

BOOK REVIEWS

An Index of Mathematical Tables (2nd ed.). By A. Fletcher, J. C. P. Miller, L. Rosenhead, L. J. Comrie. Vol. 1, Introduction, Part 1. Index According to Functions; Vol. 2, Part 2. Bibliography, Part 3. Errors, Part 4. Index to Introduction and Part 1. 994 pp. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962. \$42.00 per set. Reviewed by Joseph Hilsenrath, National Bureau of Standards, Washington, D. C.

EIGHTEEN years have passed since the appearance of the first edition of this work which was described at that time as monumental, remarkably comprehensive, and extraordinarily accurate. This edition is no less remarkable in quality while being slightly over twice as large in quantity. Whatever may be the future impact on mathematical tables of the continued proliferation and improvement of digital computers, they have not as yet rendered the mathematical table obsolete. They have not lessened, nor will they lessen, the need for accurate tables.

Even in a subject as seemingly narrow as mathematical tables, the authors had to exercise restraint and judgment in what was included. Their discussions of this and various other choices and decisions make most interesting reading. To attempt to convey the scope of this encyclopedic work in even a long review would serve only to mislead. The readers will find it profitable to consult a more detailed review of this *Index* by Dr. J. W. Wrench which appears in the July issue (Volume 17, 1963) of *Mathematics of Computation*. That issue carries also a comprehensive list of corrections and additions to the *Index* supplied by the authors, the reviewer, and others.

The review of the first edition by the late Professor R. C. Archibald (*M.T.A.C.* Vol. 2, 13-18, 1946-47) should be read by anyone sufficiently motivated to consult this *Index*. In the closing paragraph of his review, he states, "... practically every table listed in the *Index* is in the Brown University Library. ..." Following his clue, this reviewer has determined that the collection of mathematical tables has continued to grow at Brown. It is therefore very likely that most, if not all, of the tables listed in the second edition of this *Index* can be obtained via interlibrary loan or in microfilm or photocopy (copyright permitting) from the Brown University Library, Providence, Rhode Island. There are undoubtedly other libraries whose holdings of mathematical tables closely approximate those listed in this *Index*.

The *Index* is primarily to tables of mathematical functions. Certain functions arising from problems in physics are treated in one or more sections. These include the Planck, Einstein, Debye, Thomas-Fermi, Fermi-Dirac, and Bose-Einstein functions. They are in-

cluded more to illustrate certain interesting applications than to provide complete coverage for tables of physical data.

Volume 1 is divided into twenty-four sections which comprise the main classified breakdown of mathematical functions and related matter. Section five, for example, is devoted largely to sources of values of the important mathematical constants, etc. Throughout, the notation and mathematical symbolism can serve as models of good practice.

Any reader confronted with the need for tables of some seemingly specialized mathematical function, will find it profitable to consult this *Index* before starting any extensive computation. Very often he will find suitable tables already available. If not, he should have learned, in the process, to consult *M.T.A.C.* and *Math. Comp.* After which, if he finds nothing suitable, he may continue his computing program with considerable confidence—if not with absolute surety—that it is not a needless duplication.

Aside from providing a very useful and comprehensive index of mathematical tables, this work does much more. It is a monument to a dedicated group of mathematicians who have devoted a large fraction of their professional lives to making accurate mathematical tables. It points out most dramatically the importance for a body of scientific literature to provide a machinery for systematic recording of errors and corrigenda. It is a gold mine of pieces of interesting and useful information of historical, editorial, or philosophical nature relating to mathematical tables. Even casual browsing in this *Index* is rewarding.

It would behoove any reader who has leaned heavily on one or more classical sets of mathematical tables to consult this *Index* to assure himself that he has not incorporated in his own work errors resulting from well documented errors in otherwise authoritative mathematical tables.

Even those of us who are old enough not to believe everything we see in print are often inclined to take mathematical tables on faith. This is not so with the authors of this *Index*. They devote a considerable fraction (150 pages) of this work to a discussion of reports of errors which they and a small group of "mathematical sleuths" have uncovered, over the years, in more or less authoritative works. In most instances references are to isolated errors or to short runs of errors. Only rarely are entire works dismissed with such a general statement as "the reliability of no table in the volume should be assumed without careful checking" or "the conclusion is that the tables are completely unreliable".

