Polarized Protons

By John H. Williams

The following article is based on this year's Retiring Presidential Address of the American Physical Society, presented on January 23 during the 1964 annual meeting of the Society in New York by John H. Williams of the School of Physics and Astronomy at the University of Minnesota. Professor Williams, whose term as president of the APS ended during the meeting, served as an Atomic Energy Commissioner in 1959-60.

John H. Williams

One of the objectives of nuclear physics is to acquire a better understanding of nuclear forces. The understanding of nuclear forces can be enhanced by investigations of the effects of the nuclear spins of the interacting particles. To make such investigations when protons are interacting with nuclei, one wishes to have available a beam of protons whose spins are oriented in a chosen direction, i.e., a beam of polarized protons. By scattering these protons from nuclei, one can investigate the nature and strength of the spin-orbit force which is required by most phenomenological theories of the nucleus, particularly the shell model.

Historically,[1] polarized proton beams were first obtained by scattering unpolarized protons (random spin orientations) of energies ~100 MeV from targets of light elements. A typical arrangement is shown in Fig. 1.[2] Here protons, accelerated in the 184-inch Berkeley cyclotron to 300 MeV, impinged on a Be target. These protons, elastically scattered at a small angle, $\theta_1 \approx 13°$, were found to be polarized by a second scattering on target 2. Polarization was established by observing the asymmetry, $(L - R)/(L + R)$, of the second scattered protons where L is the number of protons scattered to the left and R is the number of protons scattered to the right. The percentage polarization was found to be approximately 75%. Indeed, it was with such a source of polarized protons that early work was done in several laboratories to determine the spin dependent parameters of high-energy proton-proton scattering.

Schwinger[3] suggested that 1-MeV neutrons could be polarized by scattering from ^{4}He as a consequence of the interference between the scattering from the resonance states of ^{5}He ($J = 1/2$ and $J = 3/2$) and the potential scattering of the S-waves. The Minnesota group,[4] recognizing the difficulties of making observations with neutrons, performed a comparable experiment in which protons were scattered from ^{4}He. In this case, the intermediate nucleus is ^{5}Li which has similar states to those of ^{5}He. The apparatus used to measure the polarization is shown in Fig. 2. The left-right asymmetry in the double scattering of 3.25-MeV protons from ^{4}He was measured by photographic emulsions detecting the second scattered protons at approximately $\pm 90°$ CM. This experiment established that the $P_{1/2}$ level of ^{5}Li was at a higher energy than the $P_{3/2}$ level; this was important information in formulating the shell model of the nucleus.

The production of polarization in scattering processes has been described by attributing the effect to spin-orbit forces acting in the spin-orbit interactions. This expectation has been confirmed by polarization measurements on the protons and neu-

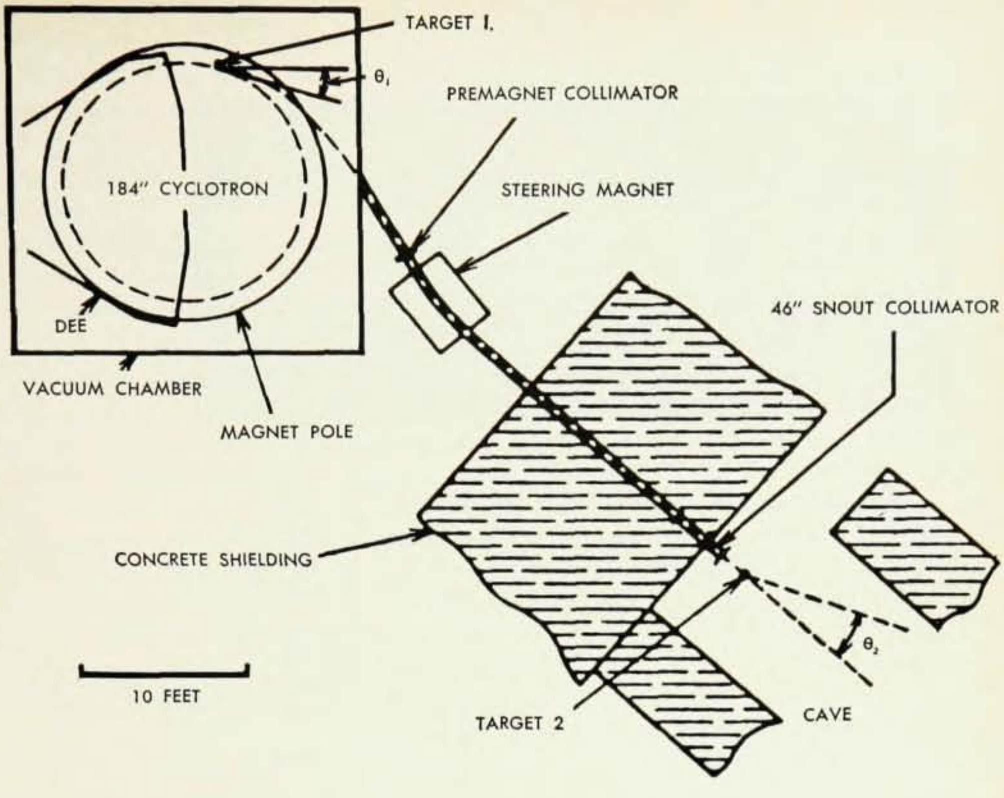

Fig. 1. Experimental arrangement used in obtaining beam of polarized protons using 184-inch cyclotron at Berkeley. (Ref. 2.)

trons resulting from the (d,d) reaction and by many other reactions.

