Scions are Fermions

A LAW OF SOCIO-PHYSICS?

By Alex E. S. Green

In a previous article* the writer drew attention to the fact that some of the models, mathematical formalism, and language used in describing nuclear phenomena can be applied to a particular sociological problem: the remuneration of scientists. In particular, the independent-particle model and its associated formalism which have been used so successfully in accounting for bound-state and scattering phenomena in nuclei, seem also to provide a basis for correlating scientific salaries. In the case of nuclear phenomena, the basic interactions between nucleons are two-body interactions. Nevertheless, it appears to a good approximation that the interaction of a particular proton or neutron with all the other particles in a complex nucleus can be characterized by a family of potential-versus-radius functions which represents the average interaction of this particular particle with all the particles in the nucleus. In the sociological problem, again scientists are people, and people essentially interact in pairs. Yet it would appear that somehow in the course of the many two-body interactions (scientist and personnel man, scientist and manager, scientist and scientist, scientist and contract monitor) some sort of overall interaction of scientist with society results which seems describable by a percentile distribution function versus rate of pay.

In this study we extend the phenomenological analysis of salary data for scientists in industry and we seek a fundamental interpretation of the observed regularities. Further, we explore the possibility that our generalized phenomenological mathematical formula might represent a basic law of socio-physics. (Socio-physics represents the aspect of sociology which depends upon physical principles or mathematical formalisms borrowed from physics.)

Scions and Nucleons

In an attempt to find the order or regularity underlying the compensation schedules for scientists in industry, the writer has made use of salary-distribution data accumulated by the Los Alamos Scientific Laboratory. A typical set of data may be graphed as distribution functions in which the vertical scale represents the percentile (z); the horizontal scale represents the rate of pay (r). The various curves represent the distribution functions for $A = 5, 10, 15$, where A represents the years which have elapsed since the BS degree. (See Fig. 2 in I) The writer's immediate response to seeing such curves was to think of the nucleon-nuclear potential functions for various values of mass number (A) used in the nuclear shell and optical models. Then the vertical scale represented the nuclear potential and the horizontal scale the nuclear radial distance (See Fig. 1 in I). Thus, it was convenient to represent the distribution func-

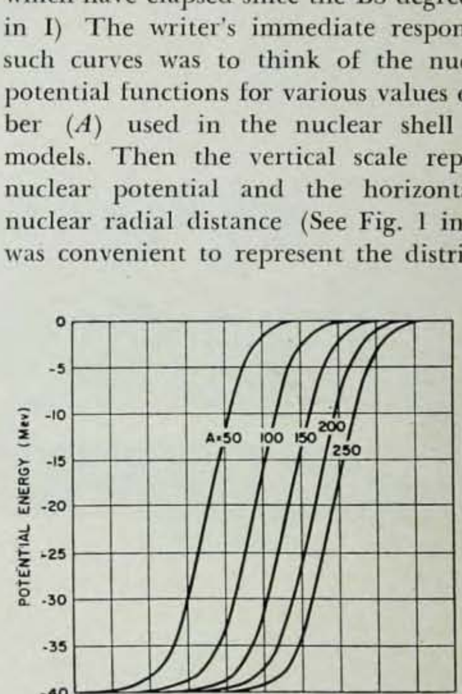

Fig. 1 (I)

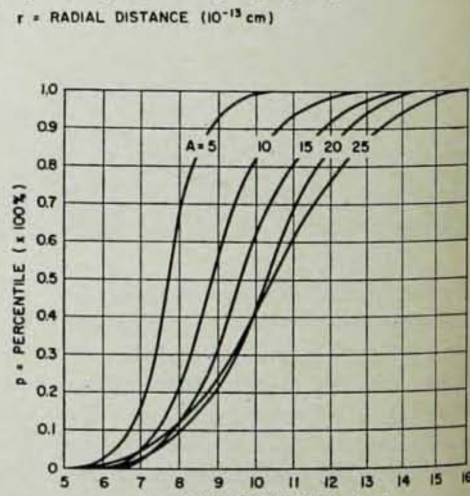

Fig. 2 (I)

The figures shown above are from the author's January 1962 article in Physics Today.

