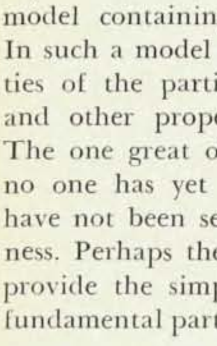
# QUARKWAYS to Particle Symmetry

*by Laurie M. Brown*

WHEN THE EDITORS OF PHYSICS TODAY asked me—or, better, challenged me—to write an article explaining unitary particle symmetries to physicists who are not fundamental-particle specialists, I hesitated. But on considering the physicist whose children may ask, "Daddy, what are fundamental particles made of?", I decided to write the article after all. That question—so innocent, so clear, so full of healthy curiosity—deserves to be answered, as far as possible, on the same level.

I have, accordingly, tried to follow the most direct pathway, which is the quark way, to reach my goal. The term "quark," introduced by Murray Gell-Mann, is used here to designate any model containing one or more unitary triplets. In such a model the symmetries and other properties of the particles follow from the symmetries and other properties of the constituent quarks. The one great objection to this approach is that no one has yet observed a quark. Perhaps they have not been searched for with sufficient seriousness. Perhaps they do not exist. Nevertheless they provide the simplest concrete way to understand fundamental particles.



This is Laurie Brown's third PHYSICS TODAY article on unitary symmetries (see "Musings in the Musey Room," April 1965, and "Tyger Hunting in the Everglades," April 1964). He is a physics professor at Northwestern, having gone there in 1950 after earning his Ph.D. under Richard Feynman at Cornell. Brown is interested in elementary particles and quantum electrodynamics.

It *does* make a great deal of sense to say that the atomic nucleus is made of nucleons: protons and neutrons. The lightest nucleus *is* a proton, the masses of the nuclear species are roughly integral multiples of the nucleon mass, the nuclear charges are integers and the nuclear shells are filled in the manner to be expected when two kinds of spin-½ particles are involved. It is perhaps of lesser importance that we can knock nucleons out of nuclei; in the electromagnetic pair-production process electron-positron pairs are "knocked" out of a high-energy photon, but a photon is not "made" of electron-positron pairs.

As it happens, the neutron and proton have almost the same mass, and the forces in nuclei are nearly charge-independent. A consequence is that the nuclear states are well classified in terms of quantum numbers that can be related (taking account of the Pauli principle) to the quantum numbers of nucleons. The procedure is to start from the symmetry limit, assuming equal proton and neutron mass and neglecting electromagnetic interaction, and then apply corrections for these symmetry-breaking effects.

Thus, assigning to the nucleon doublet a nucleon number $N$ of unity and an isospin $I$ of $\frac{1}{2}$ (with the proton assigned $I_3 = \frac{1}{2}$ and the neutron $I_3 = -\frac{1}{2}$), so that the charge $Q$ is $I_3 + N/2$, the additive *internal* quantum numbers of nuclear states are given by

$$A = \Sigma_i N^{(i)}$$
$$Z = \Sigma_i Q^{(i)}$$
$$\mathbf{I} = \Sigma_i \mathbf{I}^{(i)}$$

where $\Sigma_i$ runs over all the nucleons, and the allowed states of $I$, together with the *dynamical*

*Each time physicists think they've found the fundamental building blocks of matter, they discover that the blocks themselves have a structure. Now it seems that the blocks may be quarks—with three different kinds you can build all known mesons and baryons and fit them into symmetrical arrangements of 8 or 10 particles.*

quantum numbers of angular momentum and energy, are restricted by the Pauli principle.

This is all very familiar and scarcely needs repeating. But suppose we had never observed nucleons, but only nuclei of mass number two or greater. Would we have invented nucleons? Of course we would! And in terms of these quasi-nucleons, or spurions, or symmetry elements or whatever we might think of calling them, we would understand nuclear physics just as well, or just as poorly, as we do now with neutrons and protons. In our real world, since nuclear forces are relatively weak, it would, of course, hardly be possible to make nuclei interact without sometimes producing real protons and neutrons. (When we use higher-energy probes we also produce mesons, strange baryons and antibaryons; but it makes less sense to say nuclei are made of *them*.)

We summarize the reasons why it is commonly stated that nuclei are made of nucleons:

1. The additive internal quantum numbers of nuclei (and also the fact that nuclear spin comes in integers or half-integers) can be accounted for by the nucleon model. That is, nucleons form a *minimum set of symmetry elements* that can be combined to give the internal symmetries of nuclear states.

2. Nuclear binding energies are sufficiently weak, relative to the energy gap between the nucleon and its first excited state (nucleon plus pion), that there exist probes which excite nuclei or decompose nuclei into nucleons without exciting nucleons.

It is clear that by the same criteria we can also say that atoms are made of nuclei and electrons, molecules are made of atoms, and so on.