The scattering of protons by ^{4}He or conversely the scattering of ^{4}He by protons, forms the basis of one of the modern ways of producing nearly 100% polarized beams of protons. This method has been successfully developed and employed by L. Rosen[5] and his group at Los Alamos.

The Los Alamos group accelerated 28-MeV alpha particles in their cyclotron and allowed the extracted beam to collide with H_2 gas in a container. The 10-MeV polarized protons recoiling at 25° from the ^{4}He-proton collisions entered a second chamber filled with ^{4}He. Measurements of the asymmetry of scattering of the protons at various angles in the second chamber showed that scattered 10-MeV protons were more than 95% polarized at certain angles. This result agreed with the theoretical prediction of polarization based on the phase shifts obtained from elastic scattering data on p-^{4}He with 10-MeV protons.

Extensions of this method of producing polarized protons have been used by Brockman at Princeton,[6] by Igo at Berkeley,[7] and by others elsewhere. Excellent work with polarized protons produced in the above manner has been done, and the method has been extended to lesser energies by slowing down the protons in appropriate thicknesses of absorbers.

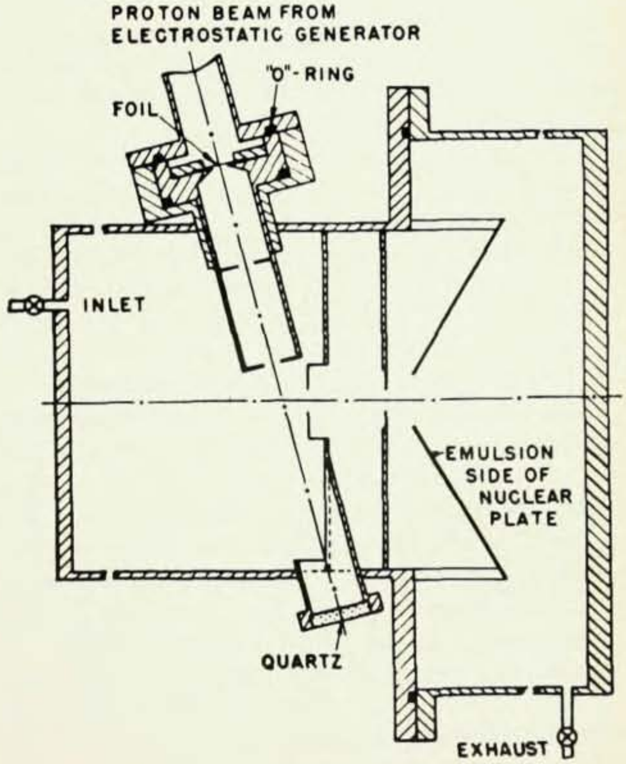

Fig. 2. Apparatus used to measure polarization of protons scattered from ^{4}He by Minnesota group (Heusinkfeld and Freier, Ref. 4).

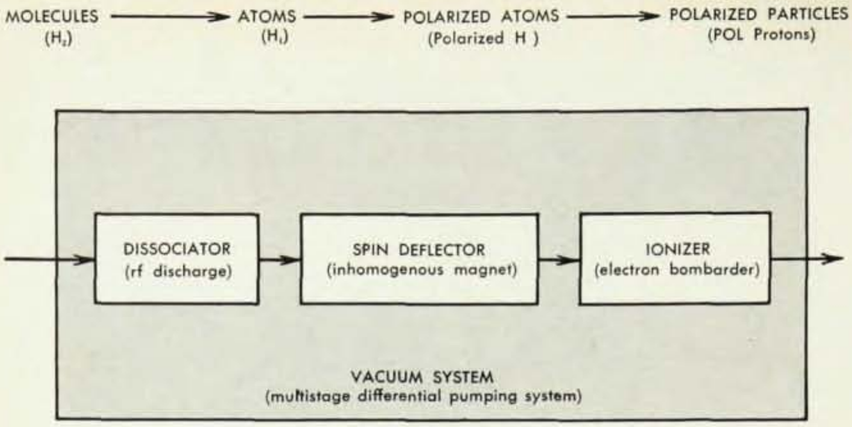

Fig. 3. Schematic drawing showing arrangement of components of polarized ion source. (Ref. 9.)

