



nuclei of low to medium mass

by Gale I. Harris and Paul Goldhammer

Among phenomena exhibited by nuclei whose masses range from 16 to 56 are pure single-particle isobaric analog states and multipolarity mixings in radiative transitions between excited states.

Gale I. Harris has been at ARL for four and one-half years. He is doing experimental nuclear structure physics, in which he received his doctorate at the University of Kansas in 1962.



Paul Goldhammer received his PhD from Washington University in 1956. He is currently professor at the University of Kansas, where his primary research interest is in theory of nuclear structure.



RECENT PROGRESS in the physics of nuclear structure is marked by a rapid increase in the amount of new data on excited states of low-to-medium-mass nuclei—those nuclei with masses roughly in the range $16 \leq A \leq 56$. Concurrent with this worldwide experimental activity a significant increase in theoretical effort is perceptible. The attraction to this area of nuclear-structure physics can possibly be related to the impressive array of physical phenomena that these nuclei exhibit. Very recent phenomenological developments include, for example, the frequent observation of unusual mixtures such as that of magnetic-quadrupole and electric-dipole multipolarities in radiative transitions between excited states and the discovery of very pure single-particle isobaric analog states at high excitation energies. In addition, electric-octupole transitions in direct competition with electric-dipole and magnetic-quadrupole transitions are becoming a persistent feature of these nuclei.

A group of 50 physicists from seven countries met to exchange ideas and hear about some recent experimental and theoretical developments in April at a symposium on the structure of low-to-medium-mass nuclei that convened at the Aerospace Research Laboratories (ARL), Wright-Patterson Air Force Base, Ohio.

What is "low-to-medium"?

We have, as the reader suspects, no precise answer to the question of which nuclei belong in the low-to-medium-

mass range. However, there appear to be some reasons of convenience, from both experimental and theoretical points of view, to apply such a designation to nuclei in the periodic table lying roughly between ^{16}O and ^{56}Ni . They are, for example, usually accessible to study by small accelerators and proton- and alpha-capture reactions. The resonance states (virtual energy levels) formed in such reactions are narrow with respect to their separations and thus can be studied in great detail. For nuclei heavier than ^{56}Ni the level density tends to become too large for detailed study by resonance reactions, and the Coulomb barrier becomes too high for convenient study by small (2–3-MeV) accelerators. In nuclei lighter than ^{16}O the levels tend to be broad with respect to their separations.

From a theoretical standpoint these nuclei provide a nice region of confluence between the ideas of the adherents, on the one hand, of the nuclear shell model and, on the other hand, of the collective model. For convenience, by borrowing from the language of the nuclear shell model, this mass range constitutes what are commonly referred to as "2s1d- and 1f2p-shell nuclei." The 2s1d-shell nuclei, for example, are those nuclei between ^{16}O and ^{40}Ca that, in the ground state, consist of a closed 1s1p-shell core (^{16}O) plus extra nucleons in the $2s_{1/2}$, $1d_{3/2}$ and $1d_{5/2}$ orbits. The nucleus ^{16}O lies at the beginning of the 2s1d shell that fills at ^{40}Ca with magic number 20 for both protons and neutrons.

Those nuclei between ^{40}Ca and ^{56}Ni represent roughly the filling of the $1f_{7/2}$ "shell." Much of the recent theoretical work with the shell model has been devoted to the $2s_{1/2}$ - and $1f$ -shell nuclei. Both rotational and vibrational structure have been identified in the excited level systems of nuclei with $16 \leq A \leq 40$. In many cases, however, collective nuclear excitations appear to become much less pronounced and much greater sophistication in model theory is evidently required even to reproduce low-lying level schemes before explanation of more detailed properties such as electromagnetic decay rates, beta decay, and static dipole or quadrupole moments can become available. The nucleus ^{28}Si , for example, which closes the $1d_{5/2}$ subshell and thus might be expected to exhibit some vibrational structure, presents only poorly defined collective effects in its excited-state structure. An exception to this statement may be provided by the recent discovery by Rolf Nordhagen (Oslo) of an apparent collective octupole state with total angular momentum $J = 3$, odd parity and 6.88-MeV excitation energy that decays by electric-octupole radiation to the spin-zero, even-parity (0^+) ground state. This transition directly competes with an electric-dipole transition to the 2^+ first-excited state of ^{28}Si !