*A. E. S. Green, Physics Today, January 1962, p. 40; April 1962, p. 60 (to be referred to in this text as "I").

The hypothesis presented in the title of this article represents an extension of the analysis offered three years ago by the same author in a paper entitled "An Independent-Particle Model of Scientific Salaries". Alex E. S. Green is graduate research professor of physics at the University of Florida in Gainesville.

tion using the Saxon form with the equation

$$1-z=\{1+\exp[(r-R)/d]\}^{-1} \quad (1)$$

where R represents the pay rate for $z = 0.5$ (i.e., the 50th percentile) for a given age, and d is the width parameter which characterizes the spread of the distribution function. If one inverts this function one finds that the pay rate is given by

$$r(A,z) = R(A) + d(A) \ \ln[z/(1-z)]. \quad (2)$$

In this article we seek to give a fundamental rather than phenomenological interpretation to the salary data. Accordingly, we take cognizance of the fact that the mathematical form of Eq. 1 is identical with the Fermi distribution function of statistical mechanics. Thus, we note the resemblance between Eq. 1 and

$$\alpha(E) = \left[\ 1+\exp\left(\frac{E-E_f}{kT}\right)\right]^{-1}, \quad (3)$$

where E is the particle energy; E_f, the Fermi energy; k, Boltzmann's constant; and T, the temperature. The possibility that Fermi statistics may provide the primitive distribution law of compensation for scientists in industry as it has evolved under the free competition of private enterprise seems worthy of detailed examination and interpretation. In a later section we shall indicate two possible derivations of this distribution law. We shall, for convenience of discussion following the tradition of nuclear and particle physicists, introduce the special name "scions" for industrial scientists whose rewards are governed by the market place. We should note that the trend of the times places the older meaning of scions (heirs of noble families) in little use. The good fit obtained in this work suggests that scions are fermions.

One of the first problems which confronts us in establishing this thesis is the fact that personnel experts tend to identify four distinct types of scions (1) nonsupervisory scientists with BS and MS degrees; (2) supervisory scientists with BS and MS degrees; (3) nonsupervisory scientists with PhD degrees; and (4) supervisory scientists with PhD degrees. To the physicist it is highly distasteful to view the various possible states of a single entity as four distinct entities. Accordingly, we shall follow the example of the nuclear and particle physicist who treats the nucleon with the aid of dichotomic variables σ and τ to characterize its four states. As in I, we use the dichotomic variable, σ, analogous to nuclear spin which here can take on two possible values: $\sigma = +\frac{1}{2}$ to represent a supervisory state, and $\sigma = -\frac{1}{2}$ to represent a nonsupervisory state. We here extend the analysis of I to encompass non-PhD level scientists by introducing a second dichotomic variable, τ (a training parameter), analogous to isotopic spin. Thus, we let $\tau = +\frac{1}{2}$ to represent a PhD state and $\tau = -\frac{1}{2}$ to represent a non-PhD state.

Having established physical analogues for these quantized degrees of freedom, it is convenient to identify a physical analogue for the variable z which is a continuous variable bounded by 0 and 1. These properties suggest that we identify z with v/c, where v represents the nucleon velocity and c represents the velocity of light, since v/c is also continuous and bounded by 0 and 1. Recent studies of nuclear and particle physics indicate that the nucleon velocity is a major variable both in the two-body problem and the many-body problem.

To arrive at a convenient analytic equation which would embrace all percentiles, we introduce a transformation to the "quality" variable suggested by Eq. 2,

$$q(z) = \ln[z/(1-z)]. \quad (4)$$

For $z = 0.5$, $q = 0$; for $z = 0.75$, $q = 2.197$; for $z = 0.25$, $q = -2.197$. This $\ln[z/(1-z)]$ law is an admitted oversimplification. It particularly tends to yield salaries for the lowest percentiles which are low when compared to the experimental evidence. At a later point we shall indicate how this law may be modified by introducing nonlinear terms in q. However, since the law is quite good between the 25th and 75th percentiles, we shall apply it for the initial purposes of this survey.