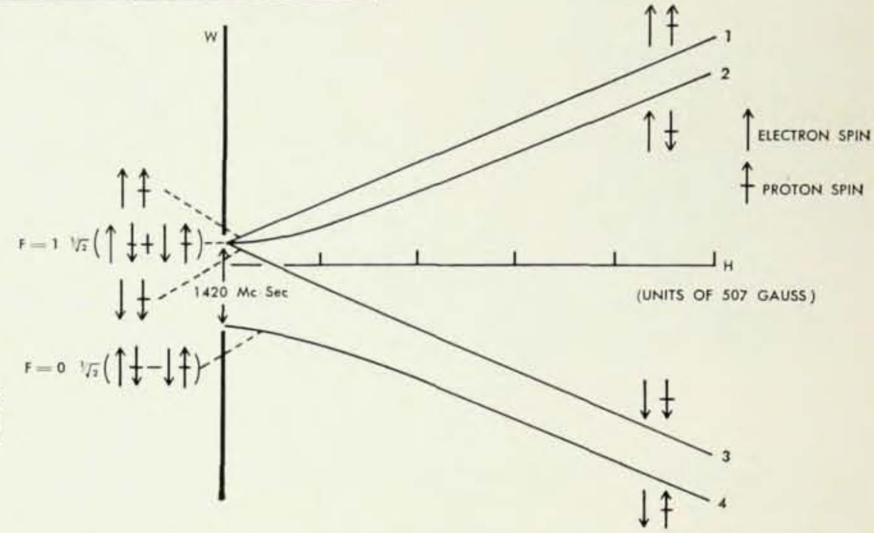

Fig. 4. Energy dependence of hyperfine states of hydrogen as a function of the external magnetic field. (Ref. 8.)

Such absorption does not influence the percentage polarization of the protons.

Alternative methods of producing polarized beams of nuclei have been developed which employ atomic-beam phenomena. These sources have been developed and used with a variety of intermediate and low-energy accelerators in several laboratories.[8-12] I wish to confine my remarks to polarized-proton ion sources such as those developed at Minnesota[9] and at the Rutherford Laboratory.[8]

The arrangement of the components of such sources is shown in Fig. 3. The first section of this diagram, the "dissociator," is an rf discharge to make atoms from the H_2 gas introduced into the system. The central section of this block diagram, labeled "spin deflector," performs the function of separating from an atomic beam, produced in the "dissociator," atoms whose electron spins are aligned in a specific direction. The "spin-deflector" action is based upon a well-known atomic phenomenon, the Stern-Gerlach experiment.

Breit and Rabi have calculated the energy dependence of the hyperfine states of hydrogen as a function of the external magnetic field in which the atom is placed. Figure 4 shows this energy dependence. The energy W of the ground state of the hydrogen atom is plotted as a function of the magnetic field H. At $H = 0$ the interaction of the internal magnetic field of the atom with the magnetic moments of the electron and proton (which result from their respective spins) serves to couple the electron and proton spins. Consequently the $H = 0$ field states are split and are labeled in Fig. 4 as $F = 0$ and $F = 1$. $F = 0$ is a singlet state and $F = 1$ is a degenerate triplet state. The arrangement of the electron spins and the nuclear spins at $H = 0$ are shown by the directional arrows representing electron spin and the barred arrows representing proton spin. As the field H increases from zero, the electron and proton spins become uncoupled and four energy states develop in which the electron spins and proton spins assume the indicated directions. For clarity, let us label these four states (1), (2), (3), (4) in order of descending energy.

Atoms produced in the dissociator of Fig. 3 drift into a region of a strong, inhomogeneous magnetic field produced by a six-pole magnet, where they are split into the four states shown in the Breit-Rabi diagram. The spin deflector produces forces on the atoms due to the interaction of the gradient of the inhomogeneous field of the sextupole magnet with the magnetic moment of the electron. Those atoms in states (3) and (4) are deflected away from the central axis of the sextupole magnet in hyperbolic paths. The atoms in states (1) and (2) perform oscillatory motion about the axis and emerge from the end of the sextupole magnet.

The atoms in states (1) and (2) which emerge from the sextupole magnet are in a strong magnetic field. They have the same electron spin, but one half of them have nuclear spin in the opposite direction to the other half. Consequently, the atoms contain unpolarized protons.

On leaving the sextupole strong field, the atoms in states (1) and (2) pass adiabatically into a region of zero magnetic field. At zero field, the state (1) atoms have a single nuclear spin direction up. In state (2) at zero field, half the atoms have a nuclear spin up and half have nuclear spin down. The number of atoms, normalized to unity in each state, with spin up is $1 + 0 = 1$, whereas the total number of atoms is 2. The nuclear spin polarization is therefore 1/2.

A more detailed and accurate statement of the theory of separation of nuclear spin states occurring in the system has been given by Clausnitzer[9] in a recent paper in *Nuclear Instruments and Methods.*

These beams then drift through the earth's magnetic field and enter the ionizer where the electrons are removed from the atoms. Surrounding the ionizer is a Helmholtz coil producing a vertical field of 4 to 6 gauss which serves to orient the proton spins in the direction of the field. By reversing the current in the Helmholtz coil, the nuclear spin is rotated through 180°.

The polarized protons with their spins in a vertical direction, as well as protons arising from background H_2, are extracted in a horizontal direction from the ionizer by electrodes and then are accelerated to high energy by the Minnesota and Harwell proton linear accelerators. To determine the percentage polarization of the protons in the Minnesota beam, they were accelerated to 10 MeV by the first section of the accelerator, collimated, and scattered from a gaseous helium target. Two counters placed at equal angles θ to the left and right of the beam direction counted scattered protons. The asymmetry is defined as $(L - R)/(L + R)$, where L is the number of protons scattered to the left and R is the number of protons scattered to the right. This asymmetry $= P_b P_s(\theta,E)$ where P_b is the polarization of the beam and $P_s(\theta,E)$ is the polarization that would be produced by scattering an unpolarized beam of energy E through an angle θ. Earlier measurements by Rosen's group[13] on the scattering of polarized protons from He gave $P_s = 44\% \pm 5\%$ for $\theta_{lab} = 40°$ and $E = 10$ MeV. With the help of Rosen's data, we have found that the P_b of our beam varies from 25% to 39%, depending on conditions of the ion source.

