BOOK REVIEWS

Asymptotic Methods in the Theory of Non-Linear Oscillations. By N. N. Bogoliubov and Y. A. Mitropolsky. Vol. 10 of Russian Monographs and Texts on Advanced Mathematics and Physics. Transl. from Russian. 537 pp. (Hindustan Publishing Corp., Delhi) Gordon & Breach, New York, 1962. $25.00 Academic and Student's Edition: $15.00. *Reviewed by J. Gillis, Weizmann Institute of Science.*

THERE are more approaches to perturbation problems than are dreamed of by most physicists, even first-order perturbation. Some of the snares that attend the use of perturbation methods were well known to Laplace's generation. However, we have managed to struggle on, usually avoiding the worst errors. There have been those, let it be admitted, who have stopped at the first-order terms out of fear that the second-order ones might upset the apple cart; and have cleared their consciences by waving the hand and muttering the magic word "asymptotic".

Fifteen years ago Krylov and Bogoliubov's *Introduction to Non-Linear Mechanics* appeared in English translation. It must have shaken quite a few theoretical physicists to realize that even so straightforward a process as linearization could be performed either scientifically or unscientifically. The ideas of that little book are much more fully developed in the one now under review, and some other basic methods are also presented.

The introduction gives a broad and lucid view of the types of problems to be encountered. Chapter 1 then presents a detailed description of the Krylov-Bogoliubov approach to "nearly linear" systems. Chapter 2 is devoted to phase-plane methods. These are valuable both as a basis for some of the approximate numerical methods of solution and also, and perhaps more important, for the insight they can provide into the singular point and cycle structure of the equation. The discussion evokes nostalgic glimpses of the heroic days of Poincaré and Liapounov, the true founders of the entire theory. The chapter ends with an account of relaxation oscillations, notably Dorodnitsin's large parameter method. This last is particularly suitable for equations of the Van der Pol or Rayleigh type when the relevant ϵ is very large instead of very small.

Chapter 3 deals with nonlinear systems under periodic external forces, including resonance phenomena, while Chapter 4 is devoted to monofrequency oscillations. In Chapters 5 and 6, the authors go into some of the basic mathematical questions which are fundamental to the methods used in the book.

It would be presumptuous to praise the mathematical power of either author. Suffice it to say that people who really need an effective idea for handling a nonlinear problem would be well advised to seek it here. They might even find, incidentally, the reason why their own earlier attacks had failed. But it is not an easy book to read, and even to consult it will require some effort.

Many Western readers will note with envy the author's remark that the first four chapters are within the scope of mathematics of a normal course at a higher technical school.

Fundamentals of the Laplace Transformation. By C. J. Savant, Jr. 229 pp. McGraw-Hill Book Co., Inc., New York, 1962. $7.75. *Reviewed by Peter L. Balise, University of Washington.*

SINCE the Laplace transformation has become such a standard technique for analyzing systems, it is now commonly taught at the undergraduate level but was not included in the college education of many engineers. This monograph is intended both for engineers and undergraduate students. Users of Dr. Savant's *Basic Feedback Control System Design* will recognize his style of writing from the students' viewpoint, with many illustrative problems. This book does not seem to be quite as empathically written as the servomechanisms text, although some of the material is excerpted from it.

Following a short review of classical solutions of differential equations, an equation is solved with the aid of the 1200-pair Laplace transform table contributed by E. C. Levy in the appendix. Having illustrated the value of the method, the author defines the transform integral and presents the common elementary transforms. He then clearly and quite thoroughly considers many specific electromechanical systems, briefly including important related topics like duality. Such a practical approach is emphasized throughout; rigor is generally preserved by noting mathematical limitations but without extensive explanations.

Most of the mathematics is in the last half of the book. Besides inversion by tables, by partial fractions, and graphically, the inversion integral is considered. Important Laplace-transform theorems are conveniently outlined in one chapter, including complex differentiation and integration, periodic functions, change of scale, and a second independent variable (but partial differential equations are not discussed). Real convolution is clearly explained, but complex convolution is not mentioned. An appropriate chapter considers s-plane analysis, including an introduction to the root locus. The appendix outlines complex-variable theory and the Fourier series and integral. Unfortunately, the extension of the Fourier transformation to the Laplace transformation is ignored.

Many texts, particularly on servomechanisms, have good introductions to the Laplace transformation; there are also excellent treatises. This book will be useful to some who want an intermediate treatment.