We now choose the most general progression function which is linear in the dichotomic variables σ and τ and the continuous variable q. Thus, we assume that the rate of pay is given by

$$r = P_a(A) + P_b(A)q + P_c(A)\sigma + P_d(A)\sigma q + P_e(A)\tau + P_f(A)\tau q + P_g(A)\tau\sigma + P_h(A)\tau q\sigma, \quad (5)$$

where each of the P's is a polynomial in the variable A. It is a simple matter to utilize the LASL data representing various scion states, $\sigma = \pm\frac{1}{2}$, $\tau = \pm\frac{1}{2}$ and $z = 0.50$, and the differences between curves representing $z = 0.75$ and $z = 0.25$ to determine the coefficients of these polynomials. Using least-square techniques analogous to those used in phenomenological fitting of nucleon phase-shift data and assuming only second-degree polynomials, we arrive at the twenty-four (24) coeffi-

Table 1. Coefficients of quadratic polynomials in Eq. 5.

(a)	731.14	35.060	−0.5887	
(b)	67.15	4.074	−0.0170	q
(c)	106.95	−5.021	+0.3013	σ
(d)	15.71	−4.263	+0.1241	$q\sigma$
(e)	187.23	0.986	+0.0950	τ
(f)	70.44	−4.317	+0.1158	τq
(g)	−76.50	+3.962	−0.0885	$\tau\sigma$
(h)	13.25	1.617	−0.0621	$\tau q\sigma$

The first column of numbers are the constants, the second the linear coefficients, and the third the quadratic coefficients.

cients given in Table 1. These polynomials in conjunction with Eq. 5 thus represent a convenient parameterization of a vast quantity of data (about 10^5 scions) relating to salaries of scientists in industry.

The Phenomenological Representation of Scion Data

The successive polynomial terms in the final universal equation developed here for 1961 Private Industry are represented in Fig. 1. To represent all the terms on the same scale we have subtracted 800 from the first polynomial. The residual is then represented by the curve labeled A in Fig. 1.

It should be clear that this term represents the major component of the progression function. This term might be regarded as the progression function for a "standard" scion ($z = 0.5$, $\sigma = 0$, $\tau = 0$).

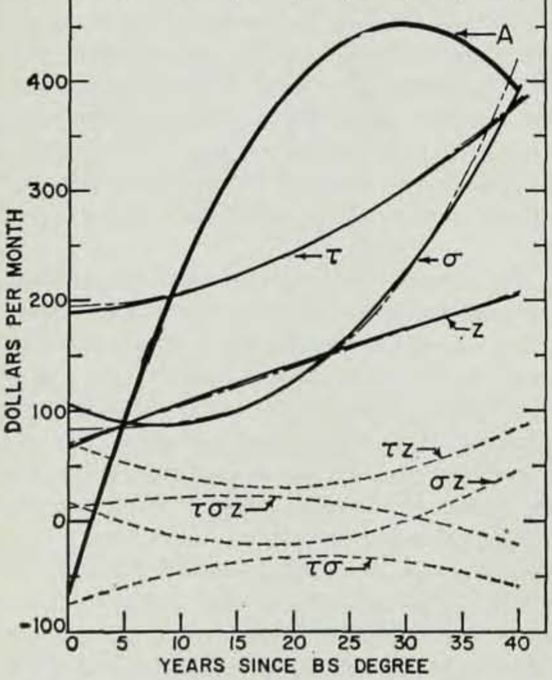

Fig. 1. Coefficients of various factors in the analytical formula for scientific salaries. The dot-dashed curves represent the simple approximation given in Eq. 6. The z factor refers to the quality correction.

The rather substantial negative quadratic term should put fear in the heart of every middle-aged scientist. Setting the derivative of this term with respect to A equal to zero, one finds that in 1961 the maximum pay rate for "standard" scientists occurred at $A = 30.0$. Taking 21.5 as the average age for the BS degree, this would correspond to age 51.5. The monthly standard salary at this maximum corresponds to \$1252.