The real existence of atoms, nuclei and nucleons is certainly a question of first importance in physics, and it has been answered affirmatively. But the explanation of complexes formed of these objects in terms of their properties is, to a certain extent, a separate question. The explanation requires only that they make sense as quasi-objects.

### *What kind of model?*

In the search for the elementary building blocks of which the world is constructed, physicists have successively sub-divided matter—and they have found that at each level of subdivision there remains at least one part that has a considerable structure. For the moment the leptons appear to be simple (though not fully understood), but the nucleons, the many other baryons and the mesons —those objects that interact strongly and are collectively called hadrons—possess a complex structure. It is natural to try to relate these complex structures to each other through a model involving more elementary structures, even though we have no assurance that these more elementary structures have an independent existence. This is actually a conservative, traditional approach that has worked many times in the history of physics. It will be interesting to see whether it will work once more.

During the last two decades we have encountered about a hundred or so hadrons other than atomic nuclei. Each of them can be assigned a baryon number $B$, which is a generalization of the nucleon number $N$—and $B$ appears to be exactly conserved. It seems to differ from the exactly conserved charge number $Q$ in not being the source of any known field. The hadrons fall into charge or isospin multiplets, which are degenerate in mass when only strong interactions are considered. They possess another additive quantum number, which is conserved when only strong and electromagnetic interactions are involved; this quantum number is either the strangeness $S$ or the hypercharge $Y$ defined as $B + S$. Particles and antiparticles always have the negatives of each other's $Q$, $B$, $S$ and $Y$.

If the hadrons are made of some minimum set of more elementary structures, these structures must have at least the following properties:

1. at least one half-integral *spin* multiplet, the minimum acceptable one being a spin-½ doublet
2. at least one half-integral *isospin* multiplet, the minimum acceptable one being an isospin-½ doublet
3. at least one element having a rational baryon number
4. at least one element having a rational strangeness.

The first three requirements are satisfied by the nucleons themselves, which have spin ½, isospin ½ and baryon number 1. Accordingly the first composite model proposed for hadrons was nuclear physics!

#### *Fermi-Yang-Sakata model*

After the discovery in 1947 of the first hadron of baryon-number zero, the pion, it was natural (as well as brilliant) for Enrico Fermi and C. N. Yang to observe in 1949 that it had quantum numbers corresponding to a very strongly bound nucleon-antinucleon system in a state of zero spin, zero orbital angular momentum and unit isotopic spin, and that this model would also account for the pion's rather puzzling odd intrinsic parity.

Again, after the discovery of strange hadrons it was natural for Shoichi Sakata, recognizing requirement 4 (above), to introduce a third symmetry element having spin one-half, isospin zero, baryon number unity and, in addition, strangeness of minus one (so that its hypercharge $Y$ would be zero). He identified this element with the baryon $\Lambda$, forming, with the neutron and proton, the Sakata triplet (P, N, $\Lambda$). This triplet of elements has the right quantum numbers for constructing all the hadrons. For example, the pseudoscalar K meson doublet ($K^+$,$K^0$) is made by binding strongly the antiparticle of the $\Lambda$, namely $\overline{\Lambda}$, with the nucleon doublet (proton and neutron) in a state of zero spin and zero orbital angular momentum.

In the symmetry limit, where mass differences within the Sakata triplet are neglected, the charge independence of the strong forces can be generalized to the requirement that there be a symmetrical interaction among the three members of the Sakata triplet.

More precisely, if charge independence of the nuclear forces is formulated in the strong form of isospin invariance, this is equivalent to the introduction of a two-dimensional complex space in which the isospin-state vectors are represented. Charge independence means that the interaction is invariant under an arbitrary rigid rotation of the axes in this space. Charge *and* hypercharge independence is formulated in terms of a three-dimensional complex charge-hypercharge space, and it means that the interactions among P, N and $\Lambda$ are invariant under an arbitrary rigid rotation of the axes in this three-dimensional space. If this were so, and P, N and $\Lambda$ were degenerate in mass, then $\pi$ and K mesons, for example, would be degenerate in mass.

The symmetry group of charge independence is the group of all $2 \times 2$ unitary matrices having determinant one. This is called the special unitary group in two dimensions, or SU(2) (it is also the symmetry group of angular momentum). The symmetry group of charge-hypercharge independence is the special unitary group in three dimensions, or SU(3). Whenever all three members of the Sakata triplet are involved, as in the hypernuclei (which are nuclei containing bound $\Lambda$'s together with nucleons), the symmetry group for classifying states is SU(3). When only *two* of the three members are involved the symmetry is that of an SU(2) subgroup of SU(3). Evidently there are three possible SU(2) subgroups of the higher symmetry SU(3). They have been called $U$-spin, $V$-spin and $I$-spin (isospin) subgroups.