In the mass region between ^{28}Si and ^{40}Ca a most noticeable accumulation of new data on excited states has occurred during the past couple of years. These nuclei represent the filling of the $2s_{1/2}$ and $1d_{3/2}$ subshells. A notable development has been identification of many odd-parity excited states in these nuclei that apparently correspond to the promotion of nucleons to $1f_{7/2}$ orbitals. Many of these levels seem to be reasonably good single-particle states and provide us with some of the nicest examples of isobaric analogs of low-lying states in neighboring nuclei.

Spectroscopy and angular correlation

Remarkable improvements in the efficiency of obtaining and interpreting data from gamma-ray spectroscopic and angular-correlation measurements have occurred during the past two or three years. Among these are the advent of the lithium-drifted germanium

detectors of sizes sufficient for study of capture gamma rays typically having energies of several MeV and the availability of large memory multi-channel and multiparameter pulse-height analyzers for data accumulation. An interesting capture spectrum, which contains gamma rays of energy up to 6.71 MeV, obtained with a 1.5 cm^3 Ge (Li) detector from the $^{37}\text{Cl} (p, \gamma) ^{38}\text{Ar}$ reaction by Pieter M. Endt (Utrecht) is shown in figure 1. That such a small detector could be used so successfully for high-energy capture gamma rays is noteworthy. The full impact of these detectors on nuclear-structure physics undoubtedly remains to be seen.

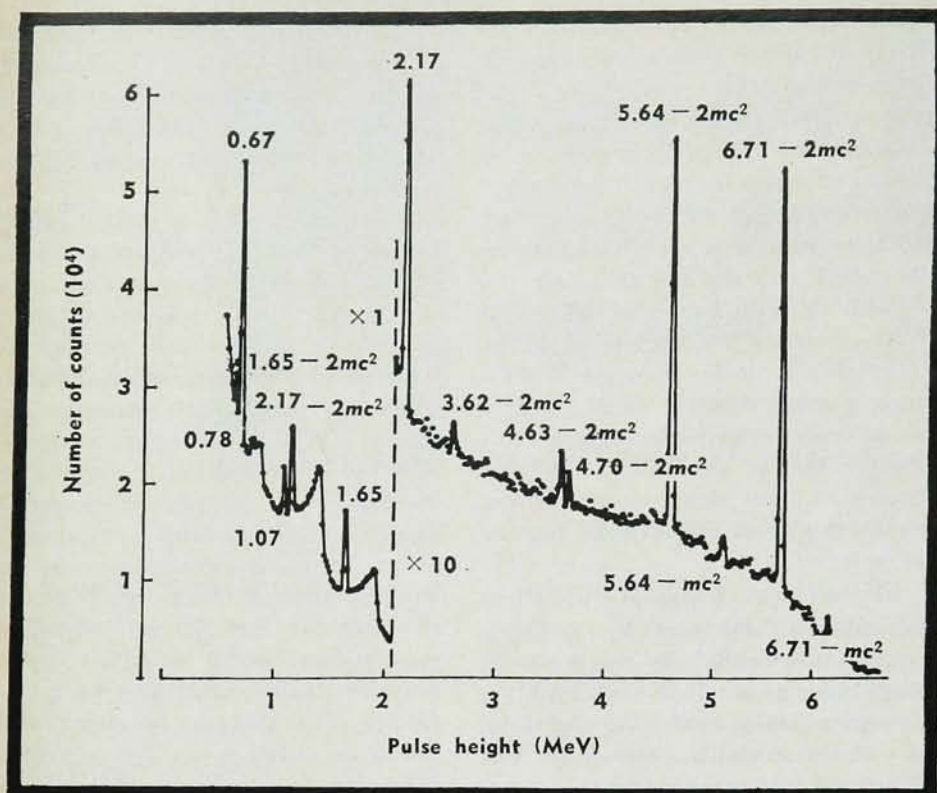
Measurement of angular correlations in radiative and particle transitions from states excited by reactions has long been a standard technique for determination of level spins and ratios of transition-matrix elements. The power of this method recently has been enhanced considerably by systematic use of more rigorous statistical methods in data analysis, extension of the general triple-correlation formalism for practical use in multiple-step cascades and use of reaction geometries that preserve axial symmetry. The advantages to be gained in nuclear spectroscopy by use of axially-symmetric reaction processes were discussed at the ARL symposium along with a survey of recent measurements in low-to-medium-mass nuclei by Frank W. Prosser (University of Kansas). The general type of correlation referred to is the "triple correlation" among the three directions specified by the bombarding beam and two outgoing radiations, one or both of which can be gamma rays. The conditions that provide axial symmetry are (a) bombardment of an unpolarized target with an unpolarized beam, (b) sharp spin and parity of the gamma emitting state a_0 and (c) detection of outgoing particles at 0 or 180 deg relative to the beam direction or nonobservation of these particles. Under these conditions angular correlations can be expressed in terms of alignment parameters of the state a_0 . (See figure 2.) These parameters are treated as unknowns to be determined from the data. By this means a knowledge of details of the state a_0 formation mech-