The next most important terms are the linear τ, σ, and q effects. In assessing the significance of these terms, it must be realized that σ and τ normally are restricted to the range of values of $\pm 1/2$ whereas q might range to much larger magnitudes, e.g., to ± 3 if one goes to the 5th and 95th percentiles. Thus, the potential reward represented in the quality parameter is greatest although it might appear somewhat less important on the graph. In view of the association of this factor with velocity dependence we can say that it embodies the well-known truism of industrial life, "you gotta hustle if you wanta get ahead".

The importance of the PhD degree is clearly indicated by the curve labeled by τ. Indeed, this curve represents directly the difference in the earning power of the median scientist ($z = 0.5$, $\sigma = 0$) associated with the PhD degree. This reflection of real status for the PhD might not be generally recognized in America.

It is rather interesting to note that the σ term, which directly reflects the difference between the rewards of supervision and nonsupervision for a $z = 0.5$, $\tau = 0$ scientist, is rather modest until later years.

The significance of the various interaction terms must be assessed with a view to the ranges of σ, τ, and q. It is interesting that the interaction between σ and z is quite modest except at advanced ages. The τ, q interaction is also rather interesting since it is always positive and it builds up with A. The fact that the τ, σ interaction is negative suggests that a PhD is penalized for going into supervision. However, remembering that ($\tau\sigma \leqq 1/4$) one sees that this term is always of small numerical magnitude. The σ, q, τ effect is also quite small and the trends represented in the curve are probably not very meaningful.

Having unfolded the various components of the compensation formula it might be appropriate to propose simplified approximations to the various coefficients. The experience factor would appear to require a full quadratic. However, the appearance of the quality dependent term suggests a linear approximation and the dot-dashed curve represents one which provides a close fit. The

coefficient of the τ factor (the training factor) can be approximated by a quadratic expression (see dot-dashed curve). Further, the coefficient of the supervisory factor can be approximated by a cubic expression (see dot-dashed curve). By virtue of the smallness of the interactions it is reasonable to use constants as approximations for them. Embodying these simplifications we have as a proposed simplified socio-physical formula

$$r = 731 + 35.06A - 0.598A^2 + 72 + 3.4A)\, q + (195 + 0.12A^2)\ \tau + (84 + .00525A^3)\ \sigma + 40\,\tau\ (q - \sigma). \qquad (6)$$

These simplified coefficients do not significantly deteriorate the fit of the formula derived directly from the least-square fits. The significance of the symmetry represented in the $\tau(q - \sigma)$ term eludes the writer, although it has a number of intriguing possibilities.

One of the weaknesses of the analysis in I was the slight inaccuracy of the ln $[z/(1-z)]$ quality correction function at low values of z. In the LASL salary survey the effect is revealed by the skewness factor in the percentile versus pay-rate distribution functions. To embody this effect in a simple way we may add a small nonlinear (in q) correction. An analysis of the skewness of the LASL data shows that this correction may be approximately represented by

$$r = [9 + 14\sigma + 2\tau + 12\sigma - 0.4A\sigma]q^2. \qquad (7)$$

The results obtained with our simplified functions with this quadratic quality correction in relation to individual least squares curves (for given τ, σ, and q) and the experimental points are shown in Figs. 2-5. It is seen that the fits are quite good for all percentiles. One could, of course, do somewhat better by using more elaborate functions of A containing additional adjustable constants. However, self-respecting curve fitters are by nature frugal in their use of adjustable constants. It is gratifying that we can provide a good phenomenological representation of the scion data with only fifteen adjustable constants. Recent phenomeno-

