed in the above fashion again belongs to B. Without some such assertion (which may be taken as a case of Russell's controversial "axiom of reducibility") it is not possible to obtain analysis in its usual form; yet it is not intuitively clear why such assertions should be true, nor even is there any a priori reason to judge that such assertions would never lead to contradictions. Indeed, if we speak uninhibitedly not only of sets of natural numbers but of sets of sets, we run into contradictions among which the simplest is Russell's on "the set of all sets of which each does not belong to itself:" call such a set s, then it can be seen that s belongs to s if and only if s does not belong to s.

According to Brouwer, the root of these paradoxical situations lies in our belief in the applicability of classical logic (in particular, the law of excluded middle) to infinite sets. And such a belief, according to him, "was caused historically by the fact that, firstly, classical logic was abstracted from the mathematics of the subsets of a definite finite set; that, secondly, an a priori existence independent of mathematics was ascribed to this logic; and that, finally, on the basis of this suppositious apriority it was unjustifiably applied to the mathematics of infinite sets." Hence, Brouwer undertook to reconstruct analysis on the solid basis of our fundamental intuition into the process of complete induction, refusing to apply the laws of classical logic unrestrictedly to infinite sets. However, the results thus obtained are disheartening. Even Weyl has to admit, "It cannot be denied, however, that in advancing to higher and more general theories the inapplica-

bility of the simple laws of classical logic eventually results in an almost unbearable awkwardness." Moreover, if we are to follow Brouwer, we have to give up many well established methods and results.

"The resolution to make this sacrifice," says Weyl, "is doubly hard in view of the historical fact that in set-theoretical analysis we find, in spite of the boldest and most elaborate combinations, complete certainty of deduction and an obvious accord among all the results." In hopes of safeguarding classical mathematics against attack, Hilbert has proposed the program of proving the consistency of the less reliable parts of mathematics with tools of the intuitively more reliable. The execution of this program has been rather unsuccessful thus far. Gödel, in defending the realistic position, suggests that classes or sets "are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions." And just as in physical theories we can never be sure whether concordance will survive further observations and experiments, we must be content if a simple axiomatic system of mathematics has met the test of our elaborate mathematical experiments thus far. We might say that the great mass of more or less intuitively evident theorems of mathematics correspond to the empirical data more or less closely related to sense-perceptions, and that the so-called axioms of mathematics correspond to the so-called laws of physical theories. But then, as Weyl is careful to point out, "A truly realistic mathematics should be conceived, in line with physics, as a branch of the theoretical construction of the one real world, and should adopt the same sober and cautious attitude toward hypothetic extensions of its foundations as is exhibited by physics." Are logicians and mathematicians going to pay more attention to the investigation of systems which are stronger than the radical constructivistic systems but weaker than systems (for example, those recently formulated by Quine or Gödel) for the whole set theory?

When we come to the empirical sciences, there are the conflicting philosophies of idealism and realism. Here, however, according to the author, the conflict is more apparent than real, because they signify principles of method which do not contradict each other: the realistic thesis is, "A difference in the perceptions offering themselves to us is always founded on a difference in the real condition;" and the idealistic thesis is, "The object image of the world may not admit of any diversities which cannot manifest themselves in some diversity of perceptions." But the author does not like radical empiricism or positivism, considering some views thereof to be either too drastic or too cheap. Of Hume he says, "To carry his approach through is as impossible as to found arithmetic on nothing but the concretely existing numerals," and again, "the sensualism of Hume and Hjelmslev—which on principle would recognize as real only the immediately given, without being able to carry this through—is deadly for science."

If readers suspect that certain issues considered by the author fall under the category of futile controversies over ill defined problems or wish that the author had at many

places written less vaguely and ambiguously, the reviewer wants only to suggest that they try to understand the author's words in context and reformulate these for themselves, when possible, in some clearer fashion. Those who are disturbed by the author's way of discussing metaphysical concepts such as those of freedom, purposiveness, and God as guarantor of reality, might perhaps be conciliated by the author's modestly expressed confession in the new preface: "With the years I have grown more hesitant about the metaphysical implication of science."

At the end of the fourth appendix, speaking of the scientific progress during the past twenty years, the author recalls that interest in the continuous aspect of nature was still dominant in 1926 and then remarks: "In the last two decades, however, discontinuous and combinatorial structures underlying the natural phenomena have become of increasing significance. Here a deeper layer seems to come to light, for the description of which our ordinary language is woefully inadequate. The preceding Appendices bear witness to this changed outlook. However, we could not do much more than assemble relevant material; the philosophical penetration remains largely a task for the future."

In the remaining portions of the book, mathematicians will likely find it delightful to read what Weyl has to say about "creative definitions," the axiomatic method, irrational numbers, complete induction, invariant relations, Riemannian geometry, relativity, Gödel's and Gentzen's results on the structure of mathematics, and combinatories. All those interested in the philosophy and methodology of natural science will no doubt enjoy reading his scattered observations on a variety of subjects such as: Kant's transcendental idealism, causality, classification, matter, probability, truth, confirmation, objectivity, the division into a priori and a posteriori, the formation of scientific theories, the hierarchy of structures from quantum mechanics to chemistry and even the laws of heredity, mutation and quantum jump, a biological principle of indeterminacy, main structures of the physical world, perseverance versus adjustment, cosmogony, geology, phylogenetics, etc., etc. Although perhaps few readers could hope to comprehend all the details in this amazing book, the reviewer is of the opinion that those interested in the philosophy of mathematics and natural science will certainly find this a highly rewarding book for careful study in toto or in part.

Hao Wang
Harvard University

Peace Potential

CONSTRUCTIVE USES OF ATOMIC ENERGY. Edited by S. C. Rothmann. 258 pp. Harper and Brothers, New York, 1949. $3.00.

The papers in this source book, as the editor points out, have been compiled from the outside looking in, that is, from the point of view of the layman seeking an insight into atomic energy rather than from the standpoint of the scientist looking out, and occasionally down, on the reader. In most of the chapters this very worthwhile aim is accomplished. Compiled by S. C. Rothmann, a research engineer on the staff of the Accident Prevention Department,