#### *Trouble with the Sakata model*

The symmetry underlying the Sakata model is that of SU(3), and the model works rather well in giving qualitative features of mesons, nuclei and hypernuclei; but it gives a rather clumsy picture of the important states of baryon number unity, the baryons. Let us carry the Sakata model a few steps further and see why this is so.

We represent any member of the Sakata triplet (P, N, $\Lambda$) by S, for Sakaton. Suppose, following Sakata, we identify this triplet with the physical proton, neutron and lambda particles. So far this is highly satisfactory since they *are* the lightest baryons we know of. Now note that a tightly bound compound of S and its antiparticle $\overline{S}$ in a state of zero spin and zero orbital angular momentum yields nine states of baryon number zero,

## Definitions

1. Baryon number $B$ and strangeness $S$ can not be defined except as numbers that can be associated with particles in such a way that conservation laws hold. **Baryon number** is always additively conserved, and **strangeness**, which may be violated in weak interactions, is additively conserved in strong and electromagnetic interactions. Mesons are hadrons of baryon number 0; other hadrons are assigned baryon numbers that are positive integers (particles) or negative integers (anti-particles). Strangeness was introduced by Gell-Mann and Nishijima in 1953 to describe the process of strong associated production, followed by weak decay, of certain hadrons (the "strange" particles). The Gell-Mann-Nishijima relation reads $Q = I_3 + \frac{1}{2}(B + S)$. The three components of isospin and the **hypercharge** $Y = B + S$ can be identified as four of the eight generators of the group SU(3), the other four being related to change of strangeness. Quarks may have fractional baryon number and fractional electric charge.

2. **Symmetry-breaking effects** *are those which, in the context of a given symmetry scheme, violate that symmetry.* For example, the charge independence of nuclear forces is broken by electromagnetic interactions, by the mass difference of neutron and proton (which may itself be an electromagnetic effect), and by the weak interactions.

3. Mesons of spin 0 and spin 1 are often designated by the space rotation and reflection properties of their fields in the rest frame. Mesons of spin-parity $J^P = 0^+, 0^-, 1^+, 1^-$ *are called, respectively,* ***scalar, pseudoscalar, axial vector*** and **vector** mesons.

total angular momentum zero and odd parity (since S and $\bar{\text{S}}$ have opposite intrinsic parity). These nine states are degenerate in the symmetry limit, and if we neglect all the symmetry-breaking effects except the outstandingly large one that exists in S —namely the relatively large mass difference between the isospin singlet Λ and the isospin doublet (P, N)—the ($\bar{\text{S}}$S) states, or linear combinations where necessary, group themselves into isospin multiplets corresponding to the pseudoscalar mesons $\pi$, K, $\eta$ and $X^0$. That is, they correspond to the known pseudoscalar mesons, all of which have masses less than 1 BeV. [In the language of the experts, the states of ($\bar{\text{S}}$S) form a nine-dimensional representation of the symmetry group SU(3) × SU(3). From the standpoint of SU(3), on the other hand, we are forming the product of representations $\bar{3}$ by 3 and obtaining thereby the self-conjugate SU(3) representations 1 and 8. The multiplication procedure is analogous to combining angular-momentum representations by using the "vector model".]

The isotopic singlet $X^0$ (the $\eta\pi\pi$ resonance at 960 MeV) is a singlet representation of SU(3), and the other mesons make up the well-known pseudoscalar octet. The most obvious symmetry-breaking device, that which we have used, is already sufficient to give the Gell-Mann–Okubo formula for the octet. This formula (if $\pi$, K and $\eta$ now represent masses) is

$$4\text{K} = 3\eta + \pi$$

This condition actually holds experimentally for the *squares* of the masses, but that is a question we shall not discuss. However, it must be pointed out that this simplest model of symmetry breaking (the "folk model", as it has been called) does not give anything like the correct mass for the $X^0$ meson. This suggests that in the Sakata model (and also in the quark model below, which is almost identical to it for the mesons) the singlet and octet must have different internal wave functions; this in turn implies that SU(3), rather than SU(3) × SU(3), is the relevant internal symmetry.

Turning now to the baryons, one can propose to make some baryons out of states ($\bar{\text{S}}$SS). A simple possibility is to combine the pseudoscalar mesons ($\bar{\text{S}}$S) with another S. For example, the $T = 1$, $Y = 0$ baryon Σ can be thought of as a pion and a lambda bound in a $P_{1/2}$ state (to get even parity). In fact, the $P_{1/2}$ bound states of $\pi$, K and $\eta$ with Λ produce the whole baryon octet including the *original* P, N and Λ. This is a puzzling and disturbing feature of the Sakata model. If this were all, it might have turned out to be the kind of puzzle that points the way to a new physical principle. However, since our successes so far depend on assuming an SU(3) invariant interaction, we must go on and require that the nucleons also form $P_{1/2}$ bound states with the members of the pseudoscalar meson octet—and this leads to many states that are simply not observed. Good examples of this deficiency are baryon states of positive strangeness, formed of K-mesons (not $\bar{\text{K}}$-mesons) and nucleons; these states do not exist.

