Group Theoretical Methods in ELEMENTARY PARTICLE PHYSICS

The following brief account of the NATO advanced study institute held July 16 to August 4, 1962, in Istanbul, Turkey, was written by two physicists who took part in the program. O. W. Greenberg is a member of the faculty of the Department of Physics and Astronomy at the University of Maryland. E. P. Wigner is professor of mathematical physics at Princeton University.

By **O. W. Greenberg** and **E. P. Wigner**

THE NATO Advanced Study Institute's summer school in Istanbul had group-theoretical methods as its principal theme. It consisted, as all such summer schools do, of lectures, seminars, and private discussions. It provided an opportunity for all participants to become acquainted with each other's work and ideas. This was probably its most useful function. There were 58 participants, from the United States, from Europe, and from Turkey and the Middle East. There was, perhaps, an overemphasis on lectures; these started, as a rule, at 9:15 a.m. and often lasted until 1:00 p.m., with a short coffee break. Many afternoons were devoted to formal seminars and some of these lasted several hours. Nevertheless, there was sufficient opportunity for individual discussion, during and after meals, in the evening hours, and on some of the afternoons. All the lectures were recorded in mimeographed notes, which were distributed with a minimum of delay. They may be published soon.

The sessions lasted three weeks. The lectures were about evenly divided between rather pure mathematics and physical applications of mathematical principles. Some of the seminars dealt with somewhat more remote subjects.

Two of the lecturers, G. Racah and D. Speiser, gave courses about semi-simple Lie groups, their classification and their representations. Racah approached this subject from the point of view of the classical theory of Cartan, starting with the study of the infinitesimal group of generators of the Lie group. After reviewing the basic notions concerning continuous groups, he described all semi-simple Lie groups and discussed their representations. Speiser approached the subject from the global point of view, and, after an introduction to the topological properties of Lie groups, developed the Hopf-Stiefel theory of semi-simple Lie groups. Speiser also gave a discussion of recent work of himself and his collaborators concerning the possible global symmetry schemes.

The lectures on the physical applications of the mathematical principles could rather sharply be divided into two classes. The lectures of the first class dealt with the dynamic consequences of symmetry properties of space-time, those of the second class endeavored to find the symmetry properties of interactions. The first class dealt with the symmetry properties of states, the second class with those of laws of physics. The latter properties, such as the charge-independence of the interaction, are all of approximate nature. The lectures of Michel and Wigner dealt with problems of the former nature; Nambu's lectures (and many of the seminars) were concerned with the symmetries of interaction.

L. Michel addressed himself with characteristic enthusiasm to a mathematical problem of great present interest in physics. The problem is: Given groups A and B to find all groups, E, such that the group E has

A as an invariant subgroup and such that the quotient of E by A is the group B. For a typical case where this problem arises in physics, B is the Poincaré group, A is a gauge group whose generators are charges, and E is the invariance group of the whole theory, which has the property that its quotient by the gauge group is the Lorentz group. The mathematical name for the problem of finding a larger group, E, which, in the sense described above, includes the groups A and B is "group extension". There is already one tantalizing formula obtained by Michel and Lurçat which relates the baryonic and leptonic charges of a particle to the value of the spin. This formula,

$$(-1)^{2j} = (-1)^{b+l},$$

seems to be confirmed by the available experimental information. It was discovered by investigating the extensions of the Lorentz group to include the gauge group of charges. By popular demand, Michel gave several two- and three-hour lectures in the evenings to supplement his daytime introduction to the new language of homological algebra.

E. P. Wigner spoke on the unitary representations of the Lorentz group with particular emphasis on the representation of the reflections. After describing the Poincaré group (that is, the inhomogeneous Lorentz group) using a 2×2 matrix representation of four vectors and Lorentz transformations, the representations of the restricted Poincaré group which we now believe to correspond to particles in nature were discussed in detail. These are representations which correspond to integral or half-integral spin and positive or zero mass. Wigner then discussed the full Poincaré group in which reflections are included, and discussed in some detail the theory of types of representations of reflection symmetries. He chose to consider those reflection symmetries which are now thought to be exact: TCP, CP, and T; and pointed out that (a) there are four inequivalent representations of these reflections, and (b) in two of these the reflections do not commute. Wigner concluded by advancing arguments for the assignment of one of the non-Abelian representations of the reflection group to particle-antiparticle systems.

E. Inönü gave a seminar on group contractions.

Y. Nambu's lectures were concerned with the symmetries of interactions, and he reviewed some of his present research interests. His first subject was the universal V-A weak interaction theory and the conserved vector current hypothesis. He then described some models for the γ_5 symmetry. Nambu discussed hidden symmetries with particular reference to the examples of ferromagnetism and superconductivity, and he outlined some consequences of chirality conservation for such phenomena as soft-pion production by pions, electrons, and neutrinos. His last topic was nonperturbative solutions of field equations.

Several speculative notions were presented by S. L. Glashow in a number of seminars. He described the

analog-symmetry model. This postulates symmetries of weak, strong, and electromagnetic interactions which are all isomorphic but conflicting—indeed, conflicting as much as possible. Glashow further reviewed more standard topics such as the Sakata model and, in particular, the eight-fold way, with its mass-sum rules and other predictions.

In addition to the lectures, there were many seminars. Some have been mentioned, and some dealt with subjects not closely related with the principal theme of the conference. A. Salam gave a brilliant report on the CERN Conference and on broken symmetries. Coleman discussed chiral symmetries. Gürsey discussed the de Sitter group, and also Mach's principle in general relativity. Emch spoke on superselection rules, and Caianiello on renormalization. The latter spoke of a renormalization group, but no group in the mathematical sense seems to have been involved. Burgoyne discussed the connection between spin and statistics on the basis of his own work, as well as that of Lüders, Zumino, Dell'Antonio, and Araki. Greenberg spoke on parastatistics, which applies to particles having properties intermediate between those of bosons and fermions. He also discussed high-energy scattering theory, including high-energy limits, and relations at high energies between cross sections for different processes. The relations in question can be derived from Pomeranchuk's two theorems: (1) that charge-exchange scattering cross sections vanish at high energy, and (2) that particle-target and antiparticle-target cross sections approach equality at high energies.

As its name indicates, the conference was chiefly sponsored by NATO, but also received support from the Ford Foundation. It owed much to the interest of W. A. Nierenberg, now at the University of California, but formerly Assistant Secretary General for Scientific Affairs of NATO. The school was organized by Dr. F. Gürsey, and it was one of the most smoothly run summer schools we ever attended. The participants —many of them from the Near East—were housed at beautiful Robert College, on a hill overlooking the Bosporus, and the lectures were in the new Engineering Building of the College. We made several excursions to the islands in the Marmora Sea and to colorful and historic Istanbul. The hospitality of the College and the cordiality of our Turkish friends—and even of strangers—was unsurpassable.

In addition to Robert College, both Istanbul and Ankara also have a state supported University and a Technical University. There is a small college in Izmir. Turkey also has its atomic energy installation, and there was an excursion to the reactor. It is of the pool type, of 1000 kW capacity. It was constructed by the American Machine and Foundry Co. Unfortunately, the pool is not watertight and this causes much irritation.

Attending the NATO Advanced Study Institute in Istanbul was a highly interesting, pleasant, and also useful experience.