FRACTURE

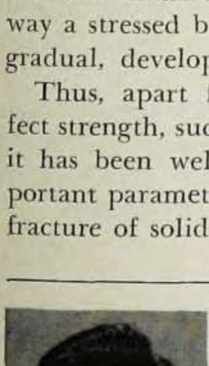
A solid body fractures not under a critical threshold tension, as once thought, but according to the complex interaction of a number of parameters. Among these parameters are the composition of the body, its temperature—and time. The physics of fracture was discussed at a recent international conference held in Sendai, Japan.

by C. C. Hsiao

ANALYSIS OF THE BREAKING STRENGTH of a solid body was based, in the remote past, on the belief that the body would break instantly under a critical threshold tensile force. Below that threshold, it was thought, the body would last without failure for a long time, if not indefinitely.

However, many relatively new experimental facts have indicated that the rupture of solids, even brittle fracture under uniaxial tension, is by no means a simple phenomenon. The magnitude of breaking tension is found to be intimately related to the duration of load application or to the rate of loading. As a general rule, for a solid body in a stressed state, the shorter the body the larger the load needed to break it. This clearly shows that we cannot assume that a solid body will fail under a critical threshold tension, but that the way a stressed body breaks is associated with some gradual, developing processes.

Thus, apart from many other factors that affect strength, such as temperature and composition, it has been well established that time is an important parameter in the study of the strength and fracture of solids. As a result, many scientists during the past fifteen years have centered their attention in investigating the time-dependent fracture phenomenon of solids. [1-15] In general, for a large variety of solids, both in experiment and in theoretical development, the logarithm of time has proved to be linearly related to the applied uniaxial state of tensile breaking or fracture stress. This linear law has been established, at least for some solids, for lengths of time ranging from microseconds to months (see figure 1).



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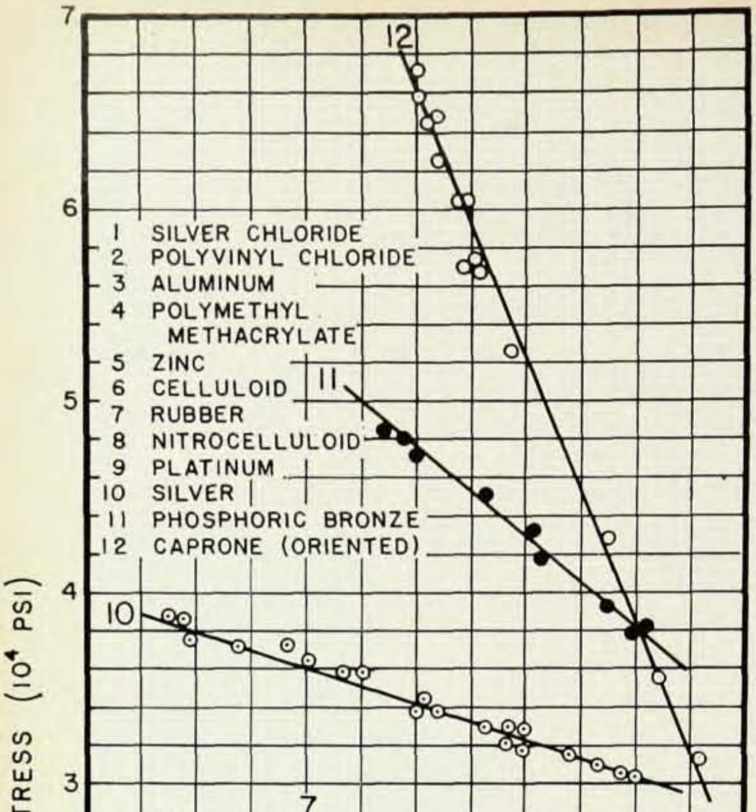
International conference

In order that these and other developments might be discussed, an international conference on fracture was held in September 1965 at Sendai on the island of Honshu, Japan. Both macroscopic and microscopic aspects of fracture were considered. The program, printed in English, included the following areas of interest:

- mathematical, physical and continuum mechanical theories
- atomistic, microstructural and macroscopic mechanics
- strength and fracture of nonmetallic materials
- fatigue and fracture with emphasis on microscopic behavior
- environmental effects, high pressure, high temperature, high strain rate, radiation damage and similar phenomena.