### *The eightfold way*

An alternative scheme of SU(3) symmetry was therefore proposed independently by Gell-Mann and Yuval Ne'eman and is known as the "eightfold

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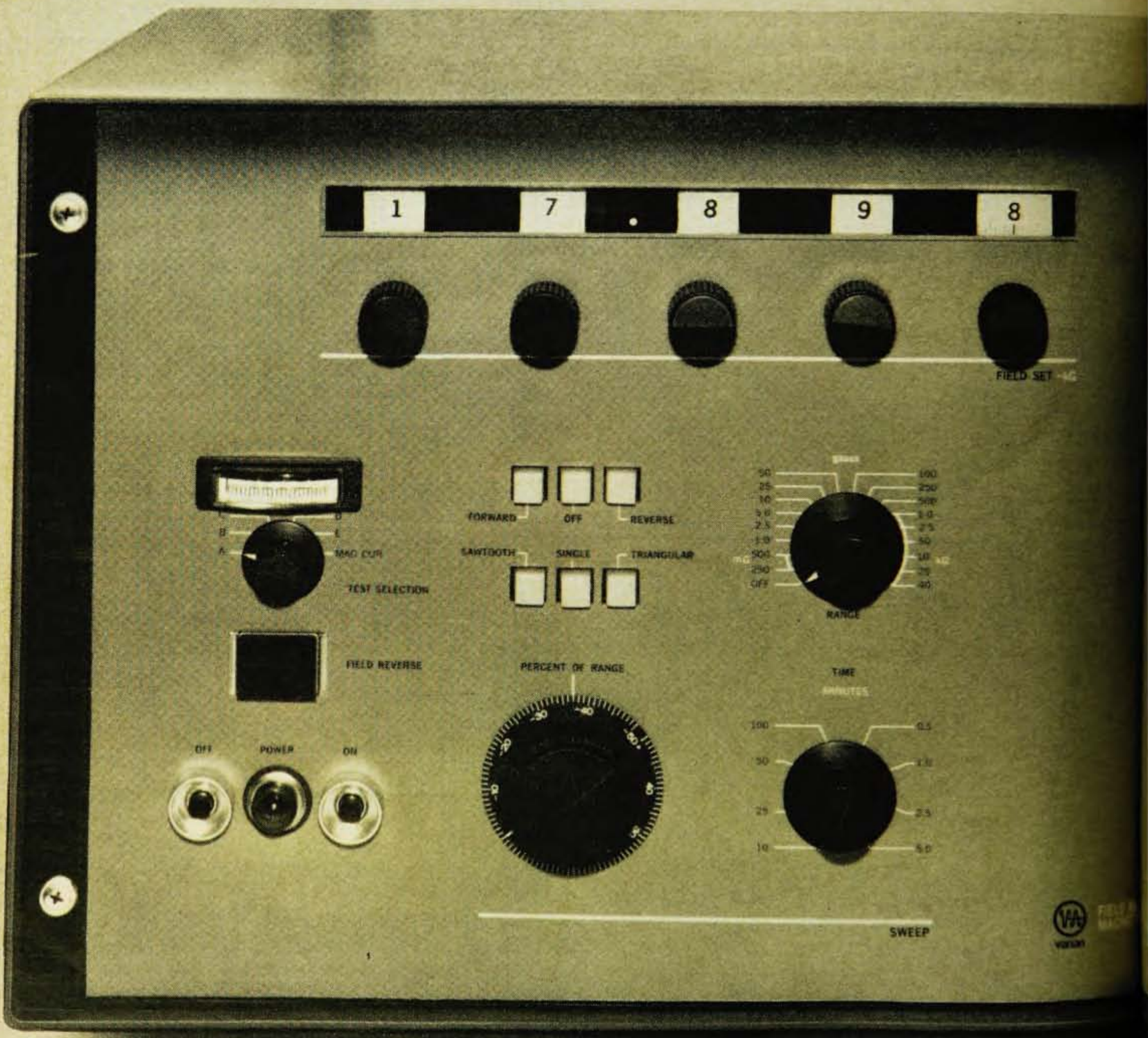

