in vacuum tubes. There is a useful series of mathematical appendices and a brief bibliography.

The author's bright and perspicuous style makes the book a pleasure to read. It is highly recommended.

Theory and Application of Liapunov's Direct Method. By Wolfgang Hahn. English ed. prepared by Siegfried H. Lehnigk. Transl. from German by Hans H. Hosenthien and Siegfried H. Lehnigk. 182 pp. Prentice-Hall, Englewood Cliffs, N. J., 1963. $9.00.

Oscillations in Nonlinear Systems. By Jack K. Hale. 180 pp. McGraw-Hill, New York, 1963. $9.00.

Nonlinear Problems. Rudolph E. Langer, ed. Symp. Proc. (Madison, Wisc., Apr.–May 1962). 321 pp. Univ. of Wisconsin Press, Madison, 1963. $7.50. *Reviewed by Peter L. Balise, University of Washington.*

NONLINEAR analysis is characterized by its mathematical intractability, its pertinence to actual systems, and its appearance as the subject of many new books. Here are three different types of books on the subject.

The English translation of *Theory and Application of Liapunov's Direct Method* is most welcome, both because of the subject's importance and the presentation's excellence. Liapunov's work is the dominant basis for analysis of nonlinear control systems in Russia, and it is now receiving much attention here. Essentially, the method states that if a positive definite function V of the system-state variables can be found such that dV/dt is negative semidefinite, the system is stable; if dV/dt is negative definite, the system is asymptotically stable. The reasonableness of this is indicated by a particular V function, a system's energy, but usually a more complex function is required. Dr. Hahn gives many examples of the construction of suitable V functions, as well as a general introduction to the stability theorems. He also considers the converse problem of inferring the existence of a V function from knowledge of a system's stability. Other interesting topics are rate of decay (degree of stability), sensitivity to perturbations, and critical cases, where the equilibrium is stable but not exponentially stable. With the best bibliography on nonlinear analysis that this reviewer has seen, the volume is warmly recommended.

Indicating the subject's importance, *Oscillations in Nonlinear Systems* is the first volume in the publisher's new series, "Advanced Mathematics with Applications," whose consulting editors are LaSalle and Lefschetz, authors of apparently the first (1961) American monograph on Liapunov's method. Most appropriately, Dr. Hale introduces his book by explaining how the applicability of superposition is the basic characteristic of a linear system, so that knowledge of its behavior near the origin applies everywhere in the state space. This seems more desirable to merely stating the form of the linear differential equation. The rest of the introductory material is much more succinct, being a good summary for one who well understands matrix representation of system behavior. A large part of the work

concerns periodic solutions of nonlinear equations with small perturbations, for which a general theory is given, based largely on the work of the author and Cesari. Interesting clarification is provided by detailed analyses of the application to several common nonlinear equations. The last part of the book considers almost periodic solutions and integral manifolds, also illustrated by interesting examples. Much of the subject is quite difficult; because the terse writing style omits explanations which this reviewer finds desirable in his classes, the book is perhaps not the introduction for graduate students that the author intended. But it certainly is a valuable contribution because of its original material and its unifying approach.

Nonlinear Problems consists of the lectures and abstracts of the papers of a symposium conducted by the Mathematics Research Center, US Army. As is to be expected from the state of nonlinear theory, the volume presents diverse research results rather than a connected study; there are almost as many different topics as there are authors. However, besides the very specialized papers, some articles survey areas of wider application, such as nonlinear hyperbolic systems of conservation laws, nonlinear eigenvalue problems, periodic solutions of ordinary differential equations, and Liapunov's method. A few papers have a physical setting: magnetogasdynamics, Navier-Stokes equations, and elastic stability. Anyone with a serious interest in nonlinear analysis will want to read parts of this volume.

As one author observed, "nonlinear" is only the negation of a field for which there is a rich theory, so it is to be expected that the literature in nonlinear analysis will continue to be characterized by a variety of approaches inspired by different physical problems or suggestive mathematical structures.

Non-Equilibrium Statistical Mechanics. By I. Prigogine. Vol. 1 of Monographs in Statistical Physics and Thermodynamics, edited by I. Prigogine. 319 pp. Interscience, New York, 1962. $12.50. *Reviewed by R. Landshoff, Lockheed Missiles & Space Company.*

STATISTICAL mechanics links the phenomenological concepts of thermodynamics to the laws of mechanics. Following Gibbs, this is achieved by the study of ensembles of points in phase space which move according to the laws of mechanics. Such an ensemble is represented by a density ρ which obeys Liouville's equation. In the original application to thermodynamic equilibrium, one restricts the investigation to stationary ensembles. In that case, the time derivative $\partial\rho/\partial t$ in Liouville's equation is zero.

To study nonequilibrium processes, one applies the Liouville equation to densities functions where $\partial\rho/\partial t$ is not zero. This idea goes back to Boltzmann who succeeded in formulating a theory which explained the second law of thermodynamics and made it possible to calculate transport coefficients.

Despite its success, this theory is not entirely satis-