anism is not necessary for determination of spins and multipolarity mixings. Prosser pointed out that although these methods have been known for some time, remarkably few papers indicate their use.

Unfortunately the directional-correlation method, as is well known, is not sensitive to parity. Despite this limitation, many parity "assignments" have been made on the basis of observed multipolarity mixings. For example a commonly employed assumption is that if a dipole-quadrupole mixing is observed in a transition, interfering radiations must be of the magnetic-dipole, electric-quadrupole type and, hence, no parity change between the levels is involved. Many exceptions to this rule are now known in low-to-medium-mass nuclei on which parity-sensitive measurements have been performed. That this may be true in self-conjugate nuclei is not too surprising since electric-dipole transitions between states of the same isobaric spin in such nuclei are expected to be strongly inhibited. Thus magnetic-quadrupole transitions may compete favorably with electric-dipole transitions. However, small but observable electric-dipole, magnetic-quadrupole admixtures have also been reported in odd-even nuclei such as ^{31}P . The result of all this is a greater suspicion of many existing parity assignments and a decidedly greater hesitation in assigning parities without the aid of polarization, elastic scattering or stripping measurements. Gamma-ray linear-polarization measurements employing, for example, the Compton scattering process are for these reasons coming into greater use as a routine method.

Electron scattering

Another general method for nuclear-structure study that, in particular, also yields information on transition character and thus energy-level parities, is high-energy electron scattering. This method and its application to the study of ^{27}Al , ^{31}P , and ^{32}S were reviewed at the symposium by George Bishop (University of Glasgow). When the condition $Z < 137$ is satisfied, the scattering cross section can be computed in the first Born approximation. The predicted cross sections have been found for many cases of



GAMMA-RAY SPECTRUM from $E_p = 1092$ -keV resonance in $^{37}\text{Cl}(p, \gamma)^{38}\text{Ar}$ reaction obtained with a 1.5-cm³ Ge (Li) detector. Figure illustrates that it is possible to study high-energy capture gamma rays with solid-state detectors that are small even in comparison with some presently available. The desirability of multichannel analyzers with large numbers of channels is also evident in this type of work. (Figure from F. Ern , W. Veltman, J. Wintermans, submitted to *Nuclear Physics*. —FIG. 1

elastic and inelastic scattering in light nuclei.

The first Born approximation corresponds physically to transfer of a single virtual photon from the electron to the scattering nuclear system. With minor modifications, this approximation has recently been extended to 2s1d-shell nuclei. The modifications consist of abandoning plane-wave descriptions for incident and outgoing electrons. The major effects of wave distortion can be handled in practice by increasing the wave number of the electron since it is accelerated on approach to the nuclear charge, and decreasing that of positrons, which are decelerated.

