LETTERS

Hubble trouble

In *Physics Today* for May 1964 (p. 21), Dr. Hong-Yee Chiu has written an admirable account of a symposium on gravitational collapse held in Dallas, Texas, in December 1963. In the course of his exposition, he has occasion to describe and make use of Hubble's law which connects the distance of a galaxy with the red shift in its spectrum. It is stated that the law is expressed by the formula, Distance = $(1/H)$ (Recession velocity), where H is the Hubble constant, that $H^{-1}\, c$ is a measure of the "dimension" of our universe and that for an object at distance $H^{-1}\, c$ "the red shift is total". In this day and age it is surprising to find that these statements are still credited. Since they give rise to much unnecessary confusion it is worth while to state the present-day situation with regard to Hubble's law. To do this, one must look back thirty years and recall what Hubble (1937) did when he established it.

In Hubble's work two points are to be noticed: firstly, the distances he calculated depended on luminosity criteria. Hence he obtained what are nowadays called luminosity-distances, D. Secondly, the red shifts of the objects he studied were small and did not really exceed $z = 0.13$. Hubble multiplied the measured red shift by c and defined the result, $V = c\,z$, as the velocity of recession, by using the classical Doppler formula. He then showed empirically that for his, cosmologically speaking, nearby objects D was proportional to V. The essential feature of this work is that *D is shown to be proportional to z* so long as objects with red shifts small compared with unity are considered. The constant of proportionality is c/H, where H is the Hubble constant, and the fundamental form of Hubble's law is: Luminosity-distance = (c/H) (Red shift), *provided that z is small compared with unity.*

The definition $V = c\,z$ is good enough when z is much less than unity. This is no longer true for most quasars or for 3C295 for example. Exact Doppler formulae should nowadays be used for calculating velocities of recession from red shifts. Each "model of the universe" has its own Doppler formula because the motion of expansion differs from model to model. Unfortunately, this leads to a number of possible definitions of the velocity of recession for an object with a given red shift (McVittie 1961) and it is therefore doubtful if the concept of the velocity of recession is nowadays very useful. This ambiguity is fortunately absent from the notions of luminosity-distance and of red shift. A particularly simple model of the universe is Milne's, which employs special relativity only. In this model it is possible to define a certain velocity of recession, U, which is connected with z by the Doppler formula of special relativity,* namely,

$$1 + z = [(1 + U/c)/(1 - U/c)]^{\frac{1}{2}}.$$

The formula shows, for example, that $z = 1$ means $U = 0.6c$.

In all models of the universe it is possible to define the most distant object as one which, for the observer, has an infinite value of z. This is presumably what is meant by Chiu's statement that "the red shift is total". But such an object does not have a luminosity-distance equal to $H^{-1}\, c$. In Table 1, five models are listed and the exact formula connecting D and z is given for each. The cosmical constant is denoted by Λ. In all five models, the most distant objects lie at infinite luminosity-distances D. It will be observed that $D = H^{-1}\, c$ corresponds to widely differing values of z in the various models. Equally obviously $D = H^{-1}\, c$ fails to give the "dimension" of the universe.

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*On p. 25, the formula is incorrectly quoted by Chiu. The correct treatment may be found, for example, in Møller (1952).

Table 1. Luminosity-distance and red shift

Model	$D/cH^{-1} =$	Value of z for $D = cH^{-1}$
Milne's	$z(1 + \frac{1}{2}z)$	$\sqrt{3} - 1 = 0.73$
Einstein—de Sitter	$2\left\{1 + z - (1 + z)^{\frac{1}{2}}\right\}$	$\sqrt{3}/2 = 0.87.$
$q_0 = 1,\ \Lambda = 0.$	z	1
$q_0 = 2,\ \Lambda = 0.$	$\frac{1}{4}\left\{2z - 1 + (1 + 4z)^{\frac{1}{2}}\right\}$	$3 - \sqrt{3} = 1.27$
Steady-state, de Sitter.	$z(1 + z)$	$\frac{1}{2}(\sqrt{5} - 1) = 0.62$

References

Chiu, Hong-Yee, *Physics Today*, May 1964, p. 21.

Hubble, E. P., 1937. *The Observational Approach to Cosmology*. Oxford: Clarendon Press.

McVittie, G. C., 1961. *Fact and Theory in Cosmology*. London: Eyre and Spottiswoode, New York: Macmillan. pp. 113-122.

Møller, C., 1952. *The Theory of Relativity*. Oxford: Clarendon Press. Sections 23 and 25.

Chiu's views

I think Professor McVittie has misunderstood the points that I made in my paper. My paper was already fairly long and it did not seem necessary to confuse readers further with the fine points of cosmological theories (for which reasons I shall discuss below). I carefully inserted an "≈" sign when I said "at a distance $\approx H^{-1}\, c$ the red shift is total". I meant an event horizon exists at roughly this distance. I never said that $H^{-1}\, c$ was *the* distance. Professor McVittie is correct in his letter in pointing out that at large distances all the fine