These experiments have led to polarized proton beams with an intensity of 1.5×10^7 protons/sec at 10 MeV, 2×10^6 protons/sec at 40 MeV, and a typical beam polarization of 35%.

Using this polarized proton ion source and the Minnesota proton linear accelerator, we have performed polarization measurements[14] to obtain $P_s(\theta)$ at an E of approximately 40 MeV, on a variety of targets. The choice of target elements was limited to those nuclei whose differential cross sections had been measured at 40 MeV in order to facilitate data analysis. The target elements measured were natural helium, lithium, carbon, aluminum, nickel, lead, and isotopically pure ^{6}Li (99.3% ^{6}Li and 0.7% ^{7}Li).

A most interesting result was found in the measurement of $P_s(\theta)$ for helium. The results are shown in Fig. 5. In this figure the experimental results, open circles with error bars, are plotted as well as the polarization calculations by Gammel and Thaler.[15] These authors used the experimental results of p-^{4}He elastic differential cross section, obtained by Brussel and Williams[16] at the same energy, to calculate the phase shifts appropriate to the scattering.

The three theoretical curves for $P(\theta)$ shown in Fig. 5 are based on the calculated phase shifts which suggest an "optical model" potential with exchange. Since our experimental results on the

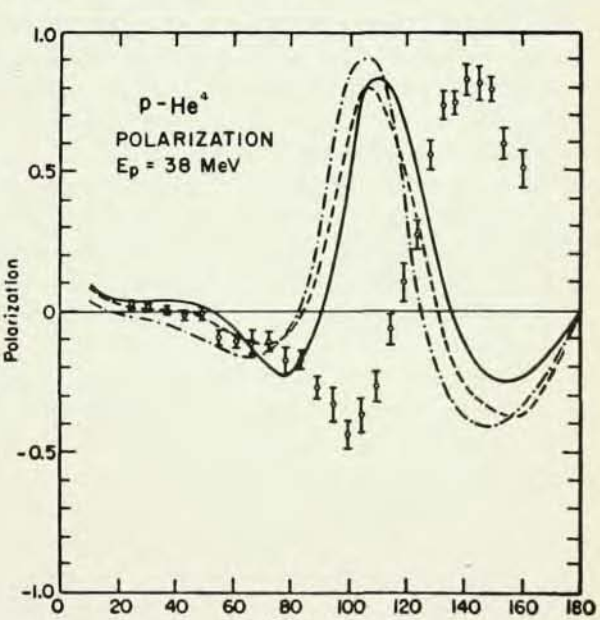

Fig. 5. Polarization produced in scattering experiments using helium target. (Ref. 14, first authors.)

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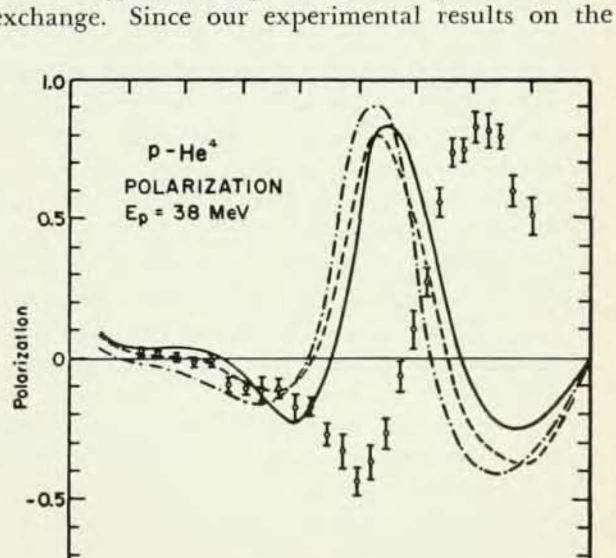

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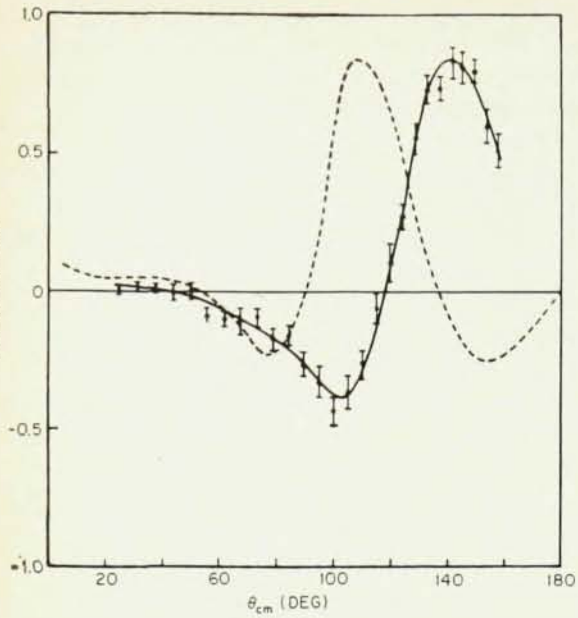