The electron scattering cross section in first Born approximation can be expressed as

$$\frac{d\sigma}{d\Omega} = \sigma_M [F_L^2(q) + \left(\frac{1}{2} + \tan^2 \frac{\theta}{2}\right) F_T^2(q)]$$

(σ_M is the Mott scattering for electrons from a point charge Z ; q is the transferred momentum; F_L and F_T are

longitudinal and transverse form factors that describe the effect of the finite extent of nuclear charge and magnetization densities). The two form factors correspond to virtual photons that have field components parallel or perpendicular to the momentum transfer vector, respectively. They are experimentally distinguishable by their different angular dependence. By means of multipole decomposition of the form factors, a comparison can be made with theoretical form factors computed from assumed nuclear wave functions with multipole operator expansions. The longitudinal form factor decomposes into a sum over electric multipoles, beginning with monopole, and is subject to usual selection rules on spin and parity change in real-photon electric transitions. These transitions are induced by the instantaneous Coulomb interaction between the electron charge and the nuclear charge distribution. On the other hand, the transverse form factor decomposes into a sum over both magnetic and electric multipoles, begin-

ning with dipole, subject to the usual real-photon selection rules. Transitions corresponding to the transverse form factor are due to the retarded interaction between the current of the scattered electron and the current and magnetization distributions of the nucleus.

In many 2s1d-shell nuclei to which this method has been applied, the dominant mode of excitation of the nucleus is through Coulomb interaction. Excitations are found to be mainly collective electric transitions considerably enhanced in comparison to single-particle transition probabilities. In light nuclei, data taken over a large range of momentum transfers can be used to determine detailed radial dependence of the transition charge distribution and thus to verify configurations in detail. For low-to-medium-mass nuclei such information is at present harder to extract because of problems of wave distortion and radiative corrections that must be applied to account for bremsstrahlung emission by the scattered electron.

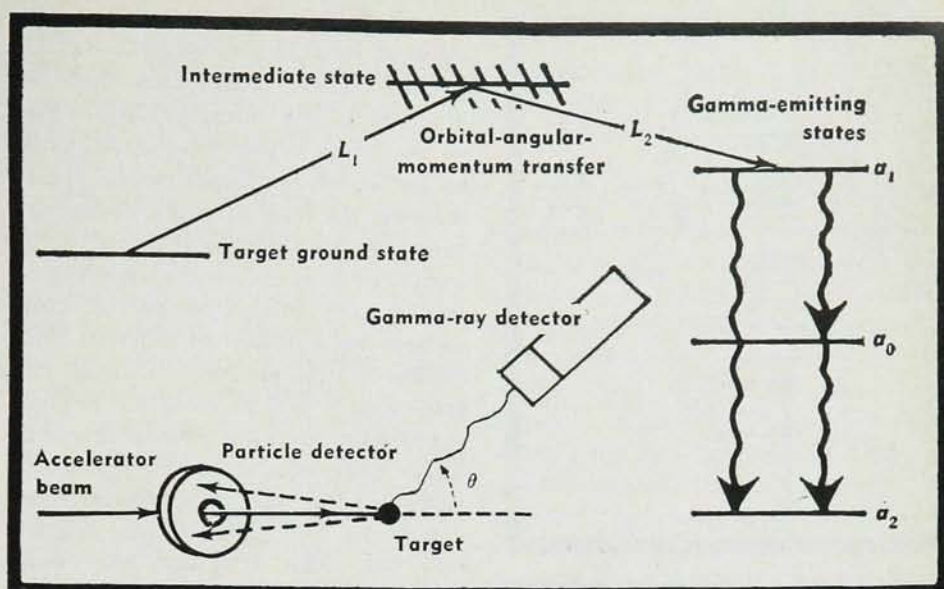
Isobaric analog resonances

Interest in the role of isobaric spin in nuclear-structure physics has recently increased considerably as a result of experimental discovery of well defined resonances corresponding to isobaric analog states in medium and heavy nuclei. Such states belong to the set of energy levels in isobaric nuclei that have identical quantum numbers, including isobaric spin (T), but differing in T_z (the z component of T) and Coulomb displacement energies. It is remarkable that one finds, when the total Coulomb energy is large relative to the (charge-independent) nucleon-nucleon interaction, well defined states in which isobaric spin clearly plays a major role. The presence of isobaric analog states in light and low-to-medium-mass nuclei is well known and not surprising because of the lower Coulomb energies. Their discovery in heavier nuclei, however, has generated great interest in the possibility that analogs of single-particle, low-lying states may systematically manifest themselves as strong single-particle resonance states at high excitation energies in neighboring isobaric nuclei. For example, such states