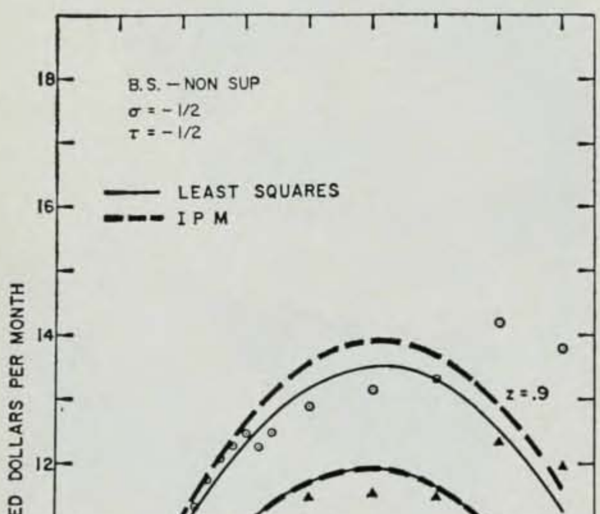

Fig. 2. Salary curves for BS-level scientists without supervisory responsibility. Solid curves represent least-square quadratic fit to the particular percentiles. Dashed curves (IPM) represent our generalized phenomenological formula.

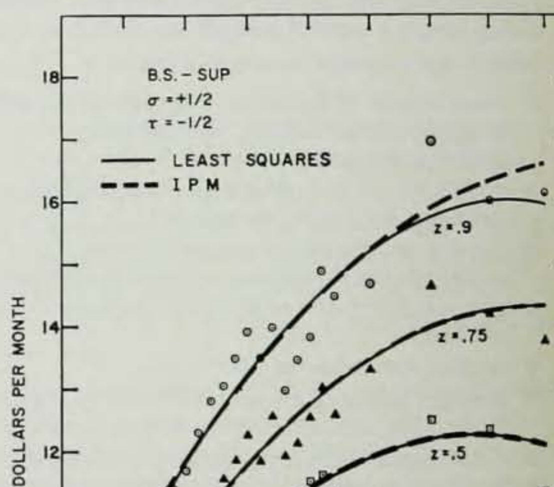

Fig. 3. Salary curves for BS-level scientist with supervisory responsibility. Solid curves represent least-square quadratic fit to the particular percentiles. Dashed curves (IPM) represent generalized phenomenological formula.

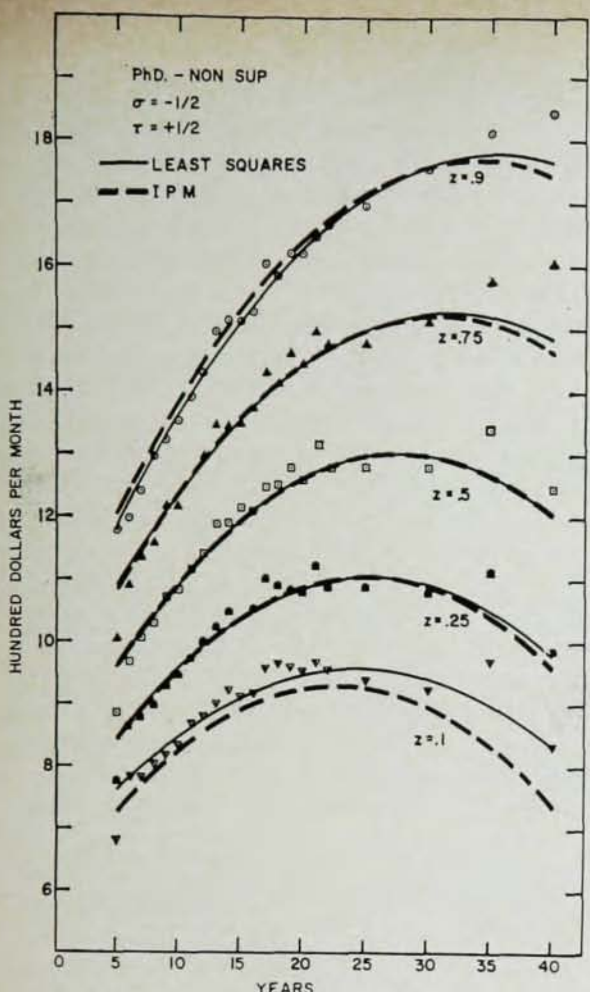

Fig. 4. Salary curves for PhD-level scientists without supervisory responsibility. Solid curves represent least-square quadratic fit to the particular percentiles. Dashed curves (IPM) represent our generalized phenomenological formula.