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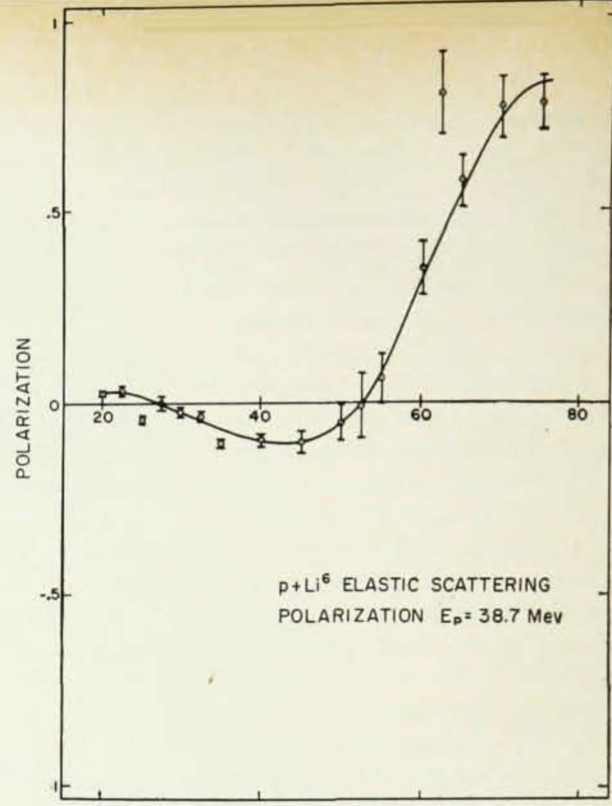

Fig. 7

polarization of ^{4}He appeared, new phase shifts have been calculated which are chosen to fit both the measured elastic differential cross section and the polarization by C. C. Giamati, V. A. Madsen, and R. M. Thaler[17] as well as by S. Suwa and A. Yokasawa[18] and by H. Kanada, S. Nagota, S. Otsuki, and Y. Sumi.[19] Suwa and Yokasawa have published a graph, shown in Fig. 6, demonstrating the agreement of their predictions, solid curve, and of Thaler's predictions, dashed curve, with the experimental data on $P_s(\theta)$ of ^{4}He. It should be clear from the last two figures that predictions of polarization from the phase shifts derived from differential elastic scattering are not necessarily trustworthy.

Figures 7, 8, 9, 10, and 11 show experimental results on ^{6}Li, ^{12}C, and ^{12*}C, Al, Ni, and Pb, all of which show distinctive features which have not yet been subjected to detailed theoretical analysis. Francis Perey and his group[20] at Oak Ridge have had remarkable success in fitting both differential cross section and polarization data with the nuclear optical model.

Observations on $P_s(\theta)$ of ^{4}He have been made at Harwell[23] at energies of 22, 29, 40, and 48 MeV. The agreement between the Minnesota 38.4-MeV results and the Harwell 40-MeV measurements of $P_s(\theta)$ is most gratifying.

Suggestions we have made[21] on the possibility of producing polarized beams of protons by scattering 40-MeV protons from carbon targets placed in the beam of a high-intensity proton accelerator have proven to be of interest to some laboratories which

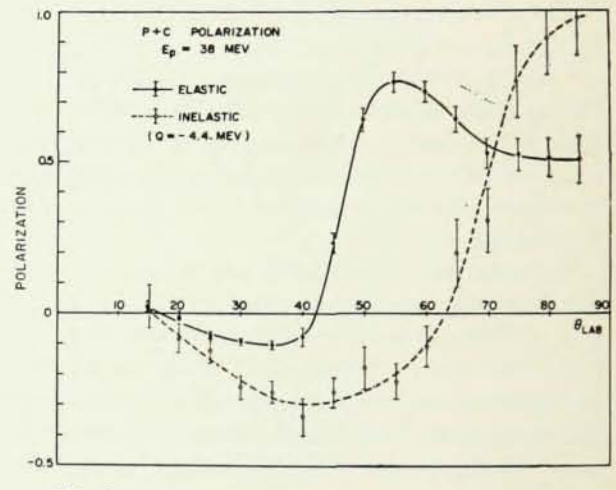

Fig. 8

have sector-focused cyclotrons. T. A. Cahill[22] has scattered 42-MeV protons from the internal beam of the UCLA cyclotron from a carbon target and observed their polarization by scattering from a second carbon target placed 21 feet away.

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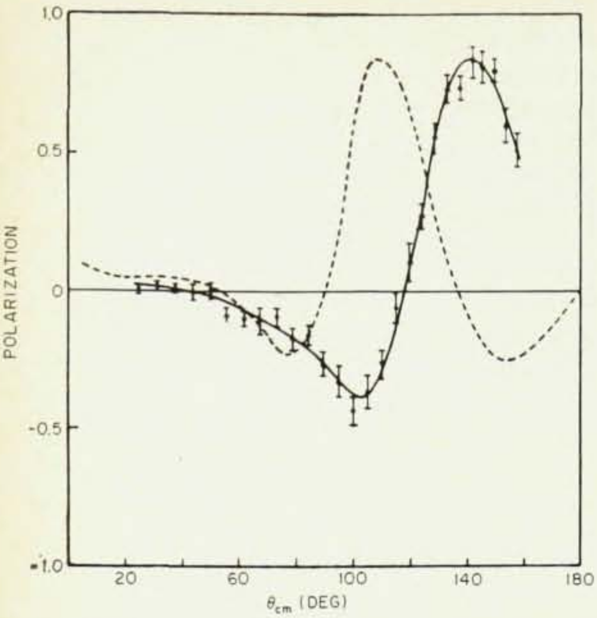