should appear as strong, rather broad resonances in proton-capture and elastic-scattering reactions in low-to-medium-mass nuclei. Systematic study of such resonances could then become a valuable spectroscopic tool for determination of properties of low-lying states in the isobaric nucleus provided reliable identification of the members of the isobaric-spin multiplet can be made. This method may be particularly valuable when states of the isobaric nucleus are inaccessible to study with usual reactions.

Characteristics of an isobaric analog resonance in a proton-capture reaction that distinguish it from others excited by the same reaction were discussed at the symposium by Endt. Let us consider, in a nucleus of ground-state isobaric spin T , the group of resonances with isobaric spin $T + 1$ that are analogs of low-lying bound states in the isobaric nucleus having reasonable single-particle character. (See figure 3.) We find three characteristic features by which they can be recognized: (a) a large reduced proton width, (b) a large gamma-ray width and, (c) in favorable cases, a simple gamma decay to a single lower state. Each of these features is due to the single-particle character of the resonance. The third feature is most interesting since the resonance decays to a lower state of the same configuration but with different isobaric spin. Examples of resonances that meet these conditions have been found in ^{27}Al , ^{30}P , ^{31}P , ^{35}Cl and ^{38}Ar in which resonances correspond to promotion of a nucleon to a $1f_{7/2}$ orbital with isobaric spin $T + 1$. The best examples are the $(f_{7/2}, T = 3/2) \rightarrow (f_{7/2}, T = 1/2)$ transitions in ^{27}Al , ^{31}P and ^{35}Cl studied at Utrecht and ARL. In each case the reduced proton width (the observed proton width corrected for barrier penetration effects) is nearly as large as the pure single-particle value; the magnetic-dipole gamma-ray transitions in question are all much stronger than the average for low-to-medium-mass nuclei, and the latter two cases are quite unusual in that each resonance decays entirely to a single lower-lying state having the same spin and parity ($7/2^-$).

It is interesting at this point to compare some specific results of this



PARTICLE-GAMMA reaction and experimental setup for an axially symmetric reaction process. —FIG. 2

work with some results of inelastic electron-scattering studies presented at the symposium by Bishop. In ^{31}P , the lower lying $7/2^-$ state to which the $f_{7/2}, T = 3/2$ resonance decays lies at 4.43-MeV excitation energy. The reduced proton width for formation of the resonance is about 60% of that expected for a pure single-particle state: the magnetic-dipole transition strength is close to that expected for a pure $(f_{7/2}, T = 3/2) \rightarrow (f_{7/2}, T = 1/2)$ transition, and the probable analog state in ^{31}Si of the resonance has a large single-particle $f_{7/2}$ component. Thus all the conditions referred to above are reasonably well met. The lower state at 4.43 MeV therefore should be largely a pure $f_{7/2}, T = 1/2$ level. A quite different interpretation of the character of the 4.43-MeV level was presented, however, as a result of the electron scattering work. In this study the 4.43-MeV level was excited by an enhanced electric-octupole excitation from the $1/2^+$ ground state of ^{31}P , and this method of excitation leads one to suspect considerable collective octupole character of the state. These data were interpreted in terms of an ^{32}S core-excitation model in which the 4.43-MeV level is considered to be a $2s_{1/2}$ proton hole coupled with a 3^- state in ^{32}S at 5.01 MeV. The ^{32}S state can probably be assumed to have considerable collective octupole character. Such a model would require a $5/2^-$ state at higher excita-

tion in ^{31}P . The scattering data suggest existence of such a level at 5.7 MeV. Thus we are confronted at the same time with two apparently divergent descriptions of the same state in ^{31}P .