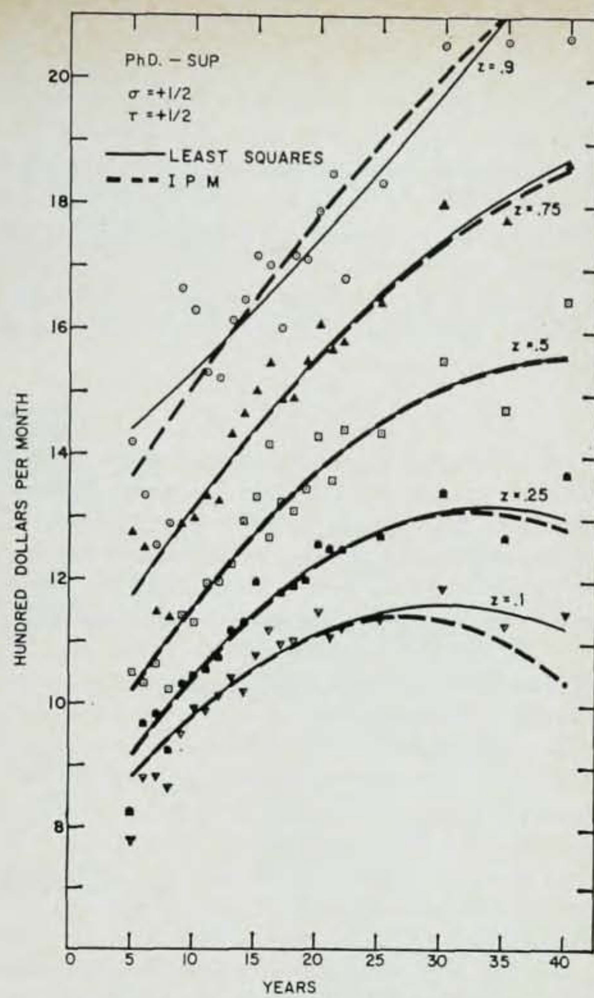

Fig. 5. Salary curves for PhD-level scientists with supervisory responsibility. Solid curves represent least-square quadratic fit to the particular percentiles. Dashed curves (IPM) represent generalized phenomenological formula.

logical representations of nucleon data have used more than twice as many. Perhaps sociological physics is simpler than particle physics.

Explanations of the Scion Interaction

It would appear that we have established the possibility of accurately characterizing scion data using nucleon formalisms. We are confronted, therefore, with the problem of explaining this successful phenomenological formulation in terms of fundamental principles. The fact that Fermi statistics are closely followed suggests that we explore the socio-physical explanation from the standpoint of the conditions which lead to Fermi distribution functions in particle physics. We do not have very far to look to see how such conditions might arise from industrial personnel practices.

One of the common practices in industry is to round off salary increments. For example, below \$800 per month it is customary to use \$10 as a minimal increment. In the \$1000 range usually \$20 serves as a minimal increment; for the \$1500 range, \$30; etc. The point to be made is that because of the labor involved in implementing wage increases, natural quantum units are employed which usually establish a quantized hierarchy of states. Another common practice which personnel managers frequently impose upon technical administrators is the value ordering of their personnel. This ordering of personnel usually tends to blend into the hierarchy of states established by the quantized system of pay raises. This establishes just the conditions for a Fermi distribution function to evolve (e.g., a quantized set of states with no more than one object per state). The dependence of remuneration upon age may now be accommodated by assuming that the temperature depends upon A and borrowing some techniques from nonequilibrium statistical mechanics. The writer has pursued this approach with Professor Irwin Oppenheim and it appears possible to achieve a semiquantitative explanation of observed distributions from this point of view.