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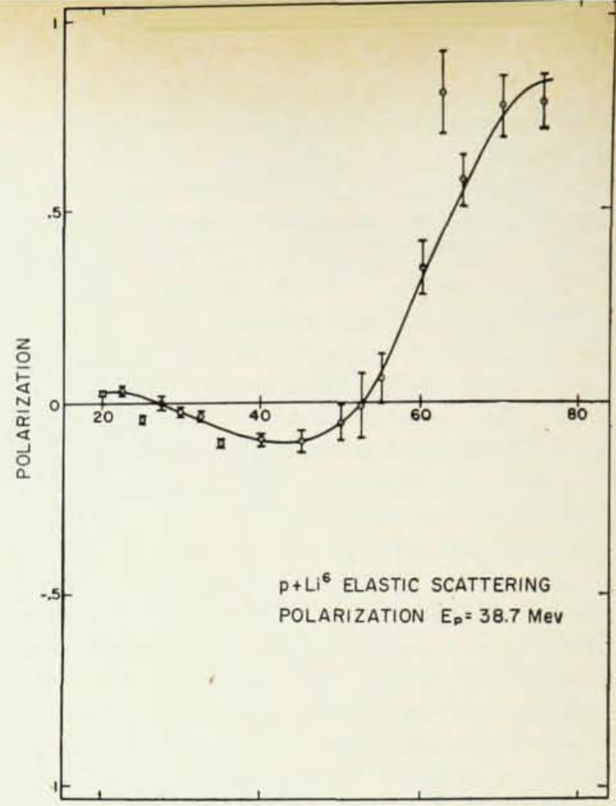

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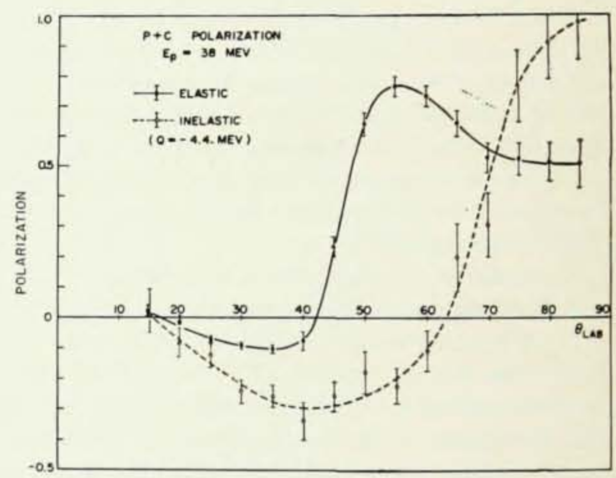

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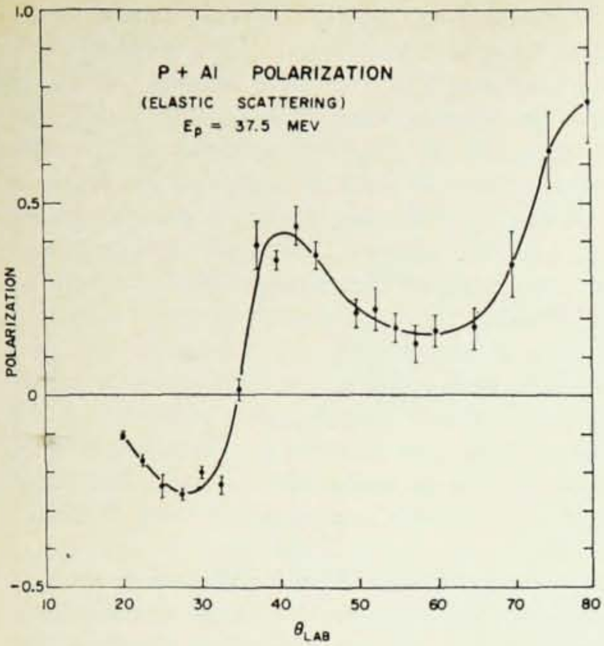

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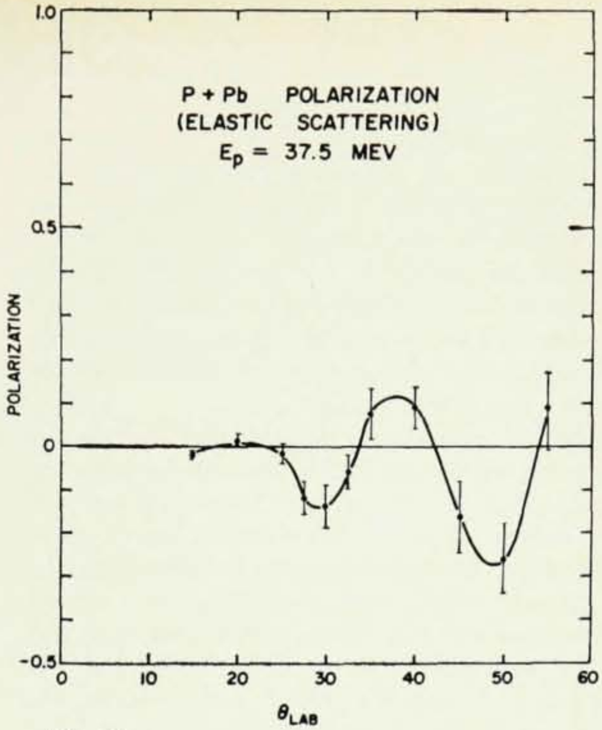