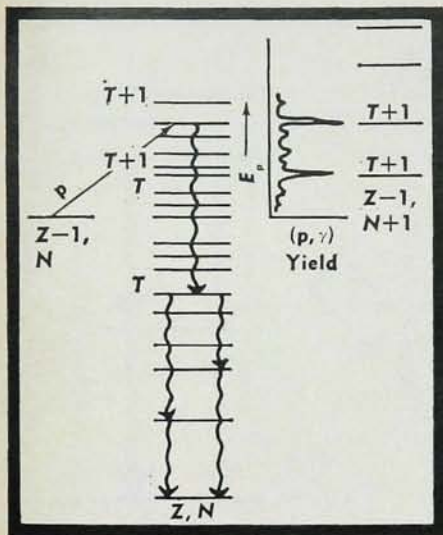
One very useful piece of work that has arisen as a byproduct of the recent interest in isobaric analog resonances is the systematic measurement of p, γ resonance strengths in s - d -shell nuclei between Na and Ca by Engelbertink and Endt. Cases are known of strength measurements reported in the literature by different authors that differ by orders of magnitude. Accurate values (say $\pm 15\%$) of the strength factor $(2J + 1) \Gamma_p \Gamma_\gamma / \Gamma$,

Single-Particle Orbitals in the Shell Model*

Nuclear shells	I_j	Number of particles in nucleus
I	$s_{1/2}$	2
II	$p_{3/2}$	6
III	$p_{1/2}$	8
IV	$d_{5/2}$	14
V	$s_{1/2}, d_{3/2}$	20
VI	$f_{7/2}$	28
VII	$p_{3/2}, p_{1/2}, f_{5/2}, g_{9/2}$	50
VIII	$d_{5/2}, s_{1/2}, d_{3/2}, g_{7/2}, h_{11/2}$	82
IX	$h_{9/2}, f_{7/2}, f_{5/2}, p_{3/2}, p_{1/2}, i_{13/2}$	126

*Table shows approximate assumed ordering. The $2s_{1/2}$ shell starts to fill when Z (or N) equals 9, and terminates at $Z = N = 20$ (Ca^{40}).

[Taken from P. Goldhammer, Rev. Mod. Phys. 35, 40 (1963).]



ANALOG RESONANCES in a proton-capture reaction. —FIG. 3

where J is the resonance spin and Γ_p , Γ_γ and Γ are the proton, gamma ray and total widths, respectively, are needed for the interpretation of transition strengths from isobaric analog resonances. As a result of this requirement, the strengths of resonances in 16 nuclei were measured to this accuracy with targets of many different chemical compounds, each containing at least two of the investigated isotopes. The final best values were determined by least-squares analysis and normalized to the strength of the $E_p = 621$ keV, $^{30}\text{Si}(p, \gamma)^{31}\text{P}$ resonance which is accurately known from gamma-ray resonant-absorption measurements. The result is that for the first time there is an accurate and reliable reference point readily available for most cases of interest in s - d -shell nuclei.

Giant dipole resonances

The role of isobaric spin in nuclear structure is also becoming more apparent in studies of the nuclear giant dipole resonance. The giant dipole resonance, which occurs in all nuclei, can be considered a property of nuclear matter. The details of the giant resonance have been studied by means of photonuclear reactions, proton-capture reactions, inelastic electron scattering, and in a few cases by alpha-capture reactions. The latter reaction in studies of the giant resonance was discussed at the symposium by Luise Meyer-Schutzmeister (Argonne National Laboratory). She pointed out that alpha-capture can play an im-

portant role in this type of work because of its ability to sort out the different reaction mechanisms that are involved in the formation and decay of the giant dipole resonance. Some of the significant experimental quantities are the total proton and neutron emissions from the giant resonance as compared with the total classical cross section, the integrated partial cross sections for emission to different final nuclear states, and for example the ground-state proton emission as compared with the total proton emission. These quantities and their systematic variations for the giant dipole resonances in the self-conjugate nuclei ^{12}C , ^{16}O , ^{20}Ne , ^{28}Si , and ^{40}Ca were discussed at the symposium. Much of the data can be explained by a particle-hole description in which one of the protons or neutrons is promoted from the ground state into the next shell with one unit change in total angular momentum.