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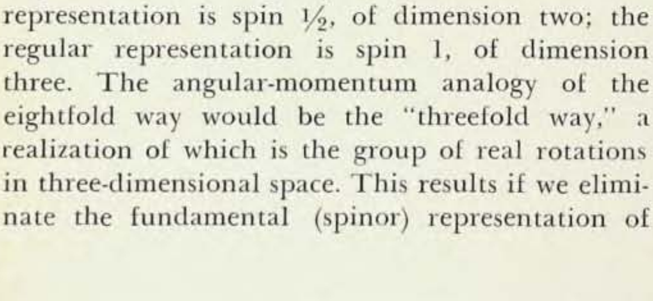
**Table 1. Quantum numbers for quarks**

| | $I$ | $I_3$ | $S$ | $B$ | $Y$ | $Q$ |
|---|---|---|---|---|---|---|
| p | 1/2 | 1/2 | 0 | 1/3 | 1/3 | 2/3 |
| n | 1/2 | −1/2 | 0 | 1/3 | 1/3 | −1/3 |
| λ | 0 | 0 | −1 | 1/3 | −2/3 | −1/3 |

way." It incorporates the successful features of the Sakata model while avoiding its difficulties. According to the eightfold way the elementary particles, in the limit of complete SU(3) symmetry, form representatives of the symmetry group SU(3) and remain identifiable when symmetry-breaking interactions are "turned on". However, the lowest nontrivial representation, which has dimension three (for example, the Sakaton), is *missing*, and the *fundamental* representation 3 is replaced by the *regular* representation 8. (The *regular* or *adjoint* representation of a Lie group like SU(n) is identified as that representation which has the same dimension and transformation properties as the generators of the group. If we consider the angular momentum group SU(2), the fundamental representation is spin ½, of dimension two; the regular representation is spin 1, of dimension three. The angular-momentum analogy of the eightfold way would be the "threefold way," a realization of which is the group of real rotations in three-dimensional space. This results if we eliminate the fundamental (spinor) representation of SU(2) and all other half-integral angular momenta.)




**SU(3) DIAGRAM FOR QUARKS. In the quark model three kinds of quarks (p, n and λ), each with charge of 1/3 or 2/3, can build all known mesons and baryons. —FIG. 1**

The well-established lower-lying states of both mesons and baryons have been successfully classified in terms of the eightfold way, and this has provided a wide range of insights and partial successes in understanding the dynamics of strong, weak and electromagnetic processes involving the hadrons. In terms of SU(3) one can concisely explain the failure of the Sakata model to give the baryon octet by the observation that the series of possible representations obtained by combining $\bar{S}$,S and S does *not* contain the representation 8.

***The quark model***

The procedure of dropping the fundamental representation in favor of the regular representation or, to put it another way, of dropping the spinor representations and retaining only the tensor representations, is sometimes called "dividing the group by its center." In doing so one is, in a sense, devouring the hand that feeds one. Throwing away the fundamental triplet used in compounding the dish has also been compared to certain procedures used in French cooking.

It is worth noting that Gell-Mann, in his paper introducing the eightfold way, made use of triplets (two pairs of them, one fermion and one boson) to generate mesons and baryons, if only as a pedagogical device. And this idea has by no means expired with the Sakata model, but continues to appear in the literature in a variety of forms; indeed, many recent developments in the theory of fundamental particles and their interactions find their most natural expression in terms of one or more sets of fundamental triplets. While it is true that no one has up to now claimed to have observed a member of such a triplet, it is such an elegant way of understanding and remembering the physical content of these theories that the remainder of this article is written as though such triplets existed.

The simplest basis for a triplet model is provided by the *quarks* (so named by Gell-Mann), the quantum numbers of which are given in table 1. (To obtain the quantum numbers of antiquarks, reverse all signs in the table.) One way of representing quarks is shown in figure 1.

The quark isospin doublet (p, n) and singlet λ have been assigned names that remind us of the Sakata triplet. However, note the one-third integral baryon number $B$, which gives the one-third integral character to the hypercharge $Y$ and the electric charge $Q$. With weak interactions turned off, so that $Y$ is conserved, and in the SU(3)

**THE MESONS** in the quark model are states of bound quarks q and antiquarks $\bar{q}$. The bound states are denoted by $(\bar{q}q)$, in which $q = (p,n,\lambda)$ and $\bar{q} = (\bar{p},\bar{n},\bar{\lambda})$. In the table the notation $\bar{p}p$, for example, indicates the product of $\bar{p}$-wave and p-wave functions. The lowest-lying states have zero angular momentum, and because q and $\bar{q}$ have opposite intrinsic parity they have $J^P = 0^-$ for quark spin $s = 0$ (pseudoscalar mesons) and $J^P = 1^-$ for quark spin $s = 1$ (vector mesons). The actual masses increase from the top to the bottom of the table.