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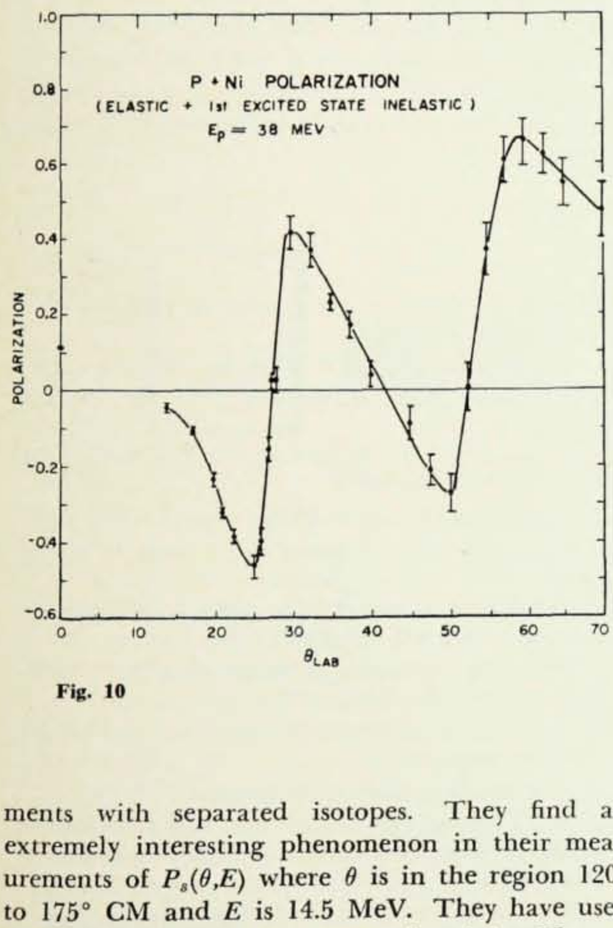

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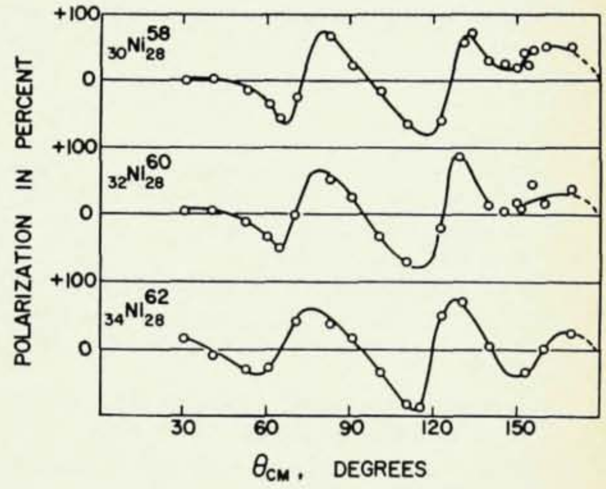

Fig. 12

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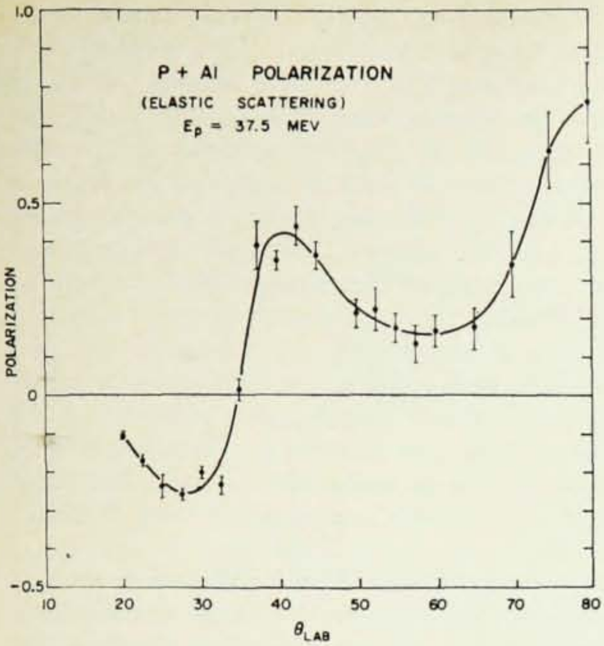