A comparison of the neutron and proton emission from the giant dipole resonance of a self-conjugate nucleus provides a test of the assumption of charge independence of nuclear forces. The proton and neutron emission leads to states in mirror nuclei (for example, ^{27}Al and ^{27}Si in the case of ^{28}Si), and thus if the forces are charge independent the two emission rates will be the same if allowance is made for the Coulomb barrier penetration of the protons. The results of such studies do, in fact, support the assumption of charge independence—the observed differences in proton and neutron emission seem to be explainable by experimental errors, by difficulties in making accurate allowance for barrier penetrability, and possibly by a volume effect caused by different spatial distributions of protons and neutrons in the nucleus due to Coulomb forces.

As one traces the change in overall shape of the giant dipole resonance in the nuclei between ^{12}C and ^{40}Ca , it is observed to vary from a rather smooth, several-MeV-wide bump to a very complex structure consisting of many separate peaks with an envelope several MeV wide. For example, the giant dipole resonance in ^{28}Si appears as a combination of the several-MeV-wide overall structure, an inter-

mediate structure with widths of approximately 300 keV, and a "fine structure" with an average 60-keV width. It is believed, although some controversy still rages, that the complex structure corresponds to actual nuclear states as opposed to interferences between unresolved states. Perhaps there is some truth to both interpretations and we are seeing a horrible mixture of phenomena. Despite these problems, it was pointed out that the structure appears very much like the structure of analog resonances seen, for example, in elastic proton scattering on ^{92}Mo where the envelope pattern of the overall analog state is broken up into a fine structure. In ^{92}Mo the structure is interpreted as an analog state with isobaric spin $T = T_z + 1$, where T_z is the isobaric spin of the ground state, superimposed upon narrow compound nuclear states with $T = T_z$. It appears that the energy of the analog state with $T = T_z + 1$ "leaks into" compound-nucleus states with $T = T_z$. If there were no mixing of isobaric spins $T_z + 1$ and T_z , there could occur no neutron emission from the analog state. Thus the neutrons appear only by virtue of the isobaric-spin mixture. This picture of analog resonances in ^{92}Mo is very similar to the giant dipole resonance in ^{28}Si which can also be considered as an analog state. In ^{28}Si , however, both neutron and proton emission are allowed since ^{28}Si is self-conjugate ($N = Z$). On the other hand, alpha-particle emission (or formation) is forbidden in the $T = 1$ giant dipole resonance if isobaric spin is to be conserved. Thus the alpha-capture reaction can be a powerful means of studying the isobaric spin $T = 0$ impurities in the giant resonance. The selective features of the alpha-capture reaction also show up in the ground-state gamma decay of the resonance. By alpha-capture, only states with $T = 0$ and natural parity (odd spin and odd parity or even spin and even parity) can be formed. Thus only electric-dipole and electric-quadrupole ground-state transitions are observed. But these can easily be distinguished by their angular distributions. The observation of electric-dipole radiation is an indicator of mixing of the $T = 0$ and $T = 1$ states in the giant dipole

resonance. The result of measurements of this type in ^{28}Si shows that the isobaric spin impurity is quite pronounced in the giant dipole resonance and is similar in many respects to the situation observed in analog states of heavier nuclei.

Astrophysical application

One very interesting application of results of the study of nuclear reactions and excited states of nuclei is in astrophysics. The production of thermal energy in a star by nuclear reactions controls the evolutionary history of the star. The relative abundances of isotopes observed in stars, as well as in our local environment, are also determined by nuclear reactions. The role of reactions and energy levels in astrophysics was discussed at the symposium by Charles A. Barnes (Cal Tech).

It is interesting to note that, as in nuclear structure physics, there are both technical and convenient reasons in astrophysics to consider separately various regions of atomic mass. These regions are the low- A elements $A \leq 20$, the elements $A \geq 70$, and the elements $20 < A < 70$. The latter region coincides closely with our definition of "low-to-medium-mass nuclei" and was pointed out as a region where much work remains to be done in the acquisition of data necessary for synthesis into a detailed picture capable of explaining the observed isotopic abundances.