A second and possibly equivalent explanation arises out of an observation by Dr. Elliot Montroll that in many distribution problems the rate of change of percentile with respect to the parameter of interest (here the pay rate) is proportional both to the percentile and the departure of the percentile from 1.0. The latter factor arises from saturation effect which may be due to the fact that the total quantity of interest (in this case, money) is limited or possibly the fact that z itself is a bounded variable. The differential equation implied by the proportionalities is

$$\frac{dz}{dr} \sim z\ (1 - z). \qquad (8)$$

This differential equation, when integrated, leads precisely to our pay-rate function given by Eq. 2. Equation 2, of course, is an inverted form of the phenomenological distribution function represented by Eq. 1 which, as we have noted, is mathematically equivalent to the Fermi distribution represented by Eq. 3. Thus, we have two plausible explanations for our phenomenological identification of scions as fermions.

The distortion of our distribution function at low values of z which we embodied in the nonlinear phenomenological correction also has a simple explanation. It is undoubtedly due to the fact that there is a threshold to the pay of scientists on the low side which is related to the minimum cost of living. Since the correction is relatively small, it can presumably be dealt with by perturbation technique. We might note that the use of perturbation techniques to account for the behavior of nucleons with meson theory has been fraught with grave divergence difficulties. We might hope, however, that scions and socio-physical theory are simpler.

Discussion and Conclusion

We have presented an analytic formula which represents the 1961 scion data. From a mathematical point of view, we might view the function $r\ (A,\ z,\ \sigma,\ \tau)$ as a function in four-space or a hypersurface in five-space. We must now emphasize that there are additional dimensions to the scientific-salary hypersurface. Some of these can be explored with the aid of earlier and later LASL surveys. For example, we could explore the time dependence of these progression schedules which, of course, are vital to a real understanding of salary practices and trends. A cursory analysis of this nature suggests that a time expansion of the schedules occurs in such a way that the rate for a standard scion with median A increases by about three to five percent per year. In detail, however, the time dependence is fairly complex and the increase with time probably propagates like a bow wave along the hypersurface from the high initial salaries offered new scions toward the older scions. Indeed, the downward trend with A observed for older scions is probably a reflection of the fact that this bow wave propagates at a velocity $(dA/dt) << 1$.

Another important variable which can be explored with the aid of the LASL surveys is the spatial (i.e., geographic) dependence of the scientific salary function. A cursory examination of the 1961 data indicates that variations of the order of six percent exist from one geographic division to another. Again, however, the dependence upon this variable is fairly complex, in that the hypersurface shows significant changes in shape from one geographic zone to another.

A third parameter that could be charted is the dependence of the salary hypersurface upon the particular industry. Here a cursory examination of the data indicates variation of the order of $\pm$ 10 percent for a median-A scion. These variations probably reflect differences in the compensation schedules associated with various scientific disciplines. Other variables undoubtedly enter the problem although in some cases additional detailed data beyond that contained in the LASL survey would be required to analyze their influence. It would, of course, be difficult to analyze in a quantitative way the influence of factors particular to individual companies such as academic atmosphere, vacation and travel policies, organizational stability, power and influence, freedom from pressure, and other intangibles which interact in a significant way with scientific salaries.

In conclusion we wish to emphasize that our entire statistical analysis pertains purely to scions, i.e., industrial scientists compensated at levels dictated by the open market place. It does not apply to university scientists, government scientists, business owners, etc. For such entities it might be necessary to use boson statistics or to borrow other aspects of particle physics, such as strangeness, hypercharge, etc. We leave it to others to bring forth the sociological counterparts of the laws governing such systems. In the meantime we hope that this analysis and interpretation of a fairly coherent body of data might serve a useful purpose and that it generates more light than heat.

The writer wishes to thank John Woodward for permission to use the data of the 1961 Los Alamos Scientific Laboratory Salary Survey. All of the statistical analyses presented in this paper were carried out while the writer was serving as Manager of Space Science, General Dynamics Convair. The final manuscript was prepared at the University of Florida following a spin-flip transition.