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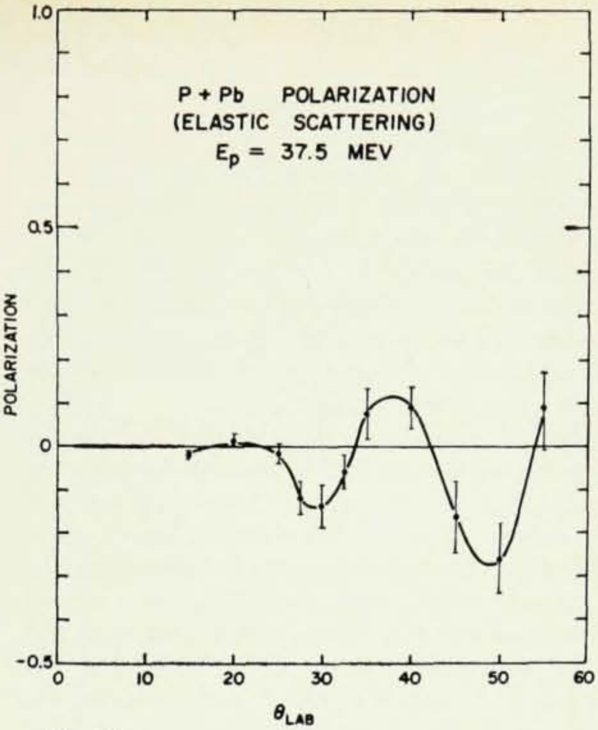

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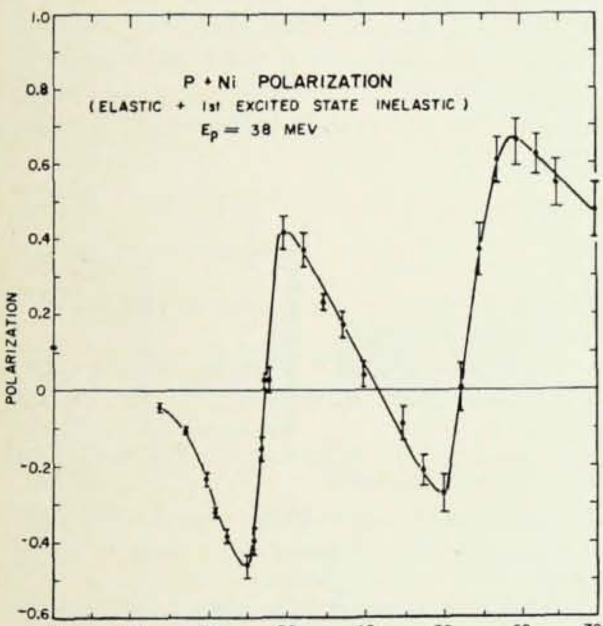

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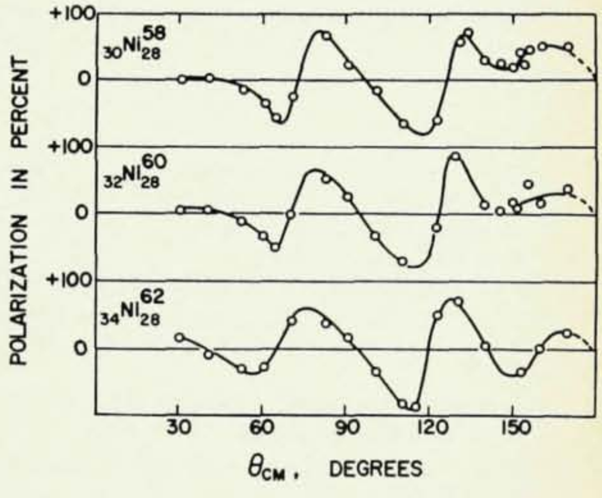

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(28,34), another two neutrons have been added. The added neutrons are seen to have produced a marked effect on P_s at large angles.

The shell model of the nucleus requires that these neutrons be added to the surface of the nucleus and so influence the spin-orbit term of the optical model of the nucleus which is assumed to exist near the surface of the nucleus. Polarization experiments of the above nature provide a much more sensitive means of probing the surface of the nucleus than measurements of the differential elastic scattering cross section.

Early predictions by L. S. Rodberg[25] suggested a connection between differential scattering cross sections and $P_s(\theta)$ by stating that $P_s(\theta)$ is proportional to the derivative of the differential scattering cross section which has been observed to have an oscillatory nature. Thus $P_s(\theta)$ would be zero at the maxima and minima of the differential cross section and a maximum in between. This "rule" has been proved by Rosen's group[24] and by others not to be correct. However, Rodberg's statement that "the spin-orbit potential V_{so} (r) $\sigma \cdot \ell$ is most important for particles with large orbital momentum, i.e., at the surface of the target" has been substantiated.

The practicality of using polarized beams depends upon both P_b and the intensity of the beam I. A good approximation to the weighting of these factors is $P_b{}^2I$. It is therefore important to develop ion sources that will give a greater polarization than the one described earlier in this paper. Such sources have been developed which contain a section, following the sextupole magnet, where rf transitions occur between states (2) and (4) of Fig. 4. It is theoretically possible[26] to make this transfer on all the atoms in state (2) so that all proton spins are in the up state, i.e., a proton polarization of 100% may be achieved. Both the Harwell[8] and Minnesota groups are currently planning to replace their existing polarized proton sources by such a polarized proton source. At Minnesota, the new source will be installed in the high-voltage terminal of a new 500-keV injector, thus eliminating the 1.5-meter drift space between the sextupole magnet and the ionizer of the present source. The latter change should result in a polarized beam of greater intensity.

This paper is limited in its discussion since it treats a very restricted portion of the whole subject of polarization. A large body of information has been obtained by using polarized neutrons produced in nuclear reactions. Also, the work with polarized protons and other particles originating in a variety of reactions has been completely omitted. For an introduction to early work in these areas, the reader is referred to the *Proceedings of the International Symposium on Polarization Phenomena, 1960*.[27]

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