Theory

If one takes seriously the shell-model assumption that the wave functions of low-lying states of nuclei in the range $16 \leq A \leq 40$ can be well described by putting valence nucleons into $2s$ and $1d$ orbitals around an ^{16}O closed shell, then the functions obtained define a basis for a unitary group of twenty-four dimensions $U(24)$. This group is "broken" by the shell-model residual interaction, which acts among the valence nucleons only and determines the nuclear level structure.

Nuclear theorists have expended considerable effort toward finding this residual interaction. Robert Lawson's discussion at the symposium of the work of the Argonne group on the

oxygen isotopes shows what can be done with a rather restricted Hilbert space. The valence neutrons were assumed to fill only $2s_{1/2}$ and $1d_{5/2}$ orbitals. If the residual interaction is invariant under rotations it can be described by eight matrix elements $\langle j_1 j_2 J \| V_{12} \| j_3 j_4 J \rangle$ for the various possible states of the two-body system. These eight matrix elements may be used as parameters to fit the level schemes in the oxygen isotopes.

The agreement obtained from such a model is somewhat spectacular. Not only is a good fit to known levels obtained, but predictions of new levels have been verified experimentally. Furthermore, some success is obtained in calculating decay rates for electric-quadrupole transitions and spectroscopic factors for stripping reactions.

Calculations of this type are complicated by the fact that the number of basis functions of a given J increases rapidly as one adds valence nucleons. Even with the aid of modern electronic computers it is impractical to go beyond about ^{20}Ne in the shell, if the entire function space defined by the individual particle model is used. An important simplification will occur whenever one can define a subgroup of $U(24)$ under which the residual interaction exhibits an approximate invariance. Two such groups are well established [rotations in three dimensions $R(3)$ and charge independence $SU(2)$]. There are two additional candidates in the $2s1d$ shell that are easy to deal with:

(a) Elliott's $SU(3)$ group, which diagonalizes the quadrupole-quadrupole (Q-Q) interaction. This interaction produces an effect similar to a spheroidal binding field. Invariance under this group would yield nuclear level schemes reminiscent of the collective model.

(b) The symplectic group $Sp(2j+1)$. This group will be important if a pairing effect dominates the residual interaction. In this scheme two nucleons in like orbitals will tend to couple to zero total angular momentum, so that low nuclear levels will exhibit a single-particle character.

Akito Arima (University of Tokyo) and Marcos Moshinsky (University of Mexico) discussed attempts to make

such a group-theoretic reduction of the shell-model problem in the $2s1d$ shell. The major portion of the residual interaction does appear to consist of the pairing plus Q-Q force, although other components cannot be neglected. Some nuclear levels (like those in ^{20}Ne) seem to be well described by a few states of maximal $SU(3)$ symmetry, while in other cases (principally in even-even nuclei) the pairing effect appears to dominate. Despite the strong competition between these two groups (which may actually be considered to be opposite extremes) our main hope of obtaining a physical insight into many-nucleon spectra seems to lie in a group-theoretic analysis.

J. Bruce French (University of Rochester) proposed that spectra may actually be too detailed a property of nuclei to look at first in the theory. The distribution of higher moments ($\langle H^2 \rangle$, $\langle H^3 \rangle$, etc.) are easy to calculate throughout a given shell, and many reveal important qualitative features of the interaction. At any rate, many participants at the symposium felt that nuclear theory is currently breaking away from old traditions, and seeking new direction. It now seems possible to derive reasonable residual interactions from realistic phenomenological potentials, and doubtless many future calculations will be made along these lines.

The fact that the good agreement obtained from effective interactions may at times be quite misleading was pointed out very forcefully by Lawson. The Argonne group constructed a model (which they dub "pseudonature") resembling the calcium isotopes in which the shell model actually provides a poor approximation. A shell-model fit is then made to these "pseudonium" isotopes and agreement entirely typical of shell-model fits to nuclei is obtained. In this model the "core" is strongly excited, primarily through pairing excitations, so that the shell model provides a very poor basis early in the shell. The good agreement obtained tells us that the ordinary tests one normally applies are very insensitive to configuration mixing of the pairing type, and acts as a warning in the interpretation of future calculations. $\square$