The conference was attended by more than two hundred Japanese scientists and almost a hundred



TIME-INDEPENDENT STRENGTH. For a large variety of solids, the logarithm of time is linearly related to the applied uniaxial state of tensile breaking or fracture stress. Each curve at left represents a different solid identified by the numbered key at top (after Zhurkov). —FIG. 1

scientists from other countries. Professor Takeo Yokobori of Tohoku University made an introductory speech, and a welcoming address was given by the mayor of Sendai.

Two simultaneous technical sessions followed immediately after the brief preliminaries. One session was concerned with continuum theories, and the other with fatigue fracture and observations of fracture surfaces. E. Kroner proposed a continuum theory dealing with the range of atomic cohesion forces and K. Kondo of Japan discussed the geometrical approach to the micromechanics of fracture. Several reports regarding dislocations and crack propagation were also discussed. Later, Japanese and foreign scientists reported on research results obtained in investigations of the mechanism of fracture.

B. L. Averbach opened the second day of the conference with a lecture on microcrack and macrocrack formation. In his presentation, Averbach outlined the mechanism of microcrack formation and the associated critical value of the crack extension force to provide the basis for a fracture-safe design criterion. One of the technical sessions on the second day was concerned primarily with continuum-mechanics studies of fracture problems. Another session dealt essentially with microscopic aspects of fracture phenomena.

The third day began with a lecture by M. L. Williams on initiation and growth of viscoelastic fracture. Williams pointed out that Griffith theory in fracture mechanics for brittle materials might be extended to materials having dominating time-dependent flow characteristics as well as viscous dissipation mechanisms. Thus the previous Griffith critical stress results for fracture could be cast into a similar form as

$$\sigma_{cr} = K \left(\frac{E}{l_i} \Sigma T_i\right)^{1/2}$$

In this equation K is constant, E is the material modulus, l is the crack length and T_i represents the individual energy quantities associated with the particular dissipation processes for brittle, ductile, and viscoelastic materials.

The last day of the conference was devoted mainly to such aspects of fracture as growth of fatigue cracks, enviromental effects on fracture and strain rate effects on deformation and failure.

Kinetic concept of strength

Near the end of the technical sessions Professor S. N. Zhurkov, of the Physical-Technical Institute of the Academy of Sciences of the USSR, presented a special lecture on the kinetic concept of strength of solids. Zhurkov pointed out the kinetic nature of the fracture process of solids. He reviewed his earlier experimental findings[16] and considered the thermofluctuation mechanism of fracture.

According to Zhurkov, it has been easy to demonstrate that the relationship between the lifetime or time-to-break t_m, the applied constant simple tensile stress σ and the absolute temperature T could be written in the form of a kinetic operation, as

$$t_m = t_0 \exp[(U_0 - \gamma\sigma)/KT] \quad (1)$$

Here K is Boltzman's constant and t_0, U_0 and γ are material constants. This formula represented not just an ordinary empirical relationship but a significant physical process of destruction in stressed solids in general. For such varied solids as silver choloride, aluminum and polymethyl methacrylate, t_0 was found to be about 10^{-13} sec. The reciprocal of t_0 coincided with the natural oscillation frequency of atoms in solids. The quantity U_0 was interpreted as the magnitude of the energy barrier determining the probability of breakage of the bonds responsible for strength.

Experimental data collected for lattice solids indicated that U_0 fitted well with the energy of sublimation or the binding energy of atoms in the crystal lattice in metals. Similarly for polymers, U_0 corresponded with the energy of breakage of chemical bonds in macromolecular chains. The corresponding nature of the parameters t_0 and U_0 with fundamental constants of the frequency of thermal oscillations of atoms and the interatomic binding energy permitted one to consider the kinetic process in the mechanism of fracture. On the basis of this concept and the application of equation 1, calculated values of time-to-break for some polymer samples were compared with actual experimental results and were found to be quite consistent.