**Table 2. Mesons From Quarks ($B = 0$)**

| $Y$ | $I$ | Quark wave function | $J^P = 0-$ mesons | $J^P = 1-$ mesons |
|---|---|---|---|---|
| 0 | 1 | $\bar{p}n,\ (\bar{p}p - \bar{n}n)/\sqrt{2},\ \bar{n}p$ | $\pi^-, \pi^0, \pi^+$ | $\rho^-, \rho^0, \rho^+$ |
| 1 | 1/2 | $\bar{\lambda}n,\ \bar{\lambda}p$ | $K^0, K+$ | $K^{*0}, K^{*+}$ |
| −1 | 1/2 | $\bar{p}\lambda,\ \bar{n}\lambda$ | $K-, \bar{K}^0$ | $K^{*-}, \bar{K}^{*0}$ |
| 0 | 0 | $(\bar{p}p + \bar{n}n - 2\bar{\lambda}\lambda)/\sqrt{6}$ | $\eta^0$ | — |
| 0 | 0 | $(\bar{p}p + \bar{n}n + \bar{\lambda}\lambda/\sqrt{3}$ | $X^0$ | — |
| 0 | 0 | $(\bar{p}p + \bar{n}n)/\sqrt{2}$ | — | $\omega^0$ |
| 0 | 0 | $\bar{\lambda}\lambda$ | — | $\phi^0$ |

**Table 3. Baryons From Quarks ($B = 1$)**

| $Y$ | $I$ | Quark wave function | $J^P = (1/2)+$ baryons |
|---|---|---|---|
| 1 | 1/2 | nnp, ppn | N, P |
| 0 | 0 | $\lambda\ (np - pn)/\sqrt{2}$ | $\Lambda$ |
| 0 | 1 | $\lambda nn,\ \lambda(np + pn)/\sqrt{2},\ \lambda pp$ | $\Sigma^-, \Sigma^0, \Sigma^+$ |
| −1 | 1/2 | $\lambda\lambda n,\ \lambda\lambda p$ | $\Xi^-, \Xi^0$ |

| $Y$ | $I$ | Quark wave function | $J^P = (3/2)+$ baryons |
|---|---|---|---|
| 1 | 3/2 | nnn, nnp, npp, ppp | $N^{*-}, N^{*0}, N^{*+}, N^{*++}$ |
| 0 | 1 | $\lambda nn,\ \lambda np,\ \lambda pp$ | $\Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}$ |
| −1 | 1/2 | $\lambda\lambda n,\ \lambda\lambda p$ | $\Xi^{*-}, \Xi^{*0}$ |
| −2 | 0 | $\lambda\lambda\lambda$ | $\Omega^-$ |

**THE BARYONS** in the quark model are states of three bound quarks (qqq), and the antibaryons (not shown) are states of three bound antiquarks $(\bar{q}\bar{q}\bar{q})$. In the table the notation nnp, for example, indicates the product of two n-wave functions and one p-wave function. The lowest-lying baryon states have zero orbital angular momentum and therefore positive parity. The ten states of quark spin $s = 3/2$ have symmetric spin structure and symmetric quark structure (the symmetry is denoted by three aligned boxes). The eight states of quark spin $s = 1/2$ have mixed spin structure and mixed quark structure (denoted by three boxes not aligned). Unlike the examples given, the $1/2^+$ baryons do not have wave functions that can, in general, be written as the product of a quark-wave function and a spin-wave function. The actual masses increase from the top to the bottom of the table.

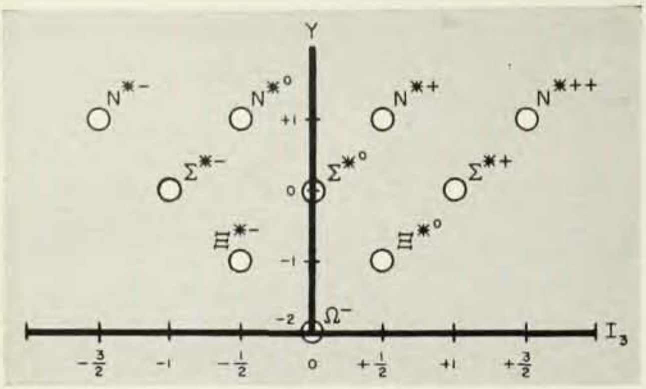
symmetric limit, where p, n and $\lambda$ are degenerate in mass, $\lambda$ is "strange" from the standpoint of $Y$; but notice that p is equally "strange" from the standpoint of $Q$. Indeed, a third conserved charge $Q'$ can be defined as $Q' = U_3 + Y/2$ where p is a $U$-spin singlet, and $(n, \lambda)$ forms a $U$-spin doublet having $U_3 = 1/2$ and $-1/2$ respectively. Then for n, $\lambda$, p we get $Q' = 2/3$, $-1/3$, $-1/3$, so that from the standpoint of $Q'$ it is n that is "strange."

### *Quark recipes*

By combining quarks with antiquarks in a spin-singlet S state one obtains the pseudoscalar meson octet and singlet, just as in the Sakata case (see table 2). The charges come out right because $Q$ and $Y$ of the quarks are simply displaced by $1/3$ unit from $Q$ and $Y$ of the Sakaton; and since $Q$ and $Y$ for the antiquarks are displaced in opposite directions from $Q$ and $Y$ of the quarks, the total displacement is canceled in the compound.