Even in a work as comprehensive as this it was not

possible to list all of the reported errors explicitly. In such instances references are given to published lists of errors. As may be expected most of these are to *Mathematics of Computation* (formerly called *Mathematical Tables and other Aids to Computation*), a quarterly publication of the National Academy of Sciences. Although *MTAC* (now *Math. Comp.*) has, for the last 20 years, been the fountainhead of information on mathematical tables, it is unfortunately still not known widely enough to the general scientific public. This *Index* provides more than an ample introduction to this valuable journal.

Advances in Computers, Volume 3. Franz L. Alt and Morris Rubino, eds. 361 pp. Academic Press Inc., New York, 1962. \$12.00. Reviewed by Peter L. Balise, University of Washington.

CERTAINLY the rapid development of computer technology provides ample justification for the continued publication of this annual series. Although the title correctly represents the contents, the books' value is not limited to reporting advances; one can get a good general view of computers, particularly the applications of digital computers, from the three volumes that have so far appeared. Volume 1 considered business applications, weather prediction, language translation, game playing, recognition of spoken words, and binary arithmetic. Volume 2 discussed parabolic differential equations, orthonormalizing, linear programming, microelectronics, and theory of automata.

Such diversity is continued in the six topics of the present volume. Samuel Conte analyzes the problems of satellite orbit computation, noting the practical requirements of various missions and carefully describing the calculation of injection parameters corresponding to a particular orbit. He compares integrating total accelerations with deviations from a reference orbit, and he also compares computer integration methods and accuracy tests. E. F. Codd examines multiprogramming: its history, details of present practice, and suggestions for future development. A clear picture is given of general problems and systems, as well as specifics of operations, such as storage allocation, queueing, interruption, and program protection. Philip Wolfe reviews nonlinear programming, the difficulty of which has so far caused its neglect compared to linear programming, which is well developed for obtaining solutions that must to some degree be approximate for real inherently nonlinear systems. By classifying and briefly describing the principal currently available nonlinear programming algorithms, including differential gradient, large-step gradient, simplex, and cutting-plane methods, this paper orients the reader and provides motivation for further study. Garrett Birkhoff, Richard Varga, and David Young survey alternating-direction implicit methods for the iterative solution of elliptic and parabolic partial difference equations. Convergence theory is emphasized, with specific recommendations on methods and selection of iteration parameters; these

are supported by numerical experiments. Harold Skramstad considers a previously neglected area in the series, simulation, in an introductory treatment of combined analog-digital techniques. Examples are briefly outlined, and there is detail on one particular application, in which a variable is handled as a number plus an analog voltage representing the least significant digits. Reed Lawlor discusses relationships between information technology and the law. He notes that mechanical data processing is just beginning to be applied to legal information, and he briefly considers such problems as possible copyright infringement through text storage in computers.

Although the articles are generally introductory and only outline their subjects, they are excellent references, especially with their extensive bibliographies.

Matrix Iterative Analysis. By Richard S. Varga. 322 pp. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962. \$10.00. Reviewed by J. Gillis, Weizmann Institute of Science.

A GREAT deal of attention has been paid in recent years to relaxation procedures, and substantial advances have been made. The practical computer will continue to use a mixture of brute force with "trial and error", and so long as he gets his results, he will continue to sneer at the theoreticians. It is when the crude methods break down that we really want to know the details of the machinery. Only the especially zealous take any interest in what goes on under the hood of their car—so long as it runs smoothly.

However, there is more to the business than that. To extend the car analogy a little further, it is the driver who understands the engine who will get the extra spurt from it. And there is no doubt that anyone interested in the speed and efficiency of a relaxation problem must really understand what actually happens.

The wealth of new ideas and results in this area is scattered over many journals of varying degrees of accessibility, and it was certainly time to have them assembled in an orderly and usable fashion. The author, whose own original contributions to the subject are notable, has done this and more.

The work begins with some basic algebra of matrices, including concepts of norm and bounds for eigenvalues. There is then an account of the various iterative methods for solving linear equations. This includes the so-called "semi-iterative" methods, i.e., those in which one interrupts the blind iteration from time to time to take steps to promote the speed of convergence. Particular reference is made in this connection to Chebyshev methods, which the author himself has helped to develop. The remaining chapters are devoted to the application of these ideas to elliptic and parabolic linear differential equations.

There is no doubt that this book contains a most complete and up-to-date account of the role of iterative methods for the solution of linear systems. The presentation is logical and lucid. If it is not easy read-