Polymer fracture

Another interesting part of Zhurkov's report was concerned with the study of kinetics of polymer fracture by electron paramagnetic resonance. Quantitative measurements of the lifetime under constant stress of nylon fibers and the rate of accumulation of radicals in samples of nylon were obtained by testing in the cavity resonator of an EPR spectrometer. It was found that the rate of bond rupture grew exponentially with the increase of applied tensile stress.

On the other hand the time-to-break under a constant tensile stress varied by the same exponential law mentioned earlier. Mathematically these relationships could be written as follows for a constant temperature T:

For the bond rupture rate v at a constant stress σ

$$v = C_1 \exp(\alpha_1 \sigma) \quad (2)$$

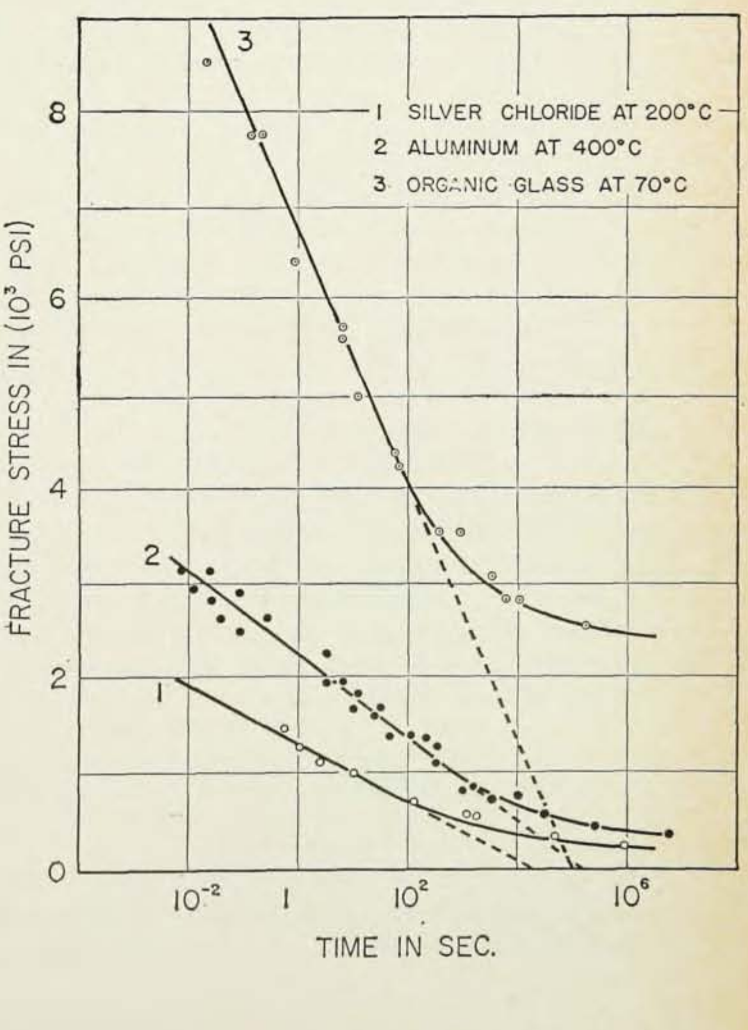
In this equation C_1 and α_1 are constants.

For the time-to-break t_m under a constant stress σ, the exponential relation is obtainable from equation 1 to give

$$t_m = C_2 \exp - (\alpha_2 \sigma) \quad (3)$$

where $C_2 = t_0 \exp(U_0/KT)$ and $\alpha_2 = \gamma/KT$ at a given temperature T are constants.

If the lifetime or time-to-break of a stressed polymer were completely determined by the rate of accumulation of the ruptured bonds, then one might expect the exponents α_1 and α_2 to be equal. In this case the lifetime of the specimen and the rupture rate of the bonds in the specimen under



DEVIATIONS from the linear stress-lifetime relationships of different solids are observed at high temperatures or if only small stresses are applied (after Zhurkov). The reason for the deviations has not been explained. —FIG. 2

a constant stress at a given temperature should be related by the equality

$$vt_{m} = C_1 C_2 = \text{constant} \qquad (4)$$

Experimental verification of this relationship for nylon fibers, at room temperature, at 50°C and at −50°C has shown good agreement with the theoretical prediction. Thus the EPR method proved to be very effective in obtaining a direct confirmation of the kinetic nature of the polymer fracture process.