The prescription for compounding baryons now differs from that for the Sakata case, since to get baryon-number unity we must combine three quarks, and not, as in the Sakata case, two Sakatons and an *anti*Sakaton (see table 3). [Group theorists are pleased to note that $3 \times 3 \times 3$ *does* contain an 8; in fact the complete decomposition is $1 + 8 + 8 + 10$.] In the SU(3) symmetric limit one would expect the states of a given space and spin structure to be degenerate. Choosing as an example the most symmetric case, namely three $\lambda$'s in relative S states having all spins parallel, we get the quantum numbers $I = 0$, $S = -3$ and $Q = -1$, and spin 3/2. This is the famous $\Omega^-$! The $\Omega^-$, with its complete spin and isospin symmetry, is representative of a class of ten members, the SU(3) baryon decuplet (or decimet), or representation 10, which is depicted in figure 2.

Since the baryon decuplet consists of ten particles of spin and parity $3/2+$ whose existence is well-established, we can mark this down as another triumph for the eightfold way. But what about the baryon octet, which proved so damaging to the Sakata model? To help understand how this fits in, let us examine the isospin structure




**SU(3) BARYON DECUPLET for baryons with spin and parity of 3/2⁺. Tenth baryon, at bottom of triangle, is $\Omega^-$, predicted by SU(3) theory. —FIG. 2**

of the decuplet. The $\Omega^-$, having strangeness $-3$, is made of three $\lambda$ quarks and thus has $I = 0$. The $\Xi^*$, of strangeness $-2$, has two $\lambda$ quarks and one or the other member of the (p, n) doublet, and thus has $I = \frac{1}{2}$. The $\Sigma^*$ has two doublet members and might have $I = 0$ or 1, but we unhesitatingly choose $I = 1$ for the following reason: we are forming an irreducible representation of SU(3), which we know must have a definite symmetry under the exchange of quarks. Since all orbital and spin states are identical for the decuplet the isospin states must all have the same symmetry, and since the $\Omega^-$ is obviously isospin symmetric we must choose the symmetric isospin state for $\Sigma^*$, and this is $I = 1$. Similarly the $N^*$, which contains three doublet members, must be in the symmetric isospin state—that is, $I = 3/2$.

The reader may have noticed that we have been somewhat vague as to what the overall symmetry of the three-quark state is. If the quarks are ordinary fermions (and not, for example, parafermions) the overall symmetry must, of course, be antisymmetric. Since we have constructed a symmetric spin-isospin wave function, the burden of antisymmetry falls on the space-wave function, which we have specified to contain only S waves. This is awkward, but not impossible: consider, for example, three orthogonal one-particle S-wave functions and form their antisymmetrized product. While not impossible, it *is* hard to understand why this should be a particularly low-lying state. There is still less reason to assume that this same peculiar space-wave function (whatever it is) is associated with other SU(3) representations. Supposing that it is, however, what other symmetric spin-isospin wave functions can be formed to multiply this space-wave function?

Although a completely antisymmetric isospin state is otherwise possible, it would have to accompany a completely antisymmetric spin state; but the antisymmetric spin state is impossible since there are only two independent spin-$\frac{1}{2}$ wave functions. But there do exist states of mixed spin symmetry (antisymmetric in one pair only) and states of mixed isospin symmetry, such that an overall symmetric product state can be formed. Obviously there is no three-$\lambda$ state of this character; the two-$\lambda$ state has $I = \frac{1}{2}$, the one-$\lambda$ states have $I = 0$ and 1 and the no-$\lambda$ state has $I = \frac{1}{2}$, for it has the same structure as the mixed-symmetry spin state (which is also formed of three spins of $\frac{1}{2}$) and this is spin $\frac{1}{2}$. The representation we have formed is the $J^P = \frac{1}{2}^+$ baryon octet, which is diagramed in figure 3.

Because we have so far restricted ourselves to having only S waves, thus avoiding the problem of spin-orbit interactions, we have been able to treat the spin effectively as an internal SU(2) symmetry group and have been constructing certain representations of the group SU(2) × SU(3). We have, in fact, exhausted the possible three-quark representations of this group that can be associated with a given antisymmetric space-wave function containing only S waves.