Deviation from linearity

However, Zhurkov also indicated that there was deviation from equation 1 when small stresses were involved in experimental studies (see figure 2). The reason for the deviation, which was found to be common for different solids, has not yet been elucidated. Besides this principal deviation, Zhurkov also frequently observed one more violation of the general linear law between the logarithm of time-to-fracture and the applied constant stress in the given general kinetic equation 1. This was not interpreted as a principal nature and was claimed to be associated with the instability of materials in mechanical tests. Stabilization of the structure would result, as a rule, in a straightening out of the nonlinear relationship between log t_m and σ, so that its total consistency with the general kinetic equation 1 would be obtained.

With regard to this point C. C. Hsiao had presented some analytical results during the second day of the conference. In his report the kinetic process was considered in the study of the ultimate behavior of solids. It appeared that on a somewhat similar basis, there would be deviation from the linearity between stress and the logarithm of time-to-fracture of a solid subjected to a simple tensile stress, whereas linear relations could be obtained only under large stresses.

Hsiao reported his findings on the basis of using the statistical theory of the absolute reaction rate.[17] The mathematical model used was a matrix of oriented elements or bonds, whether primary or secondary, embedded randomly in an arbitrary domain. If f represents the fraction of unbroken elements per unit solid angle, then the rate of change of f can be written as

$$\frac{df}{dt} = K_r\left(\frac{1}{4\pi} - f\right) - K_b f \qquad (5)$$

where $K_r = \omega_r \exp - (U/RT + \rho\psi)$.
In this equation the value K_r is the rate of reformation of broken elements, ω_r being the frequency of motion of broken elements. U is the original potential energy barrier to be crossed between two equilibrium states, R is a universal constant, T is absolute temperature, ρ is a material constant for the system and $\psi(t)$ is the stress subjected by the elements. Similarly

$$K_b = \omega_b \exp - (U/RT - \beta\psi)$$

if K_b and ω_b are respectively the rate of rupturing and frequency of motion of unbroken elements, and β is a modification constant. After a stress $\sigma(t)$ is applied to the system as a whole, the energy barrier for parallel elements becomes modified to $U/RT - \beta\psi(t)$ in the direction of stressing, and to $U/RT + \rho\psi(t)$ in the opposite direction. The *time-dependent* fracture of any medium can be studied by solving equation 5, from which

$$f(t) = \frac{1}{4\pi}\exp\left[-\int_0^t (K_r + K_b)\,dt\right] \times \left\{\int_0^t K_r \exp\left[\int_0^t (K_r + K_b)\,dt\right] dt + f_0\right\} \qquad (6)$$

In this equation f_0 is a constant. For simplicity consider a fully oriented system, in which the stress function $\psi(t)$ in each element would be given by

$$\psi(t) = \frac{\sigma(t)}{f(t)} \qquad (7)$$

Also for simplicity, the fracture under the influence of a constant stress σ was considered, and an assumption was made that the fracture strength was associated with a limiting value $\psi_m = \psi(t_m)$, beyond which every element oriented in the direction of applied stress would break. Then at a specific time-to-break

$$\psi(t_m) = \frac{\sigma}{f(t_m)} \qquad (8)$$

Now returning to equation 6, for a large value of stress ψ_m, K_b can be shown to be very large compared with K_r. Therefore to a first approximation, equation 6 can be reduced to the form

$$f(t) = \frac{1}{8\pi}\exp\left\{-\int_0^t \omega_b \times \exp\left[-U/RT - \beta\psi(\tau)\right] d\tau\right\} \qquad (9)$$

From equations 7 and 9 one can write

$$\psi(t_m) = 8\pi\sigma \exp\left[\omega_b \exp\left\{-U/RT \times \left[\int_0^{t_m} \exp\left\{\beta\left[\psi(0) + \psi'(0)\tau + \ldots\right]\right\} d\tau\right]\right\}\right] \qquad (10)$$

Here t_m is the time-to-fracture for a constant applied stress σ. From this relationship it was found that $\psi(0) = 8\pi\sigma$ together with $\psi(t_m) = \psi_m$ gives

$$\log \frac{8\pi\sigma}{\psi_m} + \omega_0 \exp(-U/RT) \times \exp(8\pi\beta\sigma) \exp(\log t_m) = 0 \qquad (11)$$

Remember that this expression is only true for large values of σ. In this case it is likely that

$$\exp(8\pi\beta\sigma) \gg \log \frac{8\pi\sigma}{\psi_m} \qquad (12)$$

Then equation 11 can be approximated to show a linear relationship between the logarithm of time $\log t_m$ and the constant applied fracture stress σ

$$\sigma \approx \frac{1}{8\pi\beta}\left(\frac{U}{RT} - \log \omega_0 t_m\right) \qquad (13)$$

This equation can be put in the form

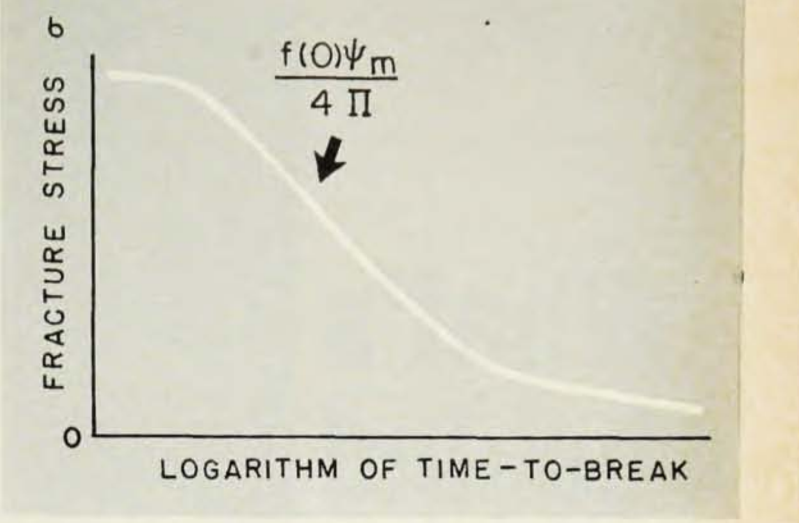
$$t_m = \frac{1}{\omega_0} \exp(U/RT - 8\pi\beta\sigma) \qquad (14)$$

which is similar to equation 1 given by Zhurkov. Assuming that the above general formulation and the approximations in the analysis are acceptable, then equation 14 cannot be said to represent a general kinetic process of fracture without qualifications. The linear relations in equation 14 can be obtained only if the range of values for σ is sufficiently large. When values of σ are small, the logarithmic term in equation 11 becomes important. By including this term, one can easily put equation 11 in the form

$$t_m = \frac{1}{\omega_0}\left(\log \frac{\psi_m}{8\pi\sigma}\right) \exp(U/RT - 8\pi\beta\sigma) \quad (15)$$

Comparing this with equation 1, it is seen that t_0 should be replaced by a function of the applied stress σ instead of being a constant as suggested by Zhurkov. In fact, even equation 15 cannot be regarded as precise when σ becomes very small. In that case K_r and K_0 will be comparable, and equation 6 must be consulted if a consistent kinetic process for fracture under all ranges of applied stress is to be maintained. A more general analytical result can be obtained by eliminating $f(t)$ from equations 5 and 8. The time-to-break can then be expressed as an integral of $\psi(t)$ and other quantities.

To illustrate the general result of this analysis, figure 3 shows schematically the variations of the fracture strength plotted against the logarithm of time. This theoretical curve, represented by equations 6, 7 and 8, appears to cover a complete range of stress-time relations and seems to fit fairly well with various published information. In addition, it can be used to explain the deviations from linear relationships between stress and time-to-break found experimentally by Zhurkov for different kinds of solids.[18,19]



STRESS-TIME RELATIONSHIPS. This curve shows the theoretical relationship between stress and the time required for fracture. The curve is represented by equations 6, 7 and 8 in the text and seems to agree with published information from various sources. —FIG. 3

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