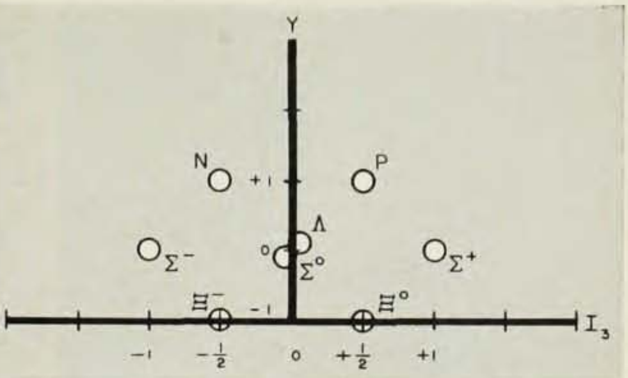
### *Higher symmetry groups*

A more far-reaching generalization was proposed by Feza Gürsey and Luigi Radicati, and by Bunji Sakita, over a year ago. They considered the allowed hadron states as representations (in the symmetry limit) of the group SU(6). This generalization has a quark interpretation in which there are six equivalent quarks, spin-up and spin-down versions of p, n and $\lambda$ being considered separately. This interpretation amounts to assuming spin and isospin independence of the strong forces. The four angular-momentum substates of a $J = 3/2$ baryon like $\Omega^-$, for example, are considered as four of the members of a larger SU(6) multiplet. The whole baryon decuplet of 40 states and the baryon octet of 16 states then combine to form one representation of SU(6), namely the 56-state representation.

An attractive feature of this proposal is that it provides a way to relate to each other the static properties of the baryon octet and decuplet, including their magnetic moments and their mass shifts under symmetry breaking. And by assuming SU(6) invariant interactions between various SU(6) multiplets, it could provide a complete dynamical basis for strong interactions. Most important, it does all this without explicitly invoking quarks, which, in spite of their pedagogical advantages, may not exist.

Relativistic versions of this theory, which can be viewed, again pedagogically, as based on quarks described by four-component Dirac wave functions, rather than on two-component non-relativistic Pauli quarks, have been proposed by a number of authors after the initial successes of SU(6). These versions go by the names $\tilde{U}(12)$, M(12), and SL(6,$c$). The details of these theories, which are still somewhat obscure, need not concern the nonspecialist reader at the present time.

Returning now to the mesons, recall that the pseudoscalar octet and the singlet $X^0$ were viewed in the SU(3) quark model as the nine states formed by putting quark and antiquark in spin-singlet S states. Following the procedure used for the baryons, we form also the nine spin-triplet S states [which make 27 states according to the SU(6) classification]. These nine spin-one mesons form the 1- and 8-vector meson representations of SU(3) and, at the same time, form with the eight states of the pseudoscalar meson octet the 35-dimensional regular representation of SU(6). [The regular representation of SU($n$) always has dimension $n^2-1$. It has been remarked that if the mesons form the regular representation of a group, this guarantees that Yukawa interactions with the baryons can be formulated that are invariant under the group, since the product of a representation by its conjugate representation always contains the regular representation.] Including the $X^0(960)$, which remains an SU(6) singlet, there are 36 meson states that might be regarded as associated (in the symmetry limit) with the lowest quark-antiquark S state of the quark model, if the mass splittings are associated with symmetry-breaking effects. On the other hand, the $X^0$ might have a different S-wave function.




**SU(3) BARYON OCTET.** This diagram represents baryons whose spin and parity are $1/2^+$. —FIG. 3

### *Summing up symmetries*

This very brief review has, it is hoped, made certain points that are relevant to understanding the unitary symmetries of elementary particles, but its shortcomings will be readily apparent to anyone at all familiar with the field. For one thing it has left out most of the interesting physics. While a list of what has been omitted would be longer than this article, a few of the major omissions are these:

1. It is believed by many that the most beautiful feature of the unitary-symmetry theories is not the predictions of the dimensions of representations, but the elegant way they allow the symmetry to be broken by medium strong, electromagnetic and weak symmetry-breaking interactions, giving mass spectra and selection rules for transitions. This, of course, is the heart of the physics.

2. The questions of the possible real existence of quarks or other triplets, what other properties they may possess and how they can be searched for have not been discussed.

3. The assignment of higher baryon and meson resonances, other than the SU(6) representations 1, 35 and 56, has not been touched upon, partly because the experimental situation is not yet clear.

Nothing of the foregoing has been original with the author, except for possible errors that may have crept in. The hope is that the article will provide for the nonspecialist some framework for appreciating new discoveries that will be made. As an example of what we might perhaps expect, a trivial extension of the above ideas leads to the following speculation:

Putting quark and antiquark in a P state, one predicts SU(3) octets and singlets, one each for $J^P = 0^+$, $1^+$ and $2^+$, obtained by adding to the unit orbital angular momentum a quark spin of unity. The $I = 0$ states of these multiplets (by analogy with the corresponding states of positronium) should be even under charge conjugation and should decay predominantly into $\pi^+ + \pi^-$. In addition there should be an octet (and perhaps a singlet) of quark-spin zero, hence with $J^P = 1^+$, which (by the same analogy) should have an $I = 0$ state that is odd under charge conjugation and therefore forbidden to decay into two pions. The developing pattern of higher meson resonances appears to the author to have a good chance to